Math 2071 - Linear Algebra and Matrices

Problem Set 2 – Chapter 2

DO ANY 4

1. Suppose that A is a 3×3 matrix and that

$$A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \text{and } A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Find the second row of A.

2. Consider the system

$$\begin{aligned} 5x + y &= 1 \\ 5y + z &= 1 \\ 5z + x &= 1 \end{aligned}$$

a)Write the system as a matrix equation $A\mathbf{x} = \mathbf{b}$
b) What is the augmented matrix of the system
c) Use elimination of solve the system. Show carefully all steps.

3. Consider the system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

and

$$\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

a) Use Gaussian Elimination to transform A to an upper triangular matrix T.
b) What are the elementary matrices corresponding to your elimination steps in part a.
c) Find a matrix $\mathcal{E}$ which is the product of these elementary matrices, and so that

$$\mathcal{E}A = T$$

.
d) What is the column vector $\mathbf{b}'$ so that $T\mathbf{x} = \mathbf{b}'$ has the same solutions as $A\mathbf{x} = \mathbf{b}$.

4. Consider the system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

and

$$\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

a) Use Gaussian–Jordan Elimination to transform A to an upper triangular matrix D.

b) What are the elementary matrices corresponding to your elimination steps in part a.

c) Find a matrix $\mathcal{E}$ which is the product of these elementary matrices, and so that

$$\mathcal{E}A = D$$

.

d) What is the column vector $\mathbf{b}''$ so that $D\mathbf{x} = \mathbf{b}''$ has the same solutions as $A\mathbf{x} = \mathbf{b}$.

5. Consider the matrix A given by

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

a) Use Gaussian–Jordan on a triply augmented matrix to compute A^{-1}.

b) Use A^{-1} to solve

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

6. Consider the matrices.

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) Compute C_1^{-1}.

b) Compute $(C_1(C_2^{-1}))^{-1}$.

c) Compute $(C_1(C_2^{-1})(C_3C_4))^{-1}$.

7. Compute the LU factorization of

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$