Problem	Score
1	
2	
3	
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PH 1130: INTRODUCTION TO 20TH CENTURY PHYSICS
C TERM, 2008
KOLECI

NAME (please include last name):____________________________
SECTION NUMBER:______________

SAMPLE EXAM II
FEBRUARY 2008
2 PM – 2:50 PM
CLOSED BOOK EXAM

PLEASE ANSWER ALL THREE QUESTIONS IN THE SPACE PROVIDED. QUESTIONS ONE AND TWO ARE WORTH 35 POINTS, EACH. QUESTION THREE IS WORTH 30 POINTS. PLEASE BE SURE TO SHOW ALL WORK AND JUSTIFY ALL YOUR ANSWERS. GRADING WILL BE BASED ON EVIDENCE OF YOUR UNDERSTANDING OF THE BASIC CONCEPTS AND PRINCIPLES—PLEASE PROVIDE THOROUGH SOLUTIONS TO THE PROBLEMS. GOOD LUCK!

Possibly Useful Information:

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$
q_e = charge of electron = 1.6×10^{-19} C
m_e = mass of electron = 9.11×10^{-31} kg
m_p = mass of proton = 1.67×10^{-27} kg
q_p = charge of proton = 1.6×10^{-19} C
$c = 3.00 \times 10^8$ m/s
$F = (1/4\pi\varepsilon_o)(q_1 q_2/r^2) = k(q_1 q_2/r^2)$
$E = k(q/r^2) = (1/4\pi\varepsilon_o)(q/r^2)$
$h = 6.626 \times 10^{-34}$ Js
$\mathbf{E} = \mathbf{F}/q$
1 eV = 1.6×10^{-19} J
$k = 1.381 \times 10^{-23}$ J/K
$\sigma = 5.67 \times 10^{-8}\ W/(m^2 K^4)$
$\gamma = (1 - (u^2/c^2))^{-1/2}$
$x = x' + ut,\ y' = y,\ z' = z$
$\Delta t = \gamma(\Delta t_o) = (\Delta t_o)/(1-(u^2/c^2))^{1/2}$
$l = l_o/\gamma = l_o(1-(u^2/c^2))^{1/2}$
$x' = \gamma(x - ut),\ y' = y,\ z' = z$
$t' = \gamma(t - (ux/c^2))$
$v_x = (v_x' + u)/(1 + (u v_x'/c^2))$
$\vec{p} = \gamma m\vec{v}$ $\quad a = (F/m)(1-(v^2/c^2))^{3/2}$ $\quad F = \gamma^3 ma$ (F & v along same line)
$F = \gamma ma$ (F & v perpendicular) $\quad E = K + mc^2 = \gamma(mc^2)$ $\quad K = (\gamma - 1)(mc^2)$
$E^2 = (mc^2)^2 + (pc)^2$ $\quad (pc)^2 = K^2 + 2K(mc^2)$ $\quad E = hf = h(c/\lambda)$
$\frac{1}{2}mv^2 = eV_o = hf - \varphi$ $\quad P = E/c = (hf)/c = h/\lambda$ $\quad hf = E_i - E_f$
$R = 1.097 \times 10^7\ m^{-1}$
Balmer Series: $(1/\lambda) = R(1/2^2 - 1/n^2)$ n = 3,4,5…
$E_n = (-hcR)/n^2$
Lyman Series: $(1/\lambda) = R(1/1^2 - 1/n^2)$ n = 2,3,4…
Paschen Series: $(1/\lambda) = R(1/3^2 - 1/n^2)$ n = 4,5,6…
Brackett Series: $(1/\lambda) = R(1/4^2 - 1/n^2)$ n = 5,6,7… Pfund Series: $(1/\lambda) = R(1/5^2 - 1/n^2)$ n = 6,7,8…
$L_n = mv_n r_n = nh/(2\pi)$ $\quad r_n = \varepsilon_o[(nh)^2/(\pi me^2)]$ $\quad v_n = e^2/(2nh\varepsilon_o)$
$eV_{AC} = hf_{max} = (hc)/\lambda_{min}$
$\lambda' - \lambda = [h/(mc)](1 - \cos\varphi)$
$I = \sigma T^4$
$\lambda_m T = 2.90 \times 10^{-3}$ m K
$I(\lambda) = (2\pi hc^2)/(\lambda^5)[e^{hc/\lambda kT} - 1]^{-1}$

