MA196X Problem Set 1

Instructions: Please first read the rules on the presentation of assignments in the course. Then complete as many of these as you can by Tuesday, March 24th. After that, I will still accept problems until the sample solutions have been distributed.

For each of the following problems, first state the problem precisely and then give a proper proof of the statement using English sentences.

1. The following hold for any integers:
   (a) If $a|b$ and $b|a$, then $b = \pm a$;
   (b) If $a$ and $b$ are positive and $a|b$, then $a \leq b$;
   (c) $a|a$;
   (d) If $a$ and $b$ are positive and $a|b$ and $b|a$, then $b = a$;
   (e) If $a|b$ and $b|c$, then $a|c$.

2. The following hold for any integers:
   (a) If $a|b$ then $a|bx$ for any integer $x$;
   (b) If $a|b$ and $a|c$, then $a|(bx + cy)$ for any integers $x$ and $y$;
   (c) If $a|b$ and $c|d$, then $ac|bd$.

3. The following are all false:
   (a) For all integers $a, b, c$, if $a|bc$ then either $a|b$ or $a|c$;
   (b) For all integers $a, b, c, d$, if $a|b$ and $c|d$ then $(a + c)|(b + d)$;
   (c) For all integers $a, b, c$, if $a \nmid b$ and $a \nmid c$ then $a \nmid bc$;
   (d) For all integers $a, b, c$, if $a \nmid b$ and $b \nmid c$ then $a \nmid c$.

4. If $n$ is an odd integer, then $8|(n^2 - 1)$.

5. If $p$ is prime and $p|ab$, then $p|a$ or $p|b$.
   [HINT: For this more challenging problem, you may use the following theorem without proof: If integers $c$ and $d$ are relatively prime, then there exist integers $x$ and $y$ such that $cx + dy = 1$. Two integers are relatively prime if they have no common divisor larger than one, as is true for $c = 15$ and $d = 28$ for example. For this same example, the values $x = -13$, $y = 7$ give $15x + 28y = 1$ as desired.]
6. Consider the following two conjectures:

**Conjecture A:** For every positive integer $n$, there exists a prime number between $n$ and $n^2$.

**Conjecture B:** For every positive integer $n$, there exists a prime number between $n$ and $2n$.

While we currently do not know if either of these is true, one implies the other. Figure out which implies which and prove this implication.