1 Basic definition

1. **Each set of ordered pairs comes from a mapping. Decide (yes or no) if the mapping could be a function or not.**

   (a) $\{(2,1),(3,4),(4,-2),(5,3),(6,3),(7,2),(8,2)\}$
   Yes. This could be a function. Every input here has only one output.

   (b) $\{(1,1),(3,1),(4,-2),(1,3),(7,6),(6,7),(0,2)\}$
   No. This cannot be a function because the input 1 has two outputs, 1 and 3.

   (c) $\{(2,1),(3,1),(4,1),(5,1),(6,1),(7,1),(8,1),(9,1),(10,1)\}$
   Yes. This could be a function, even though every input has the same output.

2. **For each part in Problem 1 that was a function, specify its domain and range.**

   (a) Domain $= \{2,3,4,5,6,7,8\}$, Range $= \{1,4,-2,3,2\}$

   (b) This was not a function.

   (c) Domain $= \{2,3,4,5,6,7,8,9,10\}$, Range $= \{1\}$

2 Notation

1. **For the given function, determine each quantities stated next to it.**

   (a) $f(x) = 5x^2 + 7x + 1$ $\quad f(y);\ f(x+a);\ f(f(x))$
   $f(y) = 5y^2 + 7y + 1$
   $f(x+a) = 5(x+a)^2 + 7(x+a)$
   $f(f(x)) = f(5x^2 + 7x + 1) = 5(5x^2 + 7x + 1)^2 + 7(5x^2 + 7x + 1) + 1$

   (b) $f(z) = z^3 + 1$ $\quad f(x);\ f(z+h);\ f(1/x);$
   $f(x) = x^3 + 1$
   $f(z+h) = (z+h)^3 + 1$
   $f(1/x) = (1/x)^3 + 1$

   (c) $f(x) = 2$ $\quad f(x+h);\ f(z);\ f(3);\ f(x+h) = 2$
   $f(z) = 2$
   $f(3) = 2$

   (d) $f(x) = 1/x$ $\quad f(2+h);\ f(3/2);\ f(1/y);\ f(2+h) = \frac{1}{2+h}$
   $f(3/2) = \frac{1}{3/2} = \frac{2}{3}$
   $f(1/y) = \frac{1}{1/y} = y$

2. **For each function below, state what its domain and range are.**

(a) $f(x) = |x| + 2$
Domain = all real numbers, range = $\{x : x \geq 2\}$

(b) $f(x) = 3\sin(x)$
Domain = all real numbers, range = $\{x : -3 \leq x \leq 3\}$

(c) $f(x) = \sqrt{4x - 24} + 9$
Domain = $\{x : x \geq 6\}$, range = $\{x : x \geq 9\}$

(d) $f(x) = x^2 + 4$
Domain = all real numbers, range = $\{x : x \geq 4\}$

3 Graphing functions

Determine if each function below is even, odd or neither.

1. $x^4 + 6x^2 + 1$
   Even. $(-x)^4 + 6(-x)^2 + 1 = x^4 + 6x^2 + 1$

2. $x\cos(x) + x^3$
   Odd. Since x and x^3 are odd, and $\cos(x)$ is even, the combined function is odd.

3. $\frac{1}{x}$
   Odd. $\frac{1}{-x} = -\frac{1}{x}$

4. $3x + 9$
   Neither even nor odd since $3(-x) + 9 = -3x + 9 \neq -(3x + 9)$

5. $\sin(x^2)$
   Even since $\sin((-x)^2) = \sin(x^2)$.

6. $x^3 + 1$
   Neither even nor odd since $(-x)^3 + 1 = -x^3 + 1 \neq -(x^3 + 1)$.

7. $\sin^3(x)$
   Odd since $\sin^3(-x) = (\sin(-x))^3 = (-\sin(x))^3 = -\sin^3(x)$.

8. $\sin^2(x^2) + x^2 + 1$
   Even since $\sin^2((-x)^2) + (-x)^2 + 1 = \sin^2(x^2) + x^2 + 1$

4 Piecewise functions

1. **For each function below, determine the value stated.**

   (a) $f(x) = \begin{cases} x^2 & \text{for } x < 3 \\ x + 7 & \text{otherwise} \end{cases}$ What is $f(4)$? $f(0)$? $f(f(0))$?

