A Valentine’s Gift that Keeps on Giving

As a Valentine’s Day present, Sandy got Terry a set of infinitely many cubes of chocolate, having side-lengths 1, 1/2, 1/3, \ldots, one cube of each side. But how to gift-wrap them? Sandy knows that the harmonic series diverges, so Sandy cannot stack one on top of another in finite space. But Sandy also knows that the total volume of the cubes is finite, since

\[ 1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots < 1 + \int_1^\infty \frac{1}{x^3} \, dx = 1.5. \]

So the total volume is less than 1.5 and it seems as if someone could just pack them all into a gift-box of dimensions \(1 \times 1 \times \frac{3}{2}\). At this point Sandy realized that a better idea would be to give Terry the chocolate cubes and the gift box and make Terry do the packing.

Show how to arrange the cubes so that they fit into a \(1 \times 1 \times \frac{3}{2}\) box.

Note: This problem originally appeared as “Pack Them In”, the Macalester College Problem of the Week #1071.