

Worcester Polytechnic Institute
Mechanical Engineering Department
ME 612 Computational Fluid Dynamics
Homework 3. Due March 15, 2010

Problem 9.

The following two-step finite difference scheme is given:

$$f_i^* = f_i^n - \frac{C\Delta t}{4\Delta x}(f_{i+1}^n - f_{i-1}^n)$$
$$f_i^{n+1} = f_i^n - \frac{C\Delta t}{2\Delta x}(f_{i+1}^* - f_{i-1}^*)$$

Derive the modified equation and find:

- The equation that is being simulated.
- The accuracy of the scheme.
- Use von Neuman's method to determine the stability of the scheme

Problem 10.

(a) Write down the characteristics for the following equation

$$\frac{\partial f}{\partial t} + f^2 \frac{\partial f}{\partial x} = -f$$

(b) Sketch the evolution of a sine wave with time. It is easiest to sketch it at time 0 and one subsequent times along with the characteristics in the $x-t$ plane.

(c) Is the equation in conservative form? If not, write down the conservative form.

Problem 11.

Write the equation

$$\frac{\partial^2 f}{\partial x^2} = \alpha \frac{\partial^2 f}{\partial y^2} + \frac{\partial f}{\partial y}; \quad \alpha > 0$$

as a system of first order equations. What type of equation is this?

Problem 12.

Show that the one-dimensional diffusion equation is an ill-posed initial value problem if the diffusion coefficient is negative.