IV. CONCLUSIONS

New technical results concerning the interactor of strictly proper and right invertible rational matrices have been exhibited. These results are proved to be useful in many areas of multivariable systems theory; especially in Theorem 2.1, which will play a key technical role in the polynomial approach of the decoupling problem, while Lemma 2.1 could be useful in adaptive model matching.

REFERENCES


On the Robust Control of Robot Manipulators

Mark W. Spong

ABSTRACT—In this note, we derive a simple robust nonlinear control law for n-link robot manipulators using the well-known Lyapunov based theory of guaranteed stability of uncertain systems. The novelty of our result lies in the fact that the uncertainty bounds needed to derive the control law to prove uniform ultimate boundedness of the tracking error depend only on the inertia parameters of the robot. Previous applications of the Leitmann approach to robot manipulators [13], [3] have required uncertainty bounds that depend not only on the inertia parameters but also on the reference trajectory and on the manipulator state vector. As a result, precise bounds on the uncertainty have been difficult to compute. Moreover, some further assumptions regarding "closeness in norm" of the computed inertia matrix to the actual inertial matrix have generally been required as well [1], [13]. The design in this note removes these assumptions. The result is achieved by exploiting the skew-symmetry property and linear parameterizability of robot dynamics [9], [13]. Thus, while previous robust controllers have been based upon the idea of robust feedback linearization, our controller uses the skew-symmetry property and linearity in parameters of the robot dynamics in a fundamental way.

II. DERIVATION OF THE CONTROL LAW

The basics of robot dynamics and control are sufficiently well known by now that we will be brief in our derivation of the control algorithm. Thus, given the Euler–Lagrange dynamic equations for an n-link robot [13]

\[ M(q)\ddot{\theta} + C(q, \dot{\theta})\dot{\theta} + g(q) = \tau \]

we note only that the matrix

\[ N(q, \dot{\theta}) = M(q) - 2C(q, \dot{\theta}) \]

is skew-symmetric and

\[ M(q)\ddot{\theta} + C(q, \dot{\theta})\dot{\theta} + g(q) = \dot{\theta} + \theta \]

where \( \theta \) is a constant \( p \)-dimensional vector of parameters and \( Y \) is an \( n \times p \) matrix of known functions of the generalized coordinates and their higher derivatives. The first equation is related to the passivity of the robot dynamics and the second equation says that the Lagrangian dynamic equations are linearly parameterizable.

We suppose only that the parameter vector \( \theta \) is "uncertain" by which we mean that there exists \( \theta_0 \in \mathbb{R}^p \) and \( \rho \in \mathbb{R}_+ \), both known, such that

\[ \| \dot{\theta} \| = \| \theta - \theta_0 \| \leq \rho. \]

4 With suitable definition of \( C(q, \dot{\theta}) \).

This with in mind we define a "nominal" control vector \( \tau_0 \) as

\[ \tau_0 = M_0(q)\ddot{\theta} + C_0(q, \dot{\theta})\dot{\theta} + g_0(q) - K \theta = Y(q, \dot{\theta}, v, \tau) + \theta_0 - K \theta \]

where the quantities \( v, a, \) and \( r \) are given by

\[ \dot{v} = q_q - \lambda \dot{q}; \quad a = \ddot{v}; \quad r = \dot{q} + \lambda \dot{q}; \quad \ddot{q} = q - q^d. \]

(5)

(6)

(1)

(2)

(3)

(4)

(5)

(6)
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they would be in an indirect adaptive control strategy. The advantage gained here is that we will not have the attendant problems of parameter drift, etc., associated with parameter a priori.

Next, we define the control input $\tau$ in terms of the nominal control vector $\tau_0$ as

$$\tau = \tau_0 + Y(q, \dot{q}, v, a)u = Y(q, \dot{q}, v, a)(\theta_0 + u) - KR$$

(7)

where $u$ is an additional control input that will be designed to achieve robustness to the parametric uncertainty represented by $\theta$. Substituting the control law (7) into (1) we obtain after some algebra

$$M(q)\ddot{r} + C(q, \dot{q})\dot{r} + K\ddot{r} = Y(q, \dot{q}, v, a)(\theta + u).$$

(8)

Then, the control law (7) above is continuous and the closed-loop system is uniformly ultimately bounded (u.u.b.) as defined in [2].

