QUIZ 2—2009

Problem 1. (50%)

1. The finite difference expression $\frac{f_{j+1}-2f_j+f_{j-1}}{h^2}$ is:
   - ☐ a second order approximation for the first derivative
   - ☐ a second order approximation for the second derivative
   - ☐ a first order approximation for the first derivative
   - ☐ a first order approximation for the second derivative
   - ☐ a first order approximation for the third derivative

2. The finite difference expression $\frac{f_{j+1}-f_{j-1}}{2h}$ is:
   - ☐ a second order approximation for the first derivative
   - ☐ a second order approximation for the second derivative
   - ☐ a first order approximation for the first derivative
   - ☐ a first order approximation for the second derivative
   - ☐ a first order approximation for the third derivative

3. The finite difference expression $\frac{f_{j+1}-f_j}{h}$ is:
   - ☐ a second order approximation for the first derivative
   - ☐ a second order approximation for the second derivative
   - ☐ a first order approximation for the first derivative
   - ☐ a first order approximation for the second derivative
   - ☐ a first order approximation for the third derivative

4. The finite difference equation $f_j^{n+1}=f_j^n+\left(\frac{D\Delta t}{h^2}\right)\left(f_{j+1}^n-2f_j^n+f_{j-1}^n\right)$ is:
   - ☐ an O(dt,h) approximation for the advection equation
   - ☐ an O(dt^2,h) approximation for the advection equation
   - ☐ an O(dt^2,h^2) approximation for the advection equation
   - ☐ an O(dt^2,h) approximation for the diffusion equation
   - ☐ an O(dt,h^2) approximation for the diffusion equation

5. The finite difference equation $f_j^{n+1}=f_j^{n-1}-\left(\frac{U\Delta t}{h}\right)\left(f_{j+1}^n-f_{j-1}^n\right)$ is:
   - ☐ an $O(\Delta t,h)$ approximation for the advection equation
   - ☐ an $O(\Delta t^2,h^2)$ approximation for the advection equation
   - ☐ an $O(\Delta t,h^2)$ approximation for the advection equation
   - ☐ an $O(\Delta t^2,h^2)$ approximation for the diffusion equation
   - ☐ an $O(\Delta t,h^2)$ approximation for the diffusion equation

6. The CFL (Courant, Fredrich, Levy) number is defined as
   - ☐ velocity divided by time step and grid spacing
   - ☐ velocity times time step divided grid spacing
   - ☐ velocity times grid spacing divided by delta time
   - ☐ delta time divided by velocity and grid spacing
   - ☐ grid spacing times delta time divided by velocity

7. The first order upwind method for the linear advection equation is always stable
   - ☐ when the CFL number is smaller than 1
   - ☐ when the CFL number is larger than 1
   - ☐ when the CFL number is smaller than 2
   - ☐ does not depend on the CFL number
   - ☐ never stable

8. The leap-frog method for the diffusion equation is
   - ☐ always stable
   - ☐ stable when the CFL number is smaller than 1
   - ☐ stable when the CFL number is larger than 1
   - ☐ stable when the CFL number is smaller than 2
   - ☐ never stable

9. The three main categories of second-order partial differential equations are:
   - ☐ first-order, second order, third-order
   - ☐ elliptic, parabolic, hyperbolic
   - ☐ advection, diffusion, wave
   - ☐ parabolic, hyperbolic, diabolic
   - ☐ elliptic, advection, heat

10. Elliptic equations
- ☐ describe the evolution of wavelike phenomenon in time
- ☐ describe the propagation of heat with time
- ☐ describe the unsteady flow of fluid
- ☐ describe the steady-state equilibrium of quantities transported by diffusion
- ☐ describe everything

11. Parabolic equations
- ☐ describe the time evolution of quantities transported by diffusion
- ☐ describe the evolution of wavelike phenomenon in time
- ☐ describe the steady-state temperature distribution in a solid
- ☐ describe the unsteady flow of fluid
- ☐ describe all unsteady processes

12. Hyperbolic equations
- ☐ are inherently unstable
- ☐ describe the flow of heat
- ☐ can be written as a system of first order equations
- ☐ always lead to shocks
- ☐ have no characteristics

13. When advecting a discontinuous solution using standard finite difference/finite volume approximations for the spatial derivative, we find that
- ☐ One-sided spatial discretizations guaranties a well-behaved solution
- ☐ Second order spatial approximations smoothes the solution excessively
- ☐ First order upwind spatial approximations lead to oscillations
- ☐ Second order spatial approximations lead to oscillations
- ☐ Third order schemes lead to well-behaved solutions