   $f(4) = 4 + 7 = 11 \quad f(0) = 0^2 = 0 \quad f(f(0)) = f(0^2) = f(0) = 0$

(b) $f(x) = \begin{cases} x & \text{if } x \geq 3 \\ 3 & \text{otherwise} \end{cases}$ What is $f(3)$? $f(10)$? $f(0)$? $f(-3)$?

$f(3) = 3 \quad f(10) = 10 \quad f(0) = 3 \quad f(-3) = 3$

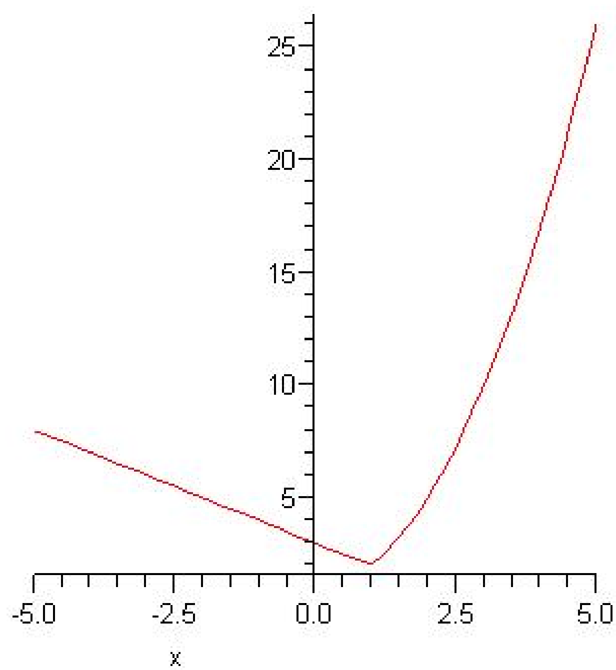
(c) $f(y) = \begin{cases} 1 & \text{if } y \geq 2 \\ 2 & \text{if } y < 2 \end{cases}$ What is $f(2)$? $f(0)$?

$f(2) = 1 \quad f(0) = 2$

(d) $f(x) = \begin{cases} 1 & \text{if } x \text{ is an integer} \\ 2 & \text{otherwise} \end{cases}$ What is $f(3)$? $f(2.2)$? $f(\pi)$? $f(1/2)$?

$f(3) = 1 \quad f(2.2) = 2 \quad f(\pi) = 2 \quad f(1/2) = 2$

2. If $f(x) = x^2 + 1$ when $x > 1$ and $-x + 3$ for all other x, what is the graph of f?

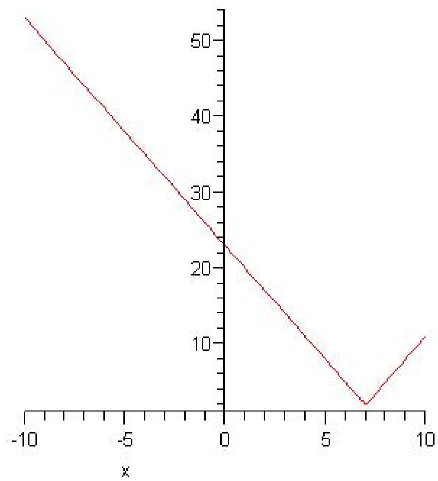


3. If $f(x) = x^2$ for $x > 3$ and $2x + b$ for all other x, what value of b would cause the graph to be unbroken for all x?
We need $3^2 = 2(3) + b \Rightarrow 9 = 6 + b \Rightarrow b = 3$

In each case, state the function in a piecewise manner, then graph it.

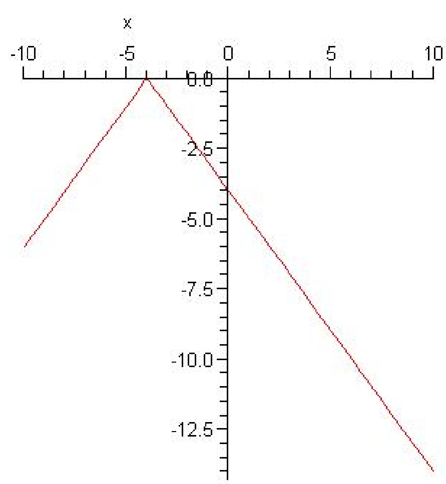
1. $f(x) = 3|x-7|+2$

$$f(x) = \begin{cases} 3(x-7)+2 & \text{if } x \geq 7 \\ -3(x-7)+2 & \text{if } x < 7 \end{cases}$$



