Problem	Score
1	
2	
3	
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PH 1121: PRINCIPLES OF PHYSICS: eLECTRICITY and MAGNETISM
B TERM, 2005
KOLECI

NAME: Solution

EXAM III
THURSDAY, DECEMBER 15, 2005
2 PM – 3:00 PM
CLOSED BOOK EXAM

PLEASE ANSWER ALL THREE QUESTIONS IN THE SPACE PROVIDED. QUESTION ONE AND TWO ARE EACH WORTH 35 POINTS, WHILE QUESTION THREE IS WORTH 30 POINTS. PLEASE BE SURE TO SHOW ALL WORK AND JUSTIFY ALL YOUR ANSWERS. GRADING WILL BE BASED ON EVIDENCE OF YOUR UNDERSTANDING OF THE BASIC CONCEPTS AND PRINCIPLES—PLEASE PROVIDE THOROUGH SOLUTIONS TO THE PROBLEMS. GOOD LUCK, HAPPY HOLIDAYS, AND BEST WISHES FOR SUCCESS!!

Possibly Useful Information:

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$

$m_e =$ mass of electron $= 9.11 \times 10^{-31}$ kg

$F = (1/4\pi\varepsilon_o)(q_1 q_2/r^2) = k(q_1 q_2/r^2)$

$dE = kdq/r^2 = 1/(4\pi\varepsilon_o)(dq/r^2)$

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

$\mathbf{p} = q\mathbf{d}$

$\Phi_e =$ electric flux $= \int_{\text{closed surface}} (\mathbf{E}\cdot d\mathbf{A})$

$e =$ charge of electron $= 1.6 \times 10^{-19}$ C

$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9\ Nm^2/C^2$

$E = k(q/r^2) = (1/4\pi\varepsilon_o)(q/r^2)$

$\mathbf{E} = \mathbf{F}/q$

$\tau = pE \sin\theta$

$U = -\mathbf{p}\cdot\mathbf{E} = -pE\cos\theta$

$\Phi_e =$ electric flux $= \int_{\text{closed surface}} (\mathbf{E}\cdot d\mathbf{A}) = q_{enc}/\varepsilon_o$

$V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{s}$

$U = \frac{1}{2} CV^2$

$q = C\mathcal{E}e^{-t/RC}$

$I = dq/dt$

$R = \rho L/A$

$\rho =$ resistivity $= E/J$

$E = \sigma/\varepsilon_o = (Q/A)/\varepsilon_o$

$Q = VC$

$V = IR$

$q = C\mathcal{E}(1 - e^{-t/RC})$

$P = IV = dW/dt$

$J = nev_d$

$\sigma =$ conductivity $= 1/\rho = 1/$(resistivity)

$C = \varepsilon_o (A/d)$

$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

$F = I\mathbf{L} \times \mathbf{B}$

$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

$U = -\boldsymbol{\tau}\cdot\mathbf{B} = -\tau B\cos\theta_{\tau,B}$

$\omega = 2\pi f = 2\pi/T$

$dB = (\mu_o i/4\pi)[(ds \sin\theta_{ds,r})/r^2]$

$F = qvB\sin\theta_{qv,B}$

$F = ILB\sin\theta_{I,B}$

$\tau = IAB\sin\theta_{\mu,B}$

$\mu = IA$

$\mathbf{dB} = (\mu_o i/4\pi)[(\mathbf{ds} \times \mathbf{r})/r^2]$

$\oint_{\text{closed loop}} \mathbf{B}\cdot d\mathbf{s} = \mu_o i_{enc} + \mu_o\varepsilon_o\, d\Phi_E/dt$

