Worcester Polytechnic Institute
Mechanical Engineering Department
ME 612 Computational Fluid Dynamics
Homework 1. Due February 3, 2010

Problem 1.
Show, by Taylor expansion, that

$$\frac{d^3 f}{dx^3} = \frac{f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}}{2h^3} + O(h^2)$$

Problem 2.
Consider the backward Euler approximation to the Diffusion Equation:

$$f_i^{n+1} = f_i^n + \frac{\Delta t\, \nu}{h^2}\left(f_{i+1}^{n+1} + f_{i-1}^{n+1} - 2f_i^{n+1}\right).$$

(a) Derive the modified equation and determine the accuracy of this scheme.
(b) Find its stability properties by von Neumann's method. How does it compare with the forward Euler scheme discussed in class?

Problem 3.
Consider the following finite difference approximation to the Diffusion Equation:

$$f_i^{n+1} = f_i^{n-1} + 2\frac{\Delta t\, \nu}{h^2}\left(f_{i+1}^n - f_i^{n+1} - f_i^{n-1} + f_{i-1}^n\right).$$

This is the so-called Dufort-Frankel scheme, where the time integration is the "Leapfrog" method, and the spatial derivative is the usual center difference approximation, except that we have replaced f_i^n by $\frac{1}{2}\left(f_i^{n+1} + f_i^{n-1}\right)$. Derive the modified equation and determine the accuracy of the scheme. Are there any surprises?

Problem 4.
Show that the predictor-corrector method:

$$f^* = f^n + \Delta t g\left(f^n\right)$$

$$f^{n+1} = f^n + \frac{\Delta t}{2}\left(g\left(f^n\right) + g\left(f^*\right)\right)$$

can be written as:

$$f^* = f^n + \Delta t g\left(f^n\right)$$

$$f^{**} = f^* + \Delta t g\left(f^*\right)$$

$$f^{n+1} = \frac{1}{2}\left(f^n + f^{**}\right)$$

That is, you simply take two explicit Euler steps and then average the solution at the beginning of the time step and the end.