

## Decision Making Assignment 1

**DUE DATE:** Thursday, March 25, by 4pm in the course mailbox, SH108. (This mail slot has a dayglo pink label “MA2210 Assignments”.)

Please carefully read the presentation rules on the back of this sheet. **Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.**

1.) [10 points] We will solve the following linear program via the simplex method:

$$\begin{array}{ll} \text{maximize} & 3x_1 + 2x_2 + 6x_3 \\ \text{subject to} & x_1 + x_2 + 2x_3 \leq 4 \\ & 2x_1 + 3x_3 \leq 7 \\ & 2x_1 + x_2 + 3x_3 \leq 5 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

(a) Introduce slack variables  $x_4, x_5, x_6$  and set up the initial tableau for this problem.

(b) Apply the simplex method **by hand**. Be sure to indicate the entering and exiting variable (as well as the relevant ratios) at each iteration.

(c) Reading the final tableau, write down the optimal solution  $\mathbf{x}^*$  and the optimal objective value  $z^*$ .

2.) [10 points] Construct a simple example of a linear programming problem involving two decision variables and three constraints in which the optimal solution is not unique. Carefully draw a diagram of the feasible region and use the ideas of the graphical method to explain why there is more than one optimal solution to your problem.

3.) [10 points] Complete Problem 14 on page 262. Use a computer. But write things up by hand.

4.) [10 points] Complete Problem 20 on page 263. Use a computer. But write things up by hand.

5.) [10 points] We consider the following (greatly simplified) asphalt mixture problem.

*A road bed is made of compacted soil, covered by compacted stones or gravel, topped with a thin layer of “asphalt concrete” which consists of stones mixed with bitumen. Bitumen is made of the same stuff as fuel oils and serves as a binder. Asphalt concrete is more flexible than ordinary concrete.* [Transit New Zealand website]

The four parameters we will consider in our mix are skid resistance, cohesion, sedimentary effect and — of course — cost.

Each ingredient in the mixture (which is called “hot mix asphalt”) has a skid resistance factor, which is an estimate based on its gripping power. Larger, more angular stones have greater grip on the wheels of a car than do smaller or smoother stones. Sand has little gripping power and bitumen has essentially none. Government regulations require that the resulting mix must have a skid resistance factor of no less than 30.

The mixture must have enough bitumen to bind once it cools. Sand binds easily as do smaller stones. The overall cohesion of the mix must be at least 0.30.

Over time, large stones floating in bitumen will settle and the road surface will become slippery. The presence of smaller stones and sand keeps these large, rough particles at the surface. The overall sedimentary effect of the mix may not exceed 4.0 units.

You must design a low-cost mix of asphalt concrete using the following ingredients:

<b>Component</b>	<b>Skid Resistance</b>	<b>Cohesion</b>	<b>Contribution to Sed. Effect</b>	<b>Cost (per yd<sup>3</sup>)</b>
Aggregate 1	44	0.1	2.0	5.00
Aggregate 2	30	0.3	5.0	4.00
Sand	10	0.4	1.0	1.00
Grade A Bitumen	0	0.7	8.0	9.00
Grade B Bitumen	0	0.8	6.0	12.00

As is standard in such mixing problems, we satisfy ourselves to form one cubic yard of our mixture. Our objective is to meet the three constraints described above at the lowest cost.

Formulate (but do not solve!) a linear program which determines the optimal mix. Be sure to define variables and label all constraints in a meaningful way.

### BASIC RULES FOR ASSIGNMENTS

- I) Each student must compose his/her assignments independently. However, rough work may be done in groups;
- II) Write legibly and use only one side of each sheet of paper;
- III) Show your work. Explain your answers using FULL SENTENCES;
- IV) No late assignments will be accepted for credit.