14. When advecting a discontinuous solution using standard finite difference/finite volume approximations for the spatial derivative, we find that
- ☐ The solution always blows up
- ☐ First order upwind spatial approximations lead to oscillations
- ☐ First order upwind spatial approximations smoothes the solution excessively
- ☐ It is necessary to use the ADI method to get well-behaved solution
- ☐ Only the method of characteristics leads to a well-behaved solution

15. When solving advection problems, it is usually advantageous to use the conservative form of the governing equations. Discrete approximations of the conservative form
- ☐ can be derived by adding an artificial viscosity
- ☐ guaranties that any discontinuities move with the correct speed.
- ☐ are identical to the nonconservative form
- ☐ guaranty conservation of the square of the solution
- ☐ only works for periodic or infinite domains

16. When advecting a discontinuous solution using finite volume approximations, it is possible to obtain relatively sharp solutions with minimum "wiggles," using
- ☐ high-order finite difference approximations
- ☐ artificial viscosity
- ☐ compact schemes
- ☐ SOR
- ☐ multigrid methods

17. Characteristics are lines in the solution domain
- ☐ that describe the propagation of waves for elliptic equations
- ☐ along which the solution is constant
- ☐ that describe the propagation of information for hyperbolic equations
- ☐ across which the solution is always discontinuous
- ☐ none of the above

18. SOR is
- ☐ used to solve hyperbolic equations
- ☐ stands for Successive Over-Relaxation
- ☐ stands for Simple Operation Reduction
- ☐ increases the accuracy
- ☐ is the best method to solve elliptic equations

19. The primary purpose of the multigrid method is to
- ☐ increase the accuracy of the solution
- ☐ allow the use of a coarse grid
- ☐ accelerate convergence
- ☐ simplify the programming
- ☐ confuse the user

20. A parabolic equation has only one characteristic. This means that
- ☐ it must be solved using an implicit method
- ☐ it always describes a steady-state problem
- ☐ the future can influence the past
- ☐ it always describes heat transfer
- ☐ none of the above

21. the ADI method
- ☐ was a breakthrough in solving parabolic equations
- ☐ was initially applied to two-dimensional problems
- ☐ uses both implicit and explicit approximations
- ☐ is unconditionally stable
- ☐ all of the above

22. In a Finite Volume Method
- ☐ we approximate the point wise value of the function
- ☐ we approximate the function using a Fourier series
- ☐ we apply the conservation principle to a control volume
- ☐ the fluxes in and out of the control volume must balance
- ☐ the fluxes are equal to the average value of the function

23. The Cell Reynolds number
- ☐ must always be less than 2
- ☐ sets the size of the time step
- ☐ shows when shock should be expected
- ☐ is used to determine the resolution for the advection diffusion equation
- ☐ none of the above

24. AMR stands for
- ☐ Another Method for Refinement
- ☐ Adaptive Method for Reactions
- ☐ Adaptive Mesh Refinement
- ☐ Accurate Mesh Refinement
- ☐ None of the above

25. Richardson extrapolation is used to
- ☐ develop higher order finite difference approximations
- ☐ estimate the accuracy of the solution
- ☐ improve the accuracy of the solution
- ☐ determine where the grid must be refined
- ☐ all of the above

26. The vorticity-streamfunction formulation of the Navier-Stokes equations
- ☐ only exists for two-dimensional flows
- ☐ only applies to invidscid flows
- ☐ only holds for very viscous flows
- ☐ always leads to conservative schemes
- ☐ none of the above

27. For incompressible flows
- ☐ density is always constant
- ☐ the flow is always irrotational
- ☐ volume is conserved
- ☐ pressure is conserved
- ☐ momentum is constant

28. In the projection method (used to solve the unsteady Navier-Stokes equations)
- ☐ the time integration must be implicit to obtain second order accuracy in time
- ☐ an elliptic equation must be solved to include the viscous terms
- ☐ an elliptic pressure equation is solved to enforce incompressibility
- ☐ the advection terms must be written in conservative form
- ☐ we use ghost points to implement boundary conditions for the pressure

29. Staggered grids
- ☐ are important for simulations of compressible flows in the aerospace industry
- ☐ are the only way to make simulations of incompressible flows work
- ☐ consists of separate control volumes for the pressure and the different velocity components
- ☐ lead to very complicated codes
- ☐ require the use of “half-indices” in programs

