

## Quiz 1 – 2009

### Problem 1. (50%)

Use the forward in time Euler method to integrate the following equation (which can arise in modeling of thin films)

$$\frac{f_j^{n+1} - f_j^n}{\Delta t} = -U \frac{3f_j^n - 4f_{j-1}^n + f_{j-2}^n}{2h} \quad U > 0$$

- Write down the modified equation
- What equation is being approximated?
- Determine the accuracy of the scheme
- Use the von Neuman's method to derive an equation for the stability conditions (you do not have to solve the equation)

### Problem 2. (20%)

- Write down the characteristics for the following equation

$$f \frac{\partial f}{\partial t} + (U + f) \frac{\partial f^2}{\partial x} = 0$$

- Consider a sine wave initial condition  $f(x, t = 0) = \sin 2\pi x$ . Sketch the characteristics and the solution at a later time.
- This equation can develop a shock. What is the speed of the shock?

### Problem 3 (30%)

Explain what the main challenges are in solving the three different types of second order partial differential methods. For **each** equation type:

Name one example of a physical situation where the equation arises;

Explain the main challenges in solving the equation;

Give one example of a numerical method suitable for solving it.

Limit your response to **less than** a page.