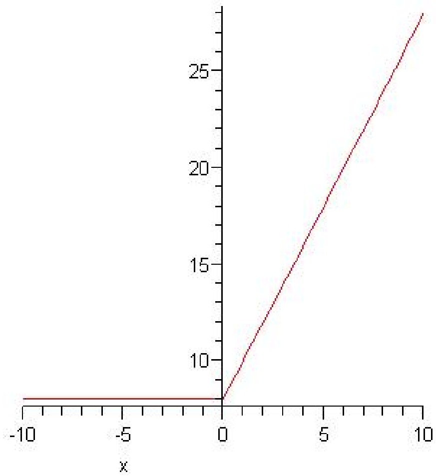
2. $f(x) = -|x+4|$

$$f(x) = \begin{cases} -(x+4) & \text{if } x \geq -4 \\ x+4 & \text{if } x < -4 \end{cases}$$



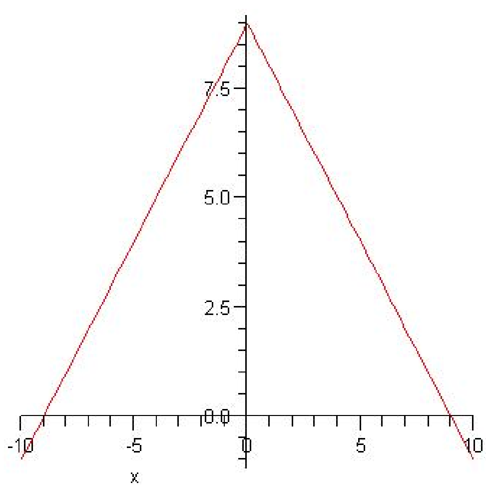
3. $f(x) = x + |x| + 8$

$$f(x) = \begin{cases} 2x + 8 & \text{if } x \geq 0 \\ 8 & \text{if } x < 0 \end{cases}$$



4. $g(x) = -|x| + 9$

$$g(x) = \begin{cases} -x + 9 & \text{if } x \geq 0 \\ x + 9 & \text{if } x < 0 \end{cases}$$



5 Greatest integer function

1. **What are actual numerical values of**

   (a) $[|2.1|]$ and $[|1.98|]$?
   $[|2.1|] = 2$, $[|1.98|] = 1$

   (b) $[|-2.1101|]$ and $[|-1.99|]$?
   $[|-2.1101|] = -3$, $[|-1.99|] = -2$

   (c) $[|\pi|]$?
   $[|\pi|] = 3$

   (d) $([|-3.11|])^2$ and $([|3.11|])^2$?
   $([|-3.11|])^2 = (-4)^2 = 16$, $([|3.11|])^2 = 3^2 = 9$

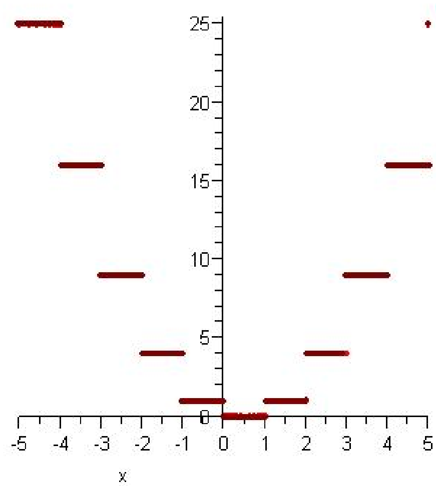
2. **What is the domain and range of each function below?**

   (a) $f(x) = [|x|]$
   Domain = all real numbers, Range = integers

   (b) $f(x) = [|x|]^2$
   Domain = all real numbers, Range = perfect squares

   (c) $f(x) = x - [|x|]$
   Domain = all real numbers, Range = $\{x : 0 \leq x \leq 1\}$

3. **What is the graph of $f(x) = [|x|]^2$?**



4. **For each function and inverse value below, determine if that value shown is correct or not:**

(a) $f(x) = 2x + 3$, $f^{-1}(9) = 3$
We need to check $f(3)$. $f(3) = 2(3) + 3 = 9$, so $f^{-1}(9) = 3$.