$\Phi_B =$ magnetic flux $= \int \mathbf{B}\cdot d\mathbf{A}$

$\varepsilon =$ induced emf $= -d\Phi_B/dt$

$L = (N\Phi_B)/i$

$i = (\varepsilon/R)[1 - e^{-tR/L}]$

$U_B = \frac{1}{2} Li^2$

$\Phi_B = \int (B\cos\theta_{B,A})dA$

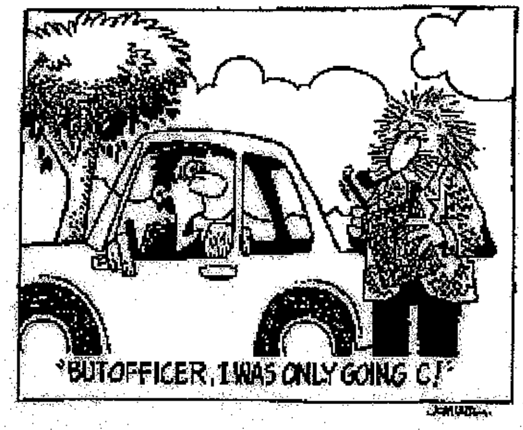
$\varepsilon = -N(d\Phi_B/dt)$

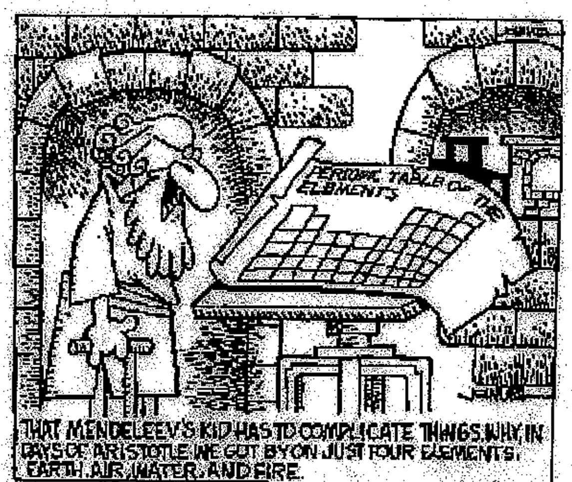
$\varepsilon = -L\, di/dt$

$i = (\varepsilon/R)e^{-tR/L}$

TABLE of POSSIBLY USEFUL INTEGRALS* , DERIVATIVES, and PROPERTIES OF THE NATURAL LOG (ln) FUNCTION

***Disclaimer:* Cartoons taken**
from: *phys.udallas.edu/C3P/cartoons.html*





$\int x^m\,dx = [x^{m+1}/(m+1)] + C$
(note: $m \neq -1$)

$\int [x/(a^2 + x^2)^{3/2}]dx = [-1/(a^2 + x^2)^{1/2}] + C$

$\int [1/(a^2 + x^2)^{3/2}]dx = \{(1/a^2)[x/(x^2 + a^2)^{1/2}]\} + C$

$\int \sin(ax)\,dx = -(1/a)\cos(ax) + C$

$\int \cos(ax)\,dx = (1/a)\sin(ax) + C$

$\int (1/x)\,dx = \ln(x) + C$

$d(e^{ax})/dx = ae^{ax}$

$d(\cos at)/dt = -a\sin(at)$

$d(\sin at)/dt = a\cos(at)$

$d(t^m)/dt = m\,t^{m-1}$

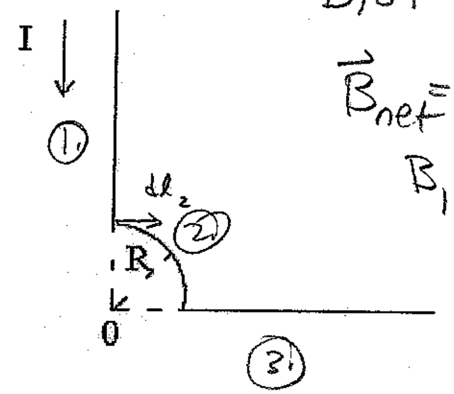
$d[\ln(x + a)]/dx = 1/(x + a)$

$\ln[(x+a)/(x-a)] = \ln(x+a) - \ln(x-a)$

***Note: These are indefinite integrals, and notice the constant of integration = C. In the above examples, a = arbitrary constant.**

TWO UNRELATED QUESTIONS:

Problem 1A: The segment of wire in the figure shown below carries a current, I = 5.00 A, where the radius of the circular arc is R = 3.00 cm. Determine the magnitude and direction of the magnetic field at the origin. (20 Points)



Biot Savart $\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

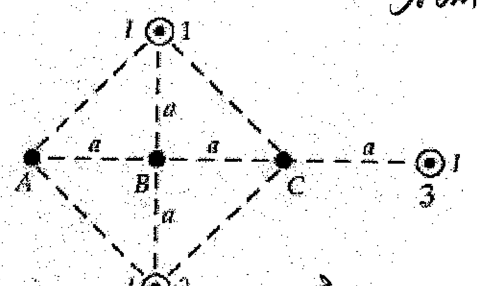
$B_1 = 0, \; B_3 = 0$ as $d\vec{\ell} \times \hat{r} = \vec{0}$.

