Math 2071 - Linear Algebra and Matrices

Problem Set 3 – Chapter 3

DO ANY 4

1. Consider the set of all vectors in $\mathbb{R}^3$ of the form (x, x^2, x^3).
Is this subset a subspace? Why or why not?

2. Consider the four vectors $\mathbf{a} = (1, -1, 0, 0)$, $\mathbf{b} = (0, 1, -1, 0)$, $\mathbf{c} = (0, 0, 1, -1)$, and $\mathbf{d} = (0, 0, 1, -1)$ in $\mathbb{R}^4$.

a) What is the rank of the subspace of all linear combinations of these four vectors.

b) Find a vector in $\mathbb{R}^4$ which is perpendicular to each of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$.

3. a) Find all solutions of the system $A\mathbf{x} = \mathbf{b}$ if A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 6 & 9 & 5 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

b) Is there vector $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has no solution?

c) Is there a solution of the form $(1, 0, x, y, z, w)$?

4. For the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 6 & 9 & 5 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix}$,
find a basis for the rowspace of A.

5. For the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 6 & 9 & 5 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix}$,
find a basis for the column space of A.

6. For the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 6 & 9 & 5 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix}$,
find a basis for the nullspace of A.

7. For the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 6 & 9 & 5 \\ 0 & 0 & 1 & 2 & 3 & 2 \end{bmatrix}$,
find a basis for the nullspace of the transpose of A.