

PH 1121: PRINCIPLES OF PHYSICS II: eLECTRICITY and Magnetism

NAME (please include last name):

Solution

SECTION:

SAMPLE EXAM ONE  
PH 1121

CLOSED BOOK EXAM

PLEASE ANSWER ALL THREE QUESTIONS IN THE SPACE PROVIDED. QUESTION ONE AND TWO ARE WORTH 35 POINTS, EACH. QUESTION THREE IS WORTH 30 POINTS. PLEASE BE SURE TO SHOW ALL WORK AND JUSTIFY ALL YOUR ANSWERS. GRADING WILL BE BASED ON EVIDENCE OF YOUR UNDERSTANDING OF THE BASIC CONCEPTS AND PRINCIPLES—PLEASE PROVIDE THOROUGH SOLUTIONS TO THE PROBLEMS. GOOD LUCK!

Possibly Useful Information:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$m_e = \text{mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$m_p = \text{mass of proton} = 1.67 \times 10^{-27} \text{ kg}$$

$$F = (1/4\pi\epsilon_0) (q_1 q_2/r^2) = k (q_1 q_2/r^2)$$

$$dE = kdq/r^2 = 1/(4\pi\epsilon_0) (dq/r^2)$$

$$\tau = \mathbf{p} \times \mathbf{E}$$

$$\mathbf{p} = q\mathbf{d}$$

$$\Phi_e = \text{electric flux} = \int_{\text{closed surface}} (\mathbf{E} \cdot d\mathbf{A})$$

$$q_e = \text{charge of electron} = 1.6 \times 10^{-19} \text{ C}$$

$$k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$q_p = \text{charge of proton} = 1.6 \times 10^{-19} \text{ C}$$

$$E = k (q/r^2) = (1/4\pi\epsilon_0)(q/r^2)$$

$$\mathbf{E} = \mathbf{F}/q$$

$$\tau = pE \sin\theta$$

$$U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos\theta$$

$$\Phi_e = \text{electric flux} = \int_{\text{closed surface}} (\mathbf{E} \cdot d\mathbf{A}) = q_{\text{enc}}/\epsilon_0$$

Table of Possibly Useful Integrals\*

$$\int x^m dx = [x^{m+1}/(m+1)] + C$$

(note:  $m \neq -1$ )

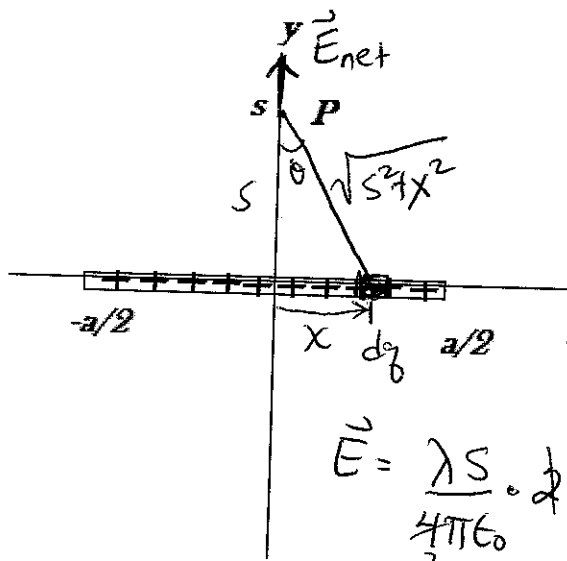
$$\int [z/(a^2 + z^2)^{3/2}] dz = [-1/(a^2 + z^2)^{1/2}]$$

$$\int [1/(a^2 + z^2)^{3/2}] dz = \{(1/a^2) [z/(z^2 + a^2)^{1/2}]\} + C$$

\*Note: These are indefinite integrals, and notice the constant of integration = C. In the above examples, a = arbitrary constant.

Problem	Score
1	
2	
3	
Net	

**Problem One:** Positive charge  $Q$  is distributed uniformly along the positive  $x$ -axis between  $x = -a/2$  and  $x = +a/2$ , as shown in the diagram below. (a) Calculate the  $x$ - and  $y$ - components of the electric field, at point  $P$  ( $y=s$ ), produced by the charge distribution (thin rod). (20 points) (b) Derive an expression for the electric field at point  $P$  when  $s \gg a/2$  (10 points). (c) Set up, but do not evaluate, an expression for the  $y$ -component of the electric field, if the thin rod now has a non-uniform linear charge density,  $\lambda$ , such that  $\lambda = \beta x^2$ , where  $\beta$  is a positive constant with units of  $C/m^3$ . (5 points).



$$a) E = \frac{1}{4\pi\epsilon_0} \int (\text{direction factor}) \frac{dq}{r^2}$$

$$1-D \Rightarrow dq = \lambda dx$$

$$E = \frac{1}{4\pi\epsilon_0} \int_{-a/2}^{a/2} \frac{s}{\sqrt{s^2+x^2}} \cdot \frac{dq}{(s^2+x^2)} \hat{j}$$

$$\vec{E} = \frac{\lambda s}{4\pi\epsilon_0} \int_0^{a/2} \frac{dx}{(s^2+x^2)^{3/2}} \hat{j}$$

