Worcester Polytechnic Institute

Mechanical Engineering Department

ME 612 Computational Fluid Dynamics

Homework 4. Due April 12, 2010

Problem 13.
Show that the equation

$$\frac{\partial f}{\partial t} = f \frac{\partial f}{\partial x}$$

in the domain $-\infty < x < \infty$, with $f \to 0$ as $x \to \pm\infty$. conserves the total amount of f. That is, show that the integral of f over the whole domain is independent of time. The equation can be written in a conservative form as

$$\frac{\partial f}{\partial t} = \frac{1}{2} \frac{\partial f^2}{\partial x}$$

Show that an "upwind" differencing of the advection term retains the conservation property of the differential equation if the conservative form is used, but not if the original form is used.

Problem 14.
Explain, in a <u>clear and concise</u> way:
(a) why it is important to compute shocks and discontinuous solutions accurately,
(b) what the fundamental difficulties with "standard" finite difference and finite volume schemes applied to the motion of shocks and sharp discontinuities are, and
(c) how these difficulties can be overcome.
Limit your response to **less than half** a page.

Problem 15.
Derive an expression for

$$f_x g_y - f_y g_x$$

in a transformed coordinate system (ξ, η).

Problem 16.
The one dimensional advection/diffusion equation

$$\frac{\partial c}{\partial t} + U \cdot \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2},$$

where *U* is given and *D* is a constant, needs to be solved in a domain whose length changes with time: $L = L_o + \varepsilon \sin(\omega t)$ (think of a one dimensional model of an engine cylinder, for example). To carry out the computations, we would like to use a fixed number of grid points at all times and map the domain into one with a fixed length.

- Propose a mapping function that generates a grid with uniform distribution of points.
- Write the linear advection equation in the mapped coordinates.