Stationary State

$d(\sin\theta) = m\lambda$ $\quad \Delta x\, \Delta p_x \geq \hbar$ $\quad \Delta E\, \Delta t \geq \hbar$
$\Psi(x,y,z) = \psi(x,y,z)e^{-iEt/h}$ $\quad e^{i\theta} = \cos\theta + i\sin\theta$
$-\hbar^2/2m(d^2\psi(x)/dx^2) + U(x)\psi(x) = E\psi(x)$ $\quad \int_{-\infty}^{\infty}|\psi(x)|^2 dx = 1$
$\psi(x) = Ae^{ikx}$ $\quad E = (\hbar k)^2/2m$ free particle $\quad E_n = (n\pi\hbar)^2/(2mL^2)$ particle in box
$\psi(x) = \sqrt{2/L}\sin[(n\pi x)/L]$ $\quad E_\infty = (\pi\hbar)^2/(2mL^2)$ $\quad \hbar\omega = \hbar\sqrt{k'/m}$
$E_n = (n + \frac{1}{2})\hbar\omega$

Free particle

harmonic oscillator

particle in box x=0 to x=L

Potential well

Problem One (*35 points*): A particle moving in one-dimension is described by the wave function:

$$\psi(x) = A\,e^{-bx}, \textit{ for } x \geq 0 \quad \textit{and} \quad \psi(x) = A\,e^{bx}, \textit{ for } x < 0.$$

Here, b, is a positive, real number, and A is also positive and real. (a) Determine A so that the function is normalized. *(10 points)*
(b) Sketch the graph of this wave function. *(10 points)* (c) Find the probability of finding this particle on the left side of the origin. *(15 points)*

a.) Normalization $\Rightarrow$ probability of finding particle in all space is 1.

$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = \int_{-\infty}^{\infty} \psi^*(x)\,\psi(x)\,dx = 1$$

$$\int_{-\infty}^{0} Ae^{bx} \cdot Ae^{bx}\,dx + \int_{0}^{\infty} (Ae^{-bx})(Ae^{-bx})\,dx = 1$$

let $u = 2bx$
$du = 2b\,dx$

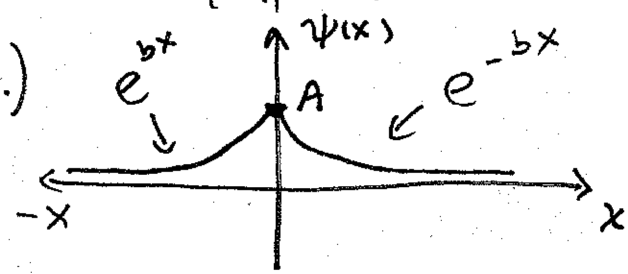
$$\int_{-\infty}^{0} A^2 e^{2bx}\,dx + \int_{0}^{\infty} A^2 e^{-2bx}\,dx = 1$$

$$\int e^u\,du = e^u + C$$

$$\frac{A^2}{2b} e^{2bx}\Big|_{-\infty}^{0} - \frac{A^2}{2b} e^{-2bx}\Big|_{0}^{\infty} = \frac{A^2}{2b} + \frac{A^2}{2b} = \frac{A^2}{b} = 1$$

$$\Rightarrow |A| = \sqrt{b}$$

b.)



$$\psi(x) = \begin{cases} Ae^{-bx} & x \geq 0 \\ Ae^{bx} & x < 0 \end{cases}$$

c.) Find particle on left side of origin

$$\int_{-\infty}^{0} |\psi(x)|^2 dx = \int_{-\infty}^{0} A^2 e^{2bx} dx = \int_{-\infty}^{0} b e^{2bx} dx$$

$$b\int_{-\infty}^{0} e^{2bx} dx = \frac{b}{2b} e^{2bx}\Big|_{-\infty}^{0} = \frac{1}{2} \checkmark$$