Proof: The proof is similar to the proof in [2] so we will merely sketch the argument. First, it is clear that (7) defines a continuous control law for any $\theta > 0$. See [11] for further discussion of this. Thus, we define a Lyapunov function candidate for the system (8) as

$$V = \frac{1}{2}r^T M(q) r + \ddot{q}^T N K \ddot{q}.$$ (10)

A simple calculation shows that along solution trajectories of (8)

$$\dot{V} = -\dot{q}^T K \ddot{q} = \dot{q}^T N K A \ddot{q} + r^T Y(\theta + u)$$

(11)

where $x^T = [\ddot{q}, \dot{q}]$ and $Q = \text{diag}(A^\top K, K)$. Mimicking the argument of Leitmann [5] we can show that $\dot{V} < 0$ for $\|x\| > \omega$ if

$$\omega^2 = \epsilon p/2 \lambda_{\text{max}}(Q)$$

(12)

where $\lambda_{\text{max}}(Q)$ denotes the minimum eigenvalue of $Q$. The proof proceeds as follows. Examining the second term in (11) we see that if $\|Y^T r\| > \epsilon$ then

$$\left( Y^T r \right)^T (\theta + u) = \left( Y^T r \right)^T \left( \theta - \rho \frac{Y^T r}{\|Y^T r\|} \right)$$

$$\leq \frac{\|Y^T r\|}{\|\theta\|} - \rho < 0$$

(13)

from the Cauchy–Schwartz inequality and our assumption on $\|\theta\|$. If $\|Y^T r\| \leq \epsilon$ we have

$$\left( Y^T r \right)^T (\theta + u) = \left( Y^T r \right)^T \left( \rho \frac{Y^T r}{\|Y^T r\|} + u \right)$$

$$- \left( Y^T r \right)^T \left( \rho \frac{Y^T r}{\|Y^T r\|} - \rho \frac{Y^T r}{\|Y^T r\|} \right).$$

(14)

This last term achieves a maximum value of $\epsilon \rho/2$ when $\|Y^T r\| = \epsilon/2$. Thus, we have that

$$\dot{V} \leq -\epsilon^2 Q x + \epsilon \rho/2.$$ (15)

To complete the proof, it suffices to notice the following. With class-K functions $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ such that

$$\gamma_1(\|x\|) \leq M(q) \leq \gamma_2(\|x\|) \quad \forall q \in R^n$$

(16)

it can be shown that there exist class-K functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that

$$\alpha_1(\|x\|) \leq V(x, t) \leq \alpha_2(\|x\|).$$

(17)

Equation (15) shows that

$$\dot{V}(x, t) \leq -\alpha_3 \|x\|^2 + \epsilon \rho/2$$

(18)

where the constant $\alpha_3 = \lambda_{\text{min}}(Q) > 0$. Uniform ultimate boundedness thus follows using the results and terminology of [2].

III. AN EXTENSION

Having a single number $p$ to measure the parameter uncertainty may lead to overly conservative design, higher than necessary gains, etc. For this reason we may be interested in assigning different "weights" or gains to the components of $u$. We can do this as follows. Suppose that we have a measure of uncertainty for each parameter $\theta_i$ separately as

$$|\theta_i| \leq \rho_i, \quad i = 1, \ldots, p.$$ (19)

Let $\xi_i$ denote the $i$th component of the vector $Y^T r$. Choose positive constants $\epsilon_i, i = 1, \ldots, p$, and define the $i$th component of the control input $u$ as

$$u_i = \begin{cases} -\rho_i \xi_i/\epsilon_i & \text{if } |\xi_i| > \epsilon_i \\ -\rho_i \epsilon_i/\xi_i & \text{if } |\xi_i| \leq \epsilon_i. \end{cases}$$

(20)

Then, it is easy to show that the same Lyapunov function candidate (10) above satisfies $\dot{V} < 0$ for

$$\|x\| > \left(\frac{1}{\lambda_{\text{max}}(Q)} \sum_{i=1}^p \rho_i \epsilon_i^2 \right)^{1/2}.$$ (21)

The proof is left to the reader.