(b) $f(x) = x^3 + 3$, $f^{-1}(30) = 3$
$f(3) = 3^3 + 3 = 27 + 3 = 30$, so $f^{-1}(30) = 3$.

(c) $f(x) = 8/x + 5$, $f^{-1}(9) = 3$
$f(3) = \frac{8}{3} + 5 = \frac{8+15}{5} = \frac{23}{3} \neq 9$, so $f^{-1}(9) \neq 3$.

(d) $f(x) = \log_{10}(x)$, $f^{-1}(.01) = -2$
Recall that the domain of $\log_{10}(x)$ is positive values of x, so $f^{-1}(.01) \neq -2$.

(e) $f(x) = \log_{10}(x)$, $f^{-1}(1000) = 2$
$f^{-1}(1000) \neq 2$

5. For each function given, decide if it has an inverse and find its inverse, for those that do. Verify your result by direct substitution.

(a) $f(x) = 9x + 2$
$f^{-1}(x) = \frac{x-2}{9}$. Check: $f^{-1}(f(x)) = f^{-1}(9x + 2) = \frac{(9x+2)-2}{9} = x$.

(b) $f(x) = x^3 + 9$
$f^{-1}(x) = (x-9)^{1/3}$. Check: $f^{-1}(f(x)) = f^{-1}(x^3+9) = [(x^3+9)-9]^{1/3} = x$.

(c) $f(x) = x^2$
No inverse. Since, for example, $f(1) = 1 = f(-1)$, then $f^{-1}(1)$ would have to be both 1 and -1, and no function can have two outputs for only one input. One could also see that this function does not pass the horizontal line test, so it doesn't have an inverse.

(d) $f(x) = 5$
No inverse. Since, for any value of x, $f(x) = 5$, $f^{-1}(x)$ would have to give all real numbers and there is no function that does that. Also, this is a horizontal line, so it clearly cannot pass the horizontal line test.

(e) $f(x) = 1/(x - 5)$ (domain all x except 5)
$f^{-1}(x) = \frac{1}{x}+5$. Check: $f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-5}\right) = \frac{1}{1/(x-5)}+5 = (x-5)+5 = x$.

6 Composition

1. **Write each expression below as the composition of two functions and identify the first as g and the second as f for purposes of answering.**

(a) $\sqrt{x^2 + 5}$
$f(x) = \sqrt{x}$, $g(x) = x^2 + 5$, $f \circ g(x) = \sqrt{x^2 + 5}$

(b) $|x^2 + 2x + 1| + 7$
$f(x) = |x| + 7$, $g(x) = x^2 + 2x + 1$, $f \circ g(x) = |x^2 + 2x + 1| + 7$

(c) $\sin^3(x) + 1$
$f(x) = x^3 + 1,\ g(x) = \sin(x),\ f \circ g(x) = \sin^3(x) + 1$

(d) $(x^2 + 2x + 5)^2 + 1$
$f(x) = x^2 + 1,\ g(x) = x^2 + 2x + 5,\ f \circ g(x) = (x^2 + 2x + 5)^2 + 1$

(e) $\cos(x^2)$
$f(x) = \cos(x),\ g(x) = x^2,\ f \circ g(x) = \cos(x^2)$

2. **Write one expression for what $\mathbf{f \circ g(x)}$ is in each case.**

(a) $f(x) = 3x^2 + 1 \quad g(x) = 9x + 1$
$f \circ g(x) = f(9x+1) = 3(9x+1)^2+1 = 3(81x^2+18x+1)+1 = 243x^2+54x+4$

(b) $f(x) = 3 \quad g(x) = x^4$
$f \circ g(x) = f(x^4) = 3$

(c) $f(x) = 3x^2 \quad g(x) = x^4$
$f \circ g(x) = f(x^4) = 3(x^4)^2 = 3x^8$

(d) $f(x) = x|x| + 9 \quad g(x) = 3$
$f \circ g(x) = f(3) = 3|3| + 9 = 18$

(e) $f(y) = 4y + 5 \quad g(z) = 3z^2$
$f \circ g(x) = f(3x^2) = 4(3x^2) + 5 = 12x^2 + 5$