Worcester Polytechnic Institute
Mechanical Engineering Department
ME 612 Computational Fluid Dynamics
Homework 2. Due February 22, 2010

Problem 5. Derive the relation

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_{i+1} + 10\left(\frac{\partial^2 f}{\partial x^2}\right)_{i} + \left(\frac{\partial^2 f}{\partial x^2}\right)_{i-1} = \frac{12}{\Delta x^2}\left(f_{i+1} - 2f_i + f_{i-1}\right) + O\left(\Delta x^4\right)$$

This equation, and other similar ones are used to construct "compact, high order methods"

Problem 6. Use the Backward Euler (implicit) method to integrate the linear advection equation

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial x} = 0 \qquad U > 0$$

in time. Use the standard second order centered difference approximation for the spatial derivative.
(a) Write down the finite difference equation.
(b) Write down the modified equation
(c) Find the accuracy of the scheme
(d) Use the von Neuman's method to determine the stability of the scheme.

Problem 7. The following finite difference approximation is given

$$f_j^{n+1} = \frac{1}{2}\left(f_{j+1}^n + f_{j-1}^n\right) - \frac{U\Delta t}{2h}\left(f_{j+1}^n - f_{j-1}^n\right) \qquad U > 0$$

(a) Write down the modified equation
(b) What equation is being approximated?
(c) Determine the accuracy of the scheme
(d) Use the von Neuman's method to examine under which conditions this scheme is stable.

Problem 8. Given the following second order partial differential equation,

$$\frac{\partial^2 f}{\partial x^2} + 3\frac{\partial^2 f}{\partial x \partial y} - \frac{1}{2}\frac{\partial^2 f}{\partial y^2} + \left(\frac{\partial f}{\partial y}\right)^2 - 2\frac{\partial f}{\partial x} + 7 = 0$$

(a) Classify the equation (hyperbolic, parabolic, or elliptic).
(b) Write it as a system of first order equations