Problem Two (***35 points***): Consider a particle in a box with rigid walls at $x = 0$ and $x = L$. Let the particle be in the ground level. (a) What are *two* boundary conditions that this wave function must obey? *(10 points)* (b) Write down the wave function which corresponds to the ground level. *(5 points)* (c) Write down the energy for the ground level. *(5 points)* (d) Calculate the probability that the particle will be found in the interval $x = 0$ to $x = L/2$ *(15 points)*

Ground level of particle in a box

a.) $\psi(x=0) = 0$

$\psi(x=L) = 0$

b.) wave function $\longleftrightarrow$ ground level

From formula sheet

$$\psi(x) = \sqrt{\frac{2}{L}}\ \sin\left(\frac{n\pi x}{L}\right)$$

Ground level $\Rightarrow n=1 \Rightarrow \boxed{\psi = \sqrt{\frac{2}{L}}\ \sin\frac{\pi x}{L}}$

c.) Energy $\rightarrow$ Ground Level! $E_1 = \frac{(1\pi\hbar)^2}{2mL^2} = \frac{\pi^2\hbar^2}{2mL^2}$

d.) Probability to find $0 \leq x \leq L/2$

$$\int_0^{L/2} |\psi(x)|^2 dx = \int_0^{L/2}\left(\frac{2}{L}\sin^2\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/2}\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx$$

$$\frac{2}{L}\left\{\int_0^{L/2}\frac{1}{2}dx - \int_0^{L/2}\cos\frac{2\pi x}{L}dx\right\} = \frac{2}{L}\left\{\frac{x}{2}\Big|_0^{L/2} - \cancel{\frac{L}{2\pi}\sin\frac{2\pi x}{L}\Big|_0^{L/2}}\right\}$$

$u = \frac{2\pi}{L}x$ $\qquad$ 0

$du = \frac{2\pi}{L}dx$

Right hand side

$\downarrow$

$$RHS = \frac{2}{L}\cdot\frac{L}{4} = \boxed{\frac{1}{2}}$$

Problem Three: Conceptual Questions: Please answer the following possibly unrelated questions, using a minimum of mathematics *(6 points each, 30 points total).*

a. What are two fundamental differences between classical mechanics and quantum mechanics?

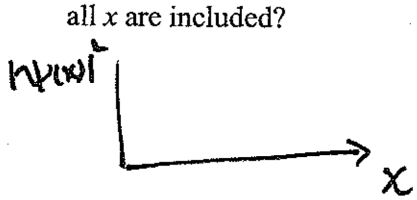
IN QM $\rightarrow$ uncertainty principle, wave nature of particles.

b. A sodium atom is in one of the states labeled "Lowest excited levels". It remains in that state for an average time of τ seconds before making a transition back to the ground state, emitting a photon with wavelength λ and energy ε *eV*. What is the uncertainty in energy of the excited state?

$\Delta E \, \Delta t \geq \hbar \Rightarrow \Delta E (\tau) \geq \hbar$

$\Rightarrow \Delta E = \dfrac{\hbar}{\tau}$

c. If $\psi(x)$ is normalized, what is the physical significance of the *area* under a graph of $|\psi(x)|^2$ *versus* x, between x_1 and x_2? What is the total area under the graph of $|\psi(x)|^2$ when all x are included?



Total area = 1 Normalized

area under graph = probability

d. Briefly explain why zero energy is NOT permitted for the quantum mechanical system of a particle in a box.

Can't have zero ψ so that prevents zero energy! $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

e. **Assessment Questions:** A guaranteed six points for answers you provide!

- ❖ Did you write your name on the exam (If No, please do so now!)?
- ❖ What was the most challenging (*aka: difficult*) topic we studied so far?
- ❖ What was the least challenging (*aka: easiest*) topic we studied so far?
- ❖ What was most helpful, in preparation for this exam?
- ❖ What was least helpful, in preparation for this exam?
- ❖ On a scale of 1-5, 5 = identical, 3 = neutral, and 1= very different, how would you rate the sample exam with the actual exam?