$B_2 = \frac{\mu_0 I}{4\pi} \int_0^{2\pi R/4} \frac{d\ell}{R^2} = \frac{\mu_0 I}{\cancel{4}_2\cancel{\pi} R^{\cancel{2}}} \left(\frac{\cancel{2}\cancel{\pi}\cancel{R}}{4} \right)$

$B_2 = \frac{\mu_0 I}{8R}$ direction $\Rightarrow d\vec{\ell} \times \hat{r} \Rightarrow \hat{i} \times -\hat{j} = -\hat{k}$

$\vec{B}_{net} = \boxed{\vec{B}_2 = -\frac{\mu_0 I}{8R} \hat{k}}$

Problem 1B: Three long, parallel conductors carry currents of I = 2.00 A. The figure shown below is an end view of the conductors, with each current coming out of the page. If a = 1.00 cm, determine the magnitude and direction of the magnetic field at point B. (15 Points) *Hint: You do not need to derive the expression for the magnetic field outside of a long wire.*



From Ampere's Law $B_{wire} = \frac{\mu_0 I}{2\pi R}$

$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$ $\quad$ ijk ijkijk

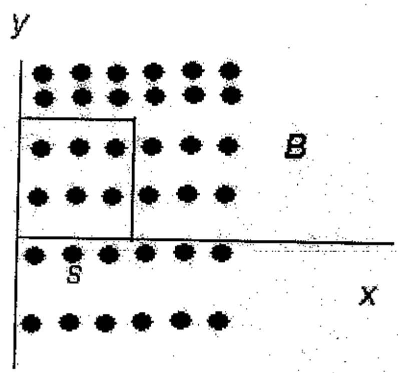
$\vec{B}_1 = \frac{\mu_0 I}{2\pi a} (\hat{k} \times -\hat{j}) = \frac{\mu_0 I}{2\pi a} \hat{i}$

$\vec{B}_2 = \frac{\mu_0 I}{2\pi a} (\hat{k} \times \hat{j}) = -\frac{\mu_0 I}{2\pi a} \hat{i}$

$\vec{B}_3 = \frac{\mu_0 I}{2\pi (2a)} (\hat{k} \times -\hat{i}) = -\frac{\mu_0 I}{4\pi a} \hat{j}$

$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \boxed{-\frac{\mu_0 I}{4\pi a} \hat{j}}$

Two: In the figure below, the square loop of wire has sides of length *s meters*. A magnetic field, *B*, is directed out of the page; its magnitude is given by: $B = 8.0t^3y^2$, where *B* is in Teslas, *t* is in seconds, and *y* is in meters. a) Briefly explain *Faraday's Law of Induction* (*5 points*), b) Determine the *emf* around the square at $t = T$ *seconds* (*20 points*). c) What is the direction of the induced current (*5 points*)? d.) Suppose you could alter the given magnetic field (magnitude) for the square loop having dimensions indicated in *part a*. Give an example of a magnetic field (magnitude) which would produce an induced *emf* of **zero**. (*5 points*)



a.) Faraday's Law of Induction states that a change in magnetic flux produces an induced Emf. i.e $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ (single loop).

b.) $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ $\quad \Phi_B = \int \vec{B} \cdot d\vec{A} = \int_0^s \int_0^s 8t^3y^2 \, dx\, dy$

$$\Phi_B = \int_0^s 8t^3 y^2 s \, dy = \frac{8t^3y^3s}{3}\Big|_0^s = \frac{8t^3s^4}{3}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{8t^3s^4}{3}\right) = -8t^2s^4$$

at $t = T$, $\mathcal{E} = -8T^2s^4$

c) Since $\dfrac{d\Phi_B}{dt} > 0$, and $\mathcal{E} < 0$, induced magnetic field will be in the opposite direction of the original B-field. B_{induced} is into the page. By Right Hand Rule, induced current is clockwise.

d.) Remove variation in 't' and 'y', so $B = 8.0\,T$ is one example (of many!)