IV. A DESIGN EXAMPLE

As an illustration, we will apply the above algorithms (9) and (20) to a two-link robot arm manipulating an unknown load. We consider the two-link planar arm from [13, example 6.4.2] shown in Fig. 1. The Lagrangian dynamic equations are as in the above cited reference. One parameterization of this robot is given by

$$\theta_1 = m_l l_1^2 + m_2 l_2^2 + I_1 \quad \theta_2 = m_2 l_2 l_1 \quad \theta_3 = m_2 l_1 \quad \theta_4 = m_2 l_1$$

(22)

With this parameterization, the dynamic equations (1) can be written as

$$Y(q, \dot{q}, \ddot{q}) \theta = \tau; \quad \theta \in R^4$$ (23)
where the components \( y_{ij} \) of \( Y \) are given as
\[

y_{11} = \dot{q}_1, \\
y_{12} = \dot{q}_2, \\
y_{13} = \cos(q_2)(2\dot{q}_1 + \dot{q}_2), \\
y_{14} = g \cos(q_1), \\
y_{15} = \cos(q_2)\dot{q}_1, \\
y_{16} = \cos(q_1 + q_2), \\
y_{21} = 0, \\
y_{22} = \dot{q}_1 + \dot{q}_2, \\
y_{23} = \cos(q_2)\dot{q}_1, \\
y_{24} = 0, \\
y_{25} = 0, \\
y_{26} = g \cos(q_1 + q_2).
\]

In the implementation of the nominal control vector \( \theta \), it is important to use the particular definition of the matrix \( C(q, \dot{q}) \) that renders \( M - 2C \) skew-symmetric. We leave it to the reader to verify that \( Y(q, q, v, a) \) in (5) has components
\[

y_{11} = a_1, \\
y_{12} = a_1 + a_2, \\
y_{13} = \cos(q_2)(2a_1 + a_2), \\
y_{14} = g \cos(q_1), \\
y_{15} = \cos(q_2)a_1, \\
y_{16} = \cos(q_1 + q_2), \\
y_{21} = 0, \\
y_{22} = a_1 + a_2, \\
y_{23} = \cos(q_2)v_1, \\
y_{24} = 0, \\
y_{25} = 0, \\
y_{26} = g \cos(q_1 + q_2).
\]

where the \( a_i, v_i \) are the additional signals defined in (6). For illustrative purposes let us assume that the parameters of the unloaded manipulator are known and are given by Table I.

For \( I_1 \) and \( I_2 \) we have used the formula \( I = 1/12mL^2 \) for the moment of inertia of a uniform thin rod. Using the values from Table I in (22) gives values of \( \dot{q}_i \) shown in Table II.

If we regard an unknown load carried by the robot as part of the second link then the parameters \( m_2, l_2, \) and \( I_2 \) will change to \( m_2 + \Delta m_2, I_2 + \Delta I_2, \) and \( I_2 + \Delta I_2, \) respectively. We will design a controller that provides robustness in the intervals
\[

0 \leq \Delta m_2 \leq 10, \\
0 \leq \Delta L_2 \leq 0.5, \\
0 \leq \Delta I_2 \leq \frac{10}{12}.
\]

We first need to choose a vector \( \theta_0 \) of nominal parameters. Choosing the mean value for the range of possible \( \theta_i \) yields the nominal parameter vector shown in Table III.

With this choice of nominal parameter vector \( \theta_0 \) and uncertainty range given by (26) it is an easy matter to calculate the uncertainty bound \( \rho \) as follows
\[

\| \hat{\theta} \|^2 - \sum_{i=1}^{6} (\theta_0 - \theta_i)^2 \leq 181.26 \quad (27)
\]

and thus \( \rho = \sqrt{181.26} = 13.46 \). Since we will also use the extended algorithm (20) we will use the uncertainty bounds for each parameter separately which are shown in Table IV. It should be obvious to the reader that the uncertainty bounds \( \rho_i \) in Table IV are simply the difference between the values from Table III and Table II, and that the value of \( \rho \) is the Euclidean norm of the vector with components \( \rho_i \). These computations should be contrasted with previous methods [1] in which the uncertainty bounds are quite difficult to compute.

The response of the manipulator using a trajectory generated from a second order critically damped reference model is shown in the following figures. Using the controller (9) the tracking errors and control signals are shown in Fig. 2. The design trade off here is between the size of \( \epsilon \), which determines the radius \( \omega \) defining the ultimate boundedness set \( S \), and the amount of chattering in the control signal. In this simulation, \( \epsilon \) was taken as 1.0, which resulted in almost no chattering and excellent transient and steady-state performance. The steady-state error, i.e., the error after two seconds, was \( \hat{q}_1 = 3.19E - 4 \) and \( \hat{q}_2 = 2.74E - 4 \). Using the control algorithm (20) with the gains from Table IV, resulted in the performance shown in Fig. 3. In this case, the steady-state errors were \( \hat{q}_1 = 4.22E - 4 \) and \( \hat{q}_2 = 3.72E - 4 \), slightly larger as a result of the lower gains. In this simulation \( \epsilon \) was taken as 0.6 for all \( i \).

