

**Worcester Polytechnic Institute**  
**Mechanical Engineering Department**  
**ME 612 Computational Fluid Dynamics**  
 Project 3. Due March 31, 2010

**One-Dimensional Euler Equations with a Shock**

Write a program to compute the solution to the one dimensional Euler equations:

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho u \\ \rho \left( e + \frac{1}{2} u^2 \right) \end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix} \rho u \\ \rho u^2 + p \\ u \left( \rho \left( e + \frac{1}{2} u^2 \right) + p \right) \end{bmatrix} = 0$$

or,

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$

where the pressure is given by the ideal gas relation  $p = (\gamma - 1)\rho e$ . The sound speed is given by  $c^2 = \gamma p / \rho$ . Use the split Lax-Wendroff method discussed in class:

$$\mathbf{f}_{j+\frac{1}{2}}^* = 0.5(\mathbf{f}_j^n + \mathbf{f}_{j+1}^n) - 0.5 \frac{\Delta t}{h} (\mathbf{F}_{j+1}^n - \mathbf{F}_j^n)$$

$$\mathbf{f}_j^{n+1} = \mathbf{f}_j^n - \frac{\Delta t}{h} (\mathbf{F}_{j+\frac{1}{2}}^* - \mathbf{F}_{j-\frac{1}{2}}^*)$$

The initial condition consists of a shock tube with two constant states, the left and the right state:

$$p_L = 10^5; \quad p_R = 10^4; \quad \rho_L = 1; \quad \rho_R = 0.125; \quad u_R = u_L = 0.$$

The tube is 10 meter long and you should assume that the initial separation between the two states is in the middle. The exact solution at time 0.005 is given in the notes

Run your program for several different spatial resolution and time steps. In general you are going to find that the solution is not very good. Large "wiggles" arise—even when the time step is small—and if the "wiggles" become negative, bad things will happen! To improve the results, add artificial viscous fluxes of the form:

$$\mathbf{F}_{AV} = -\alpha \Delta x^2 \rho \begin{bmatrix} 0 \\ 1 \\ u \end{bmatrix} \left| \frac{\partial u}{\partial x} \right| \frac{\partial u}{\partial x}$$

where  $\alpha$  is an adjustable coefficient. Experiment with the coefficient. Generally, the stability requirement become more restrictive once we add artificial viscosity, so we want the coefficient to be as small as possible.

You should hand in a discussion of what you have done and the tests that you conducted. Your report should include a printout of your code and plots of the solution at one or more times for a few different resolutions. You can also compare with the results in the lecture notes. Remember to be professional and concise.

%Code discussed in class--upwind-van Leer flux splitting

nx=256; maxstep=128

gg=1.4;p\_left=100000;p\_right=10000;r\_left=1;r\_right=0.125;  
xl=10.0;h=xl/(nx-1);time=0;for i=1:nx,x(i)=h\*(i-1);end

r=zeros(1,nx);ru=zeros(1,nx);rE=zeros(1,nx);  
rn=zeros(1,nx);run=zeros(1,nx);rEn=zeros(1,nx);  
c=zeros(1,nx);u=zeros(1,nx);m=zeros(1,nx);

for i=1:nx,r(i)=r\_right;ru(i)=0.0;rE(i)=p\_right/(gg-1);end  
for i=1:nx/2;

    r(i)=r\_left;  
    rE(i)=p\_left/(gg-1);

end

rn=r;run=ru;rEn=rE;

dt=0.465\*h/sqrt(1.4\*max([700+p\_right/r\_right,700+p\_left/r\_left]) )

for istep=1:maxstep

    for i=1:nx,c(i)=sqrt( gg\*(gg-1)\*(rE(i)-0.5\*(ru(i)^2/r(i)))/r(i) );end  
    for i=1:nx,u(i)=ru(i)/r(i);end; for i=1:nx,m(i)=u(i)/c(i);end

    for i=2:nx-1  
        %upwind  
        rn(i)=r(i)-(dt/h)\*(...  
        (0.25\*r(i)\*c(i)\*(m(i)+1)^2) - (0.25\*r(i-1)\*c(i-1)\*(m(i-1)+1)^2)+...  
        (-0.25\*r(i+1)\*c(i+1)\*(m(i+1)-1)^2) - (-0.25\*r(i)\*c(i)\*(m(i)-1)^2) );

        run(i)=ru(i)-(dt/h)\*(...  
        ( 0.25\*r(i)\*c(i) \*(m(i)+1)^2) \*(( 1+0.5\*(gg-1)\*m(i)) \*2\*c(i) /gg) - ...  
        ( 0.25\*r(i-1)\*c(i-1)\*(m(i-1)+1)^2)\*(( 1+0.5\*(gg-1)\*m(i-1))\*2\*c(i-1)/gg) + ...  
        (-0.25\*r(i+1)\*c(i+1)\*(m(i+1)-1)^2)\*((-1+0.5\*(gg-1)\*m(i+1))\*2\*c(i+1)/gg) - ...  
        (-0.25\*r(i)\*c(i) \*(m(i)-1)^2) \*((-1+0.5\*(gg-1)\*m(i)) \*2\*c(i) /gg) );

        rEn(i)=rE(i)-(dt/h)\*(...  
        ( 0.25\*r(i)\*c(i) \*(m(i)+1)^2) \*((1+0.5\*(gg-1)\*m(i))^2 \*2\*c(i)^2 /((gg^2-1)) - ...  
        ( 0.25\*r(i-1)\*c(i-1)\*(m(i-1)+1)^2)\*((1+0.5\*(gg-1)\*m(i-1))^2 \*2\*c(i-1)^2/((gg^2-1)) + ...  
        (-0.25\*r(i+1)\*c(i+1)\*(m(i+1)-1)^2)\*((1-0.5\*(gg-1)\*m(i+1))^2 \*2\*c(i+1)^2/((gg^2-1)) - ...  
        (-0.25\*r(i)\*c(i) \*(m(i)-1)^2) \*((1-0.5\*(gg-1)\*m(i))^2 \*2\*c(i)^2 /((gg^2-1)) );  
    end

    r=rn;ru=run;rE=rEn;

    time=time+dt,istep  
    hold on

end

for i=1:nx,p(i)=r(i)\*c(i)^2/gg;end

hold on;plot(x,p,'r','linewidth',2);text(7.,100000.0,'Pressure','fontsize',24),  
axis([0 xl 0 1.1\*max(p)])  
set(gca,'fontsize',24);set(gca,'linewidth',2)