

1. EXAMPLE: A PURELY COMBINATORIAL RECIPROCAL

Here is what I think is a rather interesting example.

There are two ingredients. The first ingredient is a framework in the plane and a specific resolvable self-stress. Figure 1 depicts the graph of a cube with diagonals,

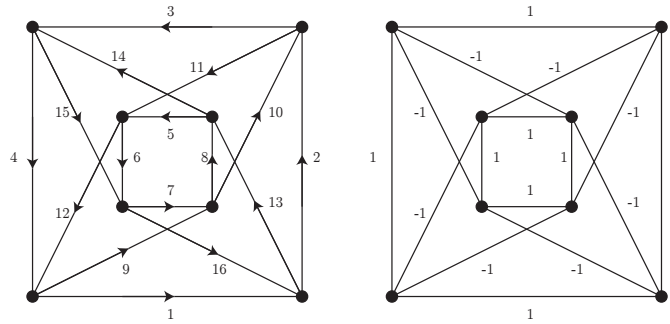


FIGURE 1. A framework with a self-stress.

in other words the complete bipartite graph  $K_{4,4}$ . The vertices are embedded in the plane at points of the integer grid. It has  $|V| = 8$  and  $|E| = 16$  so it is not a rigidity cycle. It is generically rigid, however the vertex embedding is singular so there may be an infinitesimal motion. The stresses on all the edges are  $\pm 1$ . In general the framework may have edge crossings. Since this is not a planar graph, we must have edge crossings.

The second ingredient is the embedding of the graph of the framework into some surface. Figure 2 depicts the cube with diagonals embedded on the torus.

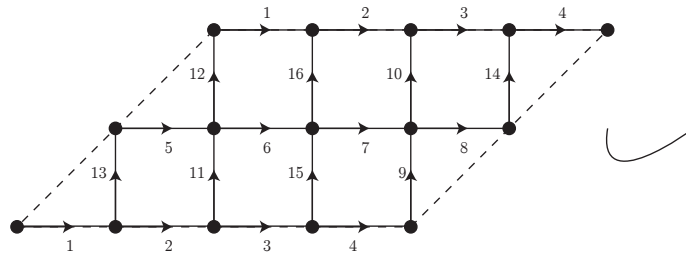


FIGURE 2. An quadrilateral embedding of  $K_{4,4}$  on the torus.

This embedding is to determine the the vertices of the reciprocal, and so is a true embedding - no edge crossings.

The dual graph *on the torus* also has eight vertices and 16 edges. In fact, we have chosen a self-dual embedding on the torus, however, that is not essential to the development.

We may now construct a reciprocal framework based on the toroidal dual of  $K_{4,4}$ . Figure 4 also possesses a resolvable stress whose values are the reciprocals

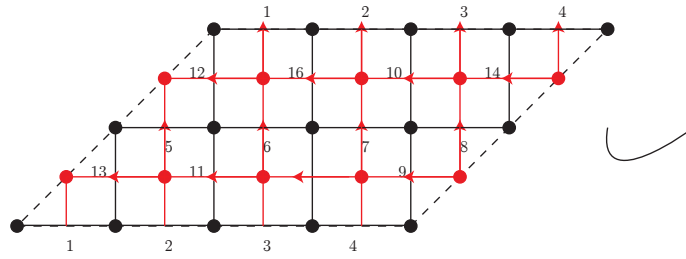


FIGURE 3.  $K_{4,4}$  has a self-dual embedding in  $T^2$ .

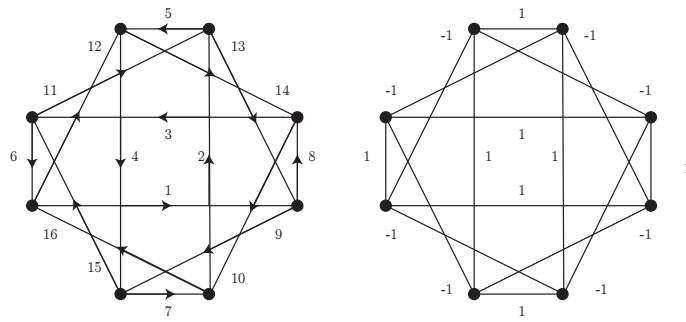


FIGURE 4. The reciprocal of non-planar framework.

of the original stress. However, this cannot be called *the* reciprocal figure since it depends on the surface in which the graph is embedded.

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