V. DISCUSSION AND CONCLUSION

In this note, we have derived a robust control law for robot manipulators using a novel modification of the so-called "Leitmann approach." Our design is simpler to compute than previous designs, particularly with respect to the computation of the uncertainty bounds. Our controller is similar to (and, in fact, based on) the adaptive control algorithm developed by Slotine and Li [8]. Recall that the adaptive algorithm of [8] is given by
\[

\tau = \dot{M}(q)a + \dot{\hat{C}}(q, \dot{q})v + \dot{\hat{g}}(q) - Kr \quad (28)
\]

\[

\dot{\theta} = -Y(q, \dot{q}, v, a)\hat{\theta} - Kr \quad (29)
\]

where \( (\cdot) \) represents estimated quantities, (29) gives the parame-
The present robust control algorithm is thus proposed as a simpler design than previous robust algorithms, and as an alternative to adaptive control which may be of use in certain cases. The present algorithm would seem to be most attractive in an
environment where the uncertainty is not too great and where robustness to disturbances and unmodeled dynamics are of concern, for example, in a grinding application using a robot equipped with harmonic drive gears and end-point force feedback.

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References


Reliable Stabilization Via Factorization Methods

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Abstract—We consider reliable stabilization of multicontroller systems composed of one plant and two controllers. Our main objective is to propose a reliability design when controllers use independent inputs and outputs of the plant. The assumption of independence is crucial if we want to increase the chance that at least one of the controllers survives the sensor and actuator failures, which otherwise could disable both controllers and result in a system breakdown.

1. INTRODUCTION

Multicontroller systems were introduced in [1] and [2] with the idea that for a reliable stabilization of a single plant, two controllers are better than one. The idea was realized in the context of the classical reliability theory by modeling controller failures as a Markov process [3] and characterizing reliability as a connective stability in the mean of the closed-loop multicontroller system. This approach has a high level of sophistication relying on a wide variety of the results available for studying jump process in systems with random parameters [4]–[7].

Recently, an alternative approach to multicontroller design has emerged in the framework of factorization methods [8]–[11]. An appealing aspect of the new approach has been the fact that it provides a systematic design procedure for building multicontroller schemes for linear systems. So far, the approach has been limited to the case one-plant-two-controllers. The controllers are obtained by splitting a single centralized controller into two controllers which use the same inputs and outputs of the plant. In case of the sensor and actuator failures which are common sources of controller breakdowns, the sharing of input and output channels may be catastrophic. The methods that produce independent controllers in a hierarchical sequence [12], [13] restrict reliability to controller or plant failures that follow the exact sequence in which the controllers are built during the design process. No such restrictions appear in the original decentralized Lyapunov-type approach to the reliable control problem [1]–[7], because independence of inputs and outputs is a standard assumption in the decentralized control systems.

The main objective of this note is to remove the sharing requirement in the multicontroller design via factorization methods and, at the same time, assure stability of the closed-loop systems whenever at least one controller is functioning. The objective is achieved by inducing a hierarchy in the multicontroller structure, which allows for the use of independent inputs and outputs of the plant, without prescribing a hierarchy of the controller failures. A solution of a wider class of reliable decentralized stabilization problems has been proposed in [14], which, however, requires generality arguments.

Notation: $C_s$ denotes the closed right-half plane ($s$: Re $s$ $\geq$ 0), and $C_{\infty}$ denotes the extended right-half plane, i.e., $C_s$ with the point at infinity. A rational function in $s$ with real coefficients is stable if it is analytic in $C_s$. $R_{ps}$ denotes the set of all matrices whose entries are all stable and proper stable rational functions, respectively. A matrix belonging to $R_{ps}$ is $R_{ps}$-unimodular if it has an inverse in $R_{ps}$.

II. PROBLEM FORMULATION

Consider a two-channel linear multivariable system:

\[
\begin{align*}
\dot{x} &= Ax + B_1u_1 + B_2u_2 \\
y_1 &= C_1x \\
y_2 &= C_2x
\end{align*}
\]  

(1)

where $x$ is the state, $u_1, u_2$ are two control inputs and $y_1, y_2$ are...