

Matrices & Linear Algebra II
 C Term, Section C01
 W. J. Martin
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Linear Algebra Assignment 1

Due Date: Wednesday, January 21, by 4pm in my mailbox, SH108. Alternatively, you may hand your completed assignment to me personally.

Please carefully read the presentation rules on the back of this sheet. Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.

1.) [10 points] Let

$$B = \begin{pmatrix} 2 & 2 & 1 & 4 & 0 & 4 & 3 \\ 6 & 1 & 0 & 1 & 3 & 1 & 3 \\ 9 & 0 & 2 & 4 & 1 & 1 & 7 \\ 4 & 1 & 1 & 1 & 3 & 4 & 5 \end{pmatrix}$$

- (a) Find a basis for the nullspace of A . Show your work.
 - (b) Find a basis for the row space of A . Show your work.
 - (c) Find a basis for the column space of A . Show your work.
- 2.) [10 points] Let A be an $m \times n$ matrix with real entries.
- (a) Prove that the nullspace of A is a subspace of \mathbb{R}^n .
 - (a) Prove that the row space of A is a subspace of \mathbb{R}^n .
 - (a) Prove that the column space of A is a subspace of \mathbb{R}^m .
- 3.) [10 points] #19, p16.
- 4.) [10 points] #21, p16.

[Problem 5 appears on the back.]

5.) [10 points] Consider the vector space over \mathbb{R} with vectors

$$V = \begin{pmatrix} x \\ y \end{pmatrix} : x, y > 0$$

and operations

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} xz \\ yw \end{pmatrix} ; \quad a \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^a \\ y^a \end{pmatrix} :$$

(Yes, this really is a real vector space!) Find a bijective linear transformation L from V to the vector space \mathbb{R}^2 . (Note that a bijection is a function which is both one-to-one and onto. See Appendix B.) Verify that your function L has the desired properties.

BASIC RULES FOR ASSIGNMENTS

- I) Each student must compose his/her assignments independently. However, rough work may be done in groups;
- II) Write legibly and use only one side of each sheet of paper;
- III) Show your work. Explain your answers using FULL SENTENCES;
- IV) No late assignments will be accepted for credit.