DO ANY FIVE:

1. Find all the non-zero residues of \( f(z) = \frac{z}{\sin(\pi z)} \).

2. Suppose \( f(z) = \frac{e^{z^4}}{z^4 + 1} \) and \( \gamma = \frac{(2 + \cos(4t))e^{it}}{2} \). Compute \( \int_{\gamma} f(z) \, dz \).

3. Find the Laurant series of \( f(z) = \frac{1}{(z - i)(z + 2)} \) centered at \( z = 0 \) which is valid in the region \( 1 \leq |z| \leq 2 \).

4. Consider the two curves \( \gamma_1(t) = e^{(1+2\pi i)t} \) and \( \gamma_2(t) = e^{(1-2\pi i)t} \), for \( 0 \leq t \leq 1 \) in the region \( \Omega = \{ z \mid z \neq 2 \} \). Decide whether or not these two curves are homotopic in \( \Omega \). If they are homotopic, give an explicit homotopy \( H(t, s) \) between them. If not, then prove why not.

5. Let \( f(z) = \frac{e^z}{z(1 + e^{2z})} \).
   a) Let \( \Omega \) be the region of analyticity for \( f(z) \). Find \( \Omega \).
   b) Compute \( \int_{\gamma} f(z) \, dz \) where \( \gamma = \left( \frac{2 + \cos(3t)}{2} \right)e^{it} \), for \( 0 \leq t \leq 2\pi \).

6. Suppose that \( \text{Res}(f(z), z_0) = \alpha \) and that \( \text{Res}(g(z), z_0) = \beta \).
   Either find a formula for \( \text{Res}(f(z)g(z), z_0) \) which depends only on \( \alpha \) and \( \beta \) or prove that no such formula exists.

7. Let \( f(z) = \cos(z + \bar{z}) \) and define
   \[
   g(z) = \int_{\zeta_1} \frac{f(\zeta)}{\zeta - z} \, d\zeta + \int_{\zeta_2} \frac{f(\zeta)}{\zeta - z} \, d\zeta
   \]
   with \( \zeta_1 = 1 + e^{it} \), and \( \zeta_2 = -1 - e^{it} \) for \( -\pi/2 \leq t \leq \pi/2 \).
   Compute \( \int_{\zeta_3} g(z) \, dz \) where \( \zeta_3 = e^{it} \), for \( 0 \leq t \leq 2\pi \).