Just Do It!

Problem 1:

a) Execute the following three commands:

```
x = 2
```

```
2
```

```
Factor[x^2 - 1]
```

```
3
```

$\text{Sum}\left[i^2 - \text{Sin}[i], \{i, 0, x\}\right]$

```
5 - Sin[1] - Sin[2]
```

and briefly explain the output that you get.

For an output, we get 5 - Sin[1] - Sin[2]. We assigned x as the number 2, so in input 12, 2 squared is 4. minus one is 3.

b) Execute the following three commands:

```
Clear[x]
```

```
Factor[x^2 - 1]
```

```
(-1 + x) (1 + x)
```

$\text{Sum}\left[i^2 - \text{Sin}[i], \{i, 0, x\}\right]$

$$\frac{1}{6} x (1 + x) (1 + 2 x) + \frac{1}{2} \left(-\text{Cos}\left[\frac{1}{2}\right] + \text{Cos}\left[\frac{1}{2} + x\right]\right) \text{Csc}\left[\frac{1}{2}\right]$$

The out puts of the last two commands are different that tbhey were in part a. THis is because in part a, x was given a value, here there is no value given to x.

and briefly explain why the outputs of the last two commands are different than they were in Part a)

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Problem 2: A local package delivery service has the following rate structure:

a) Any package weighing less than a pound costs $5.00.

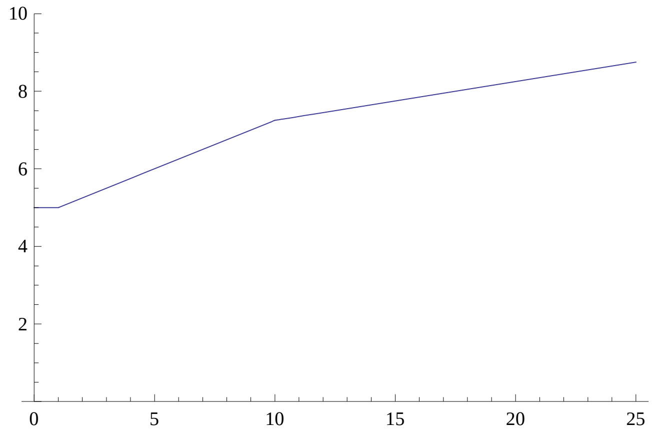
b) A package weighing at least one pound but less than 10 pounds costs 25 cents a pound for each additional pound or part of a pound over 1 pound.

c) A package weighing at least 10 pounds but less than 25 pounds costs 10 cents a pound for each additional pound or part of a pound over 10 pounds.

No packages weighing 25 pounds or more are accepted. Define function cost[x] that gives the cost of delivering a package weighing x pounds for $0 < x < 25$. Graph the function on the interval $0 < x < 25$.

```
cost[x_] := 5 /; x < 1
cost[x_] := 5 + .25 (x - 1) /; 10 > x ≥ 1
cost[x_] := 7.25 + .10 (x - 10) /; 25 ≥ x > 10
```

```
Plot[cost[x], {x, 0, 25}, PlotRange → {0, 10}]
```



Problem 3: (Defining Periodic Functions)

Recall that a function f[x] is periodic if there is a positive constant p such that

(*) f[x + p] = f[x] for all x.

The six trigonometric functions are periodic, and so is any constant function. If there is a smallest positive number p with the property (*), it is called the period of f.

The built-in Mathematica function Mod[m,n] can, among many other things, help us to define periodic functions different from the standard trigonometric functions.

For two real numbers m and n, Mod[m,n] gives the reainder for the division of the number m by the number n; that is, if

m = q n + r where r is between 0 and n but not equal to n

then Mod[m,n] = r. Run the following to check out a few values:

```
Mod[3.5, 2]

1.5

Mod[3.5, 2.5]

1.

Mod[1.5, 2]

1.5

Mod[-3.5, 2]

0.5

Mod[3.5, -2]

-0.5
```

Notice that if n is a positive number, then the remainder r is nonnegative and less than n; that is,

$n > 0 \implies 0 \leq r < n$.