Three: Conceptual Questions: Please answer the following questions. To receive full credit, remember that you must justify your answers. (*6 points each, 30 points total*).

a.) Two, long, parallel wires are separated by a distance *d*. the currents, I_1 and I_2 have the directions shown. 1.)What is the magnitude and direction of the magnetic field created by wire 2 (top wire), at point p—midway between the wires (*3 points*)? 2.)What is the magnitude and direction of the force per unit length on wire 1 due to the magnetic field created by wire 2, and is this force attractive or repulsive? (*3 points*)

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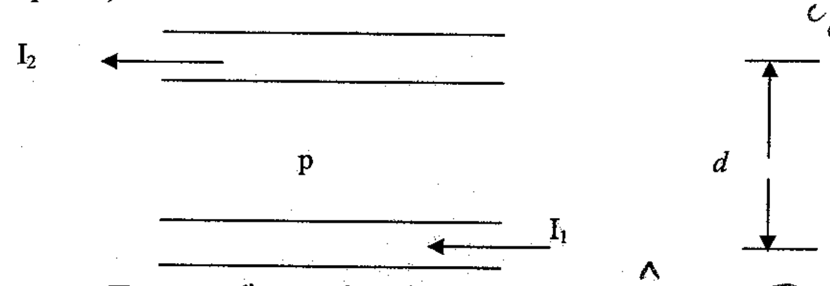


① $B_{2,p} = \frac{\mu_0 I_2}{2\pi(d/2)}(-\hat{i}\times-\hat{j}) = \frac{\mu_0 I}{\pi d}\hat{k}$ ② $\vec{F}_1 = I_1\vec{L}\times\vec{B}_2$

$B_{2,1} = \frac{\mu_0 I_2}{2\pi d}\hat{k}$ $\frac{F_{1,\text{due } 2}}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}(-\hat{i}\times\hat{k}) = \boxed{\frac{\mu_0 I_1 I_2}{2\pi d}\hat{j}}$

Force is attractive → like currents attract

b.) *Briefly* explain why magnetic forces do NO work.

$\vec{F} = q\vec{v}\times\vec{B}$ $\vec{v} = d\vec{s}/dt$

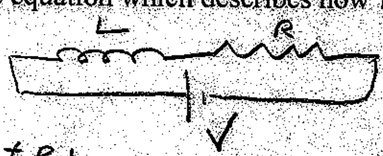
$W = \int\vec{F}\cdot d\vec{s} = \int\left(q\frac{d\vec{s}}{dt}\times\vec{B}\right)\cdot d\vec{s} = 0$

c.) *Briefly* explain Lenz's Law.

Lenz's Law states that an induced magnetic field/current is produced to countract an increase in magnetic flux. If there is a decrease in magnetic flux then an induced field acts in the same direction of the B-field,

d.) Suppose you take an inductor, of inductance *L*, and connect it to a resistor, *R*, and battery of emf *V*. Write down the equation which describes how *Voltage* across the *inductor* varies with time.

$\mathcal{E} = \left|-L\frac{di}{dt}\right|$



$i = \frac{V}{R}(1-e^{-tR/L})$

$V_L = \left|-L\left(\frac{V}{L}e^{-tR/L}\right)\right| = \left|-Ve^{-tR/L}\right| = Ve^{-tR/L}$

e.) **Assessment Questions** *(a guaranteed six points, for any answers you provide)*:

1. What was the most difficult topic we studied in PH 1121? Not Sure
2. What was the easiest topic we studied in PH 1121? Not sure
3. What was most helpful, in preparation for this exam? You tell me
4. What was least helpful, in preparation for this exam? You tell me
5. What has been the most interesting topic we studied in PH 1121? Maxwell's Equation
6. What, if anything, did you like most about this class? Why? Cool projects!