Quiz-I from 2008

Problem 1. (50%)

Use the Backward Euler (implicit) method to integrate the following equation (which can arise in modeling of thin films)

$$\frac{\partial f}{\partial t} - U\frac{\partial^3 f}{\partial x^3} = 0 \qquad U > 0$$

in time. Use a second order centered difference approximation for the spatial derivative.
(a) Write down the finite difference equation.
(b) Write down the modified equation
(c) Find the accuracy of the scheme
(d) Use the von Neuman's method to analyze the stability of the scheme.

Problem 2. (20%)

(a) Classify the following second order partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + 3\frac{\partial^2 f}{\partial x \partial y} - \frac{1}{2}\frac{\partial^2 f}{\partial y^2} + \left(\frac{\partial f}{\partial y}\right)^2 - 2\frac{\partial f}{\partial x} + 7 = 0\,.$$

Explain your reasoning.

(b) Write it as a system of first order equations

Problem 3 (30%)
Explain how a multigrid iteration method for the solution of an elliptic equation works and the basic ideas behind it. Limit your response to **less than half** a page.