Also, if f[x] is a periodic function of period p, then the value of f at any x is the same as its value at Mod[x,p]; that is,

f[x] = f[Mod[x,p]]

and 0 ≤ Mod[x,p] < p.

a) Enter the non-periodic function g[x] = x^2- 2 x.

b) Use the Mod[m,n] command to define a function f[x] of period 3 that is identical to g[x] on the interval 0 < x < 3. Graph the fuction f[x] on the interval

-6 ≤ x ≤ 6.

c) Did you need to use := to define the function f[x]? Why?

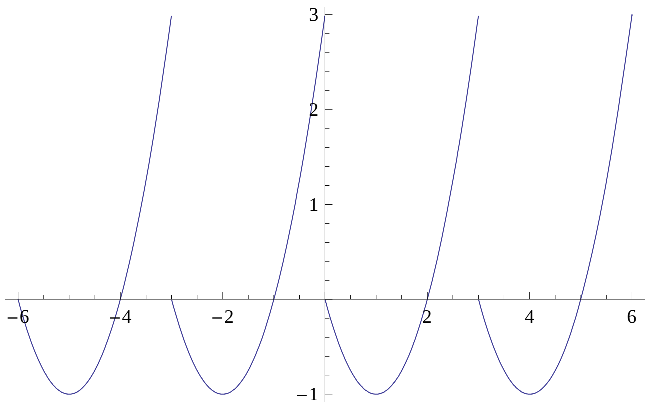
```
yes I needed to use := to define function f[x] because if I didn't use it,
it wouldn't understand the restriction that 0 ≤ Mod[x, 3] < 3
```

```
Clear[f, g]

g[x_] := x^2 - 2 x

f[x_] := g[Mod[x, 3]] /;
  0 ≤ Mod[x, 3] < 3

Plot[f[x], {x, -6, 6}]
```



Just Do It!

Problem 4: Newton's Law of Cooling states that, under suitable conditions, the temperature of a cooling body decreases at a rate proportional to the difference between the body's temperature and the (assumed constant) temperature of its surroundings. Suppose that a cake is taken from the oven at 350 degrees F. and placed in a large room that is maintained at 70 degrees to cool. Let t[n] represent the temperature of the cake n minutes after it is removed from the oven. Then application of Newton's Law of Cooling would lead to the following sort of mathematical model for t[n]:

t[0] = 350

t[n] - t[n-1] = -c (t[n-1] - 70)

where c is a positive constant that measure the rate at which this cake dissipates heat.

a) Define a function t[n] that represents the temperature of the cake n minutes after it is removed from the oven if the value of c for this particular cake is c = .1.

What is the temperature of the cake after 30 minutes?

b) Make a table of values of t[n] for n values between 0 and 60 minutes.

```
Clear[c, t, n]
c = .1
0.1
t[0] = 350
350
t[n_] := t[n] = -c (t[n - 1] - 70) + t[n - 1]
t[30]
81.8695
Table[t[n], {n, 0, 60}]
{350, 322., 296.8, 274.12, 253.708, 235.337, 218.803, 203.923,
 190.531, 178.478, 167.63, 157.867, 149.08, 141.172, 134.055, 127.65,
 121.885, 116.696, 112.026, 107.824, 104.041, 100.637, 97.5736,
 94.8162, 92.3346, 90.1011, 88.091, 86.2819, 84.6537, 83.1884, 81.8695,
 80.6826, 79.6143, 78.6529, 77.7876, 77.0088, 76.308, 75.6772, 75.1094,
 74.5985, 74.1386, 73.7248, 73.3523, 73.0171, 72.7154, 72.4438,
 72.1994, 71.9795, 71.7816, 71.6034, 71.4431, 71.2988, 71.1689,
 71.052, 70.9468, 70.8521, 70.7669, 70.6902, 70.6212, 70.5591, 70.5032}
```