30. Structured grids
- ☐ lead to relatively simple numerical schemes
- ☐ lead to relatively efficient numerical schemes
- ☐ lead to relatively accurate numerical schemes
- ☐ all of the above
- ☐ none of the above

31. Body fitted grids
- ☐ require the use of high-order finite difference schemes
- ☐ require the use of low-order finite difference schemes
- ☐ generally result in a low accuracy solution
- ☐ must contain information about the connectivity of the grid
- ☐ none of the above

32. Unstructured grids
- ☐ are always made up of triangles/tetrahedrons
- ☐ are always made up of squares/hexahedrons
- ☐ must contain information about the connectivity of the grid
- ☐ always consists of differently shaped control volumes
- ☐ all of the above

33. In a staggered grid:
- ☐ pressure and velocity are stored at the same point
- ☐ pressure is stored at one point and velocities at another point
- ☐ each velocity component and pressure are computed using different control volumes
- ☐ we only have equations for the velocites
- ☐ all of the above

34. “Colocated grid” means
- ☐ a single control volume is used to compute the pressure and the velocities
- ☐ the velocities are stored at the corner of the control volume
- ☐ the pressure is located at the same point as surface tension
- ☐ the grid is unstructured
- ☐ none of the above

35. The k-epsilon model for turbulent flows
- ☐ is ideally suited for anisotropic turbulence
- ☐ represents the state-of-the-art in turbulence modeling
- ☐ abandons the concept of turbulent viscosity and works directly with the Reynolds stresses
- ☐ all of the above
- ☐ none of the above

36. In the k-epsilon model for turbulent flow, epsilon is
- ☐ the kinetic energy of the velocity fluctuations
- ☐ the vorticity of the velocity fluctuations
- ☐ the dissipation of the velocity fluctuations
- ☐ the time-scale of the velocity fluctuations
- ☐ the length-scale velocity fluctuations

37. In the k-epsilon model for turbulent flow,
- ☐ the k equation describes the time-scale of the turbulence
- ☐ the k equation describes the length-scale of the turbulence
- ☐ the epsilon equation describes the length-scale of the turbulence
- ☐ the epsilon equation describes the intensity of the turbulence
- ☐ none of the above

38. Industrial simulations of the average motion of turbulent flows are most commonly done using
- ☐ Prandtl’s mixing length model
- ☐ Reynolds stress turbulence models
- ☐ k-epsilon turbulence models
- ☐ direct numerical simulations
- ☐ large eddy simulations

39. In combustion modeling
- ☐ for diffusion flames we usually work with an overall reaction
- ☐ in principle we need to solve O(100) equations for O(100) species
- ☐ diffusion flames can often be models using a conserved scalar and precomputed libraries
- ☐ the G-equation approach is suitable for premixed combustion
- ☐ all of the above

40. Disperse multiphase flows (flows containing particles, drops or bubbles) can be simulated using:
- ☐ Euler/Euler methods
- ☐ Euler/Lagrange methods
- ☐ finite volume methods
- ☐ finite element methods
- ☐ all of the above

Problem 2. (25%)

You need to solve the unsteady advection-diffusion equation

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial x} = D\frac{\partial^2 f}{\partial x^2}$$

in a domain given by $0<x<1$, that $U>0$, and that the boundary conditions are $f(0)=0$ and $f(1)=1$. The velocity U is high and the diffusion D is small so we expect a boundary layer near $x=1$.

(a) Sketch the steady-state solution for high U and low D.
(b) Propose a mapping function that will cluster the grid points near the x=1 boundary.
(c) Write the equation in the mapped coordinates.

Problem 3. (25%)

Your job requires you to write an "in-house" code to solve for the airflow in a "hood" used for a variety of chemical experiments. The airflow comes in through a narrow opening on the bottom and leaves through a vent at the top that has a diameter of about 20 centimeters. The hood is about a meter deep, a meter high and three meters long. The volume flow rate can be up to 0.5 cubic meters per second. In general, there will be "stuff" in the hood, including bottles, plastic containers, and equipment of various sort. Thus, although the overall geometry of the hood is simple, you will have to account for the "stuff" as internal boundaries that can change from one run to the next. The code is intended to check the airflow as experiments are planned and will therefore be used often.

(a) EXPLAIN the main issues that you must deal with.
(b) PROPOSE a method to simulate this problem. You do not have to write down any equations.
(c) ESTIMATE the time writing the code will take you.

Limit the response to less than a page!