From Integral Table on Front Page

$$I = \int \frac{dz}{(a^2+z^2)^{3/2}} = \frac{1}{a^2} \left[ \frac{z}{\sqrt{z^2+a^2}} \right] + C$$

$a \rightarrow s, z \rightarrow x$

$$\rightarrow \vec{E} = \frac{\lambda s}{2\pi\epsilon_0 s^2} \left[ \frac{x}{\sqrt{x^2+s^2}} \right]_0^{a/2} = \frac{\lambda a}{4\pi\epsilon_0 s} \cdot \frac{1}{\sqrt{\frac{a^2}{4} + s^2}} \hat{j}$$

Now  $\lambda = \frac{Q}{a}$

$$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 s \sqrt{\frac{a^2}{4} + s^2}} \hat{j}$$

(note:  $Q$  is given, not  $\lambda$ !)

b.) For  $s \gg \frac{a}{2}$  !  $(\frac{a^2}{4} + s^2)^{1/2} = [s^2(\frac{a^2}{4s^2} + 1)]^{1/2} = s[1 + \frac{a^2}{4s^2}]^{1/2}$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 s^2} \cdot [1 + \frac{a^2}{4s^2}]^{-1/2} \hat{j} \approx \frac{Q}{4\pi\epsilon_0 s^2} \hat{j} \quad (\text{point charge!})$$

1c.)  $\lambda = \beta x^2$

$$E_y = \frac{1}{4\pi\epsilon_0} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{S}{\sqrt{S^2+x^2}} \cdot \frac{\lambda dx}{(S^2+x^2)} = \frac{S}{4\pi\epsilon_0} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{\beta x^2 dx}{(S^2+x^2)^{3/2}}$$

$$E_y = \frac{2S\beta}{4\pi\epsilon_0} \int_0^{\frac{a}{2}} \frac{x^2 dx}{(S^2+x^2)^{3/2}}$$

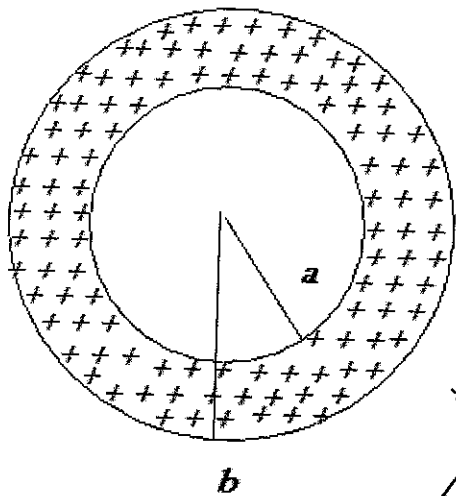
Note!  $\int_{-a/2}^{a/2} \text{even function} = 2 \int_0^{a/2} \text{even function} = 2 \int_{-a/2}^0 \text{even function}$

even function!  $f(x) = f(-x)$

$$\int_{-a/2}^{a/2} \text{odd function} = 0$$

odd function!  $f(x) = -f(-x)$


**Problem Two:** The figure below shows an insulating spherical shell with uniform volume charge density,  $\rho$ , inner radius  $a$ , and outer radius  $b$ . (a) In terms of the given information, what is the NET charge of the spherical shell? (b) Find an expression for the electric field for  $r < a$ . (c) Find an expression for the electric field for  $a < r < b$ . (d) Find an expression for the electric field for  $r > b$ . (e) Compute the electric field for  $r = b$ . (7 points for each part)

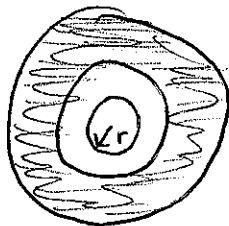


a.)  $Q = \rho V$

$$V = \frac{4}{3}\pi(b^3 - a^3)$$

$$Q_{\text{net}} = \rho \cdot \frac{4}{3}\pi(b^3 - a^3)$$

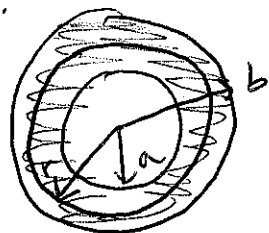
b.)  $r < a$ !



$$q_{\text{enc}} = 0$$

$$\therefore \vec{E} = 0 \hat{r} \quad r < a$$

c.)  $a < r < b$ !



$$EA_{\text{Gauss}} = \frac{q_{\text{enc}}}{\epsilon_0} \Rightarrow E = \frac{q_{\text{enc}}}{A \epsilon_0}$$

$$A_{\text{Gauss}} = 4\pi r^2$$

$$\rho_{\text{Gauss}} = \rho$$

$$\frac{q_{\text{enc}}}{4\pi r^2 \epsilon_0} = \rho \Rightarrow q_{\text{enc}} = \frac{4}{3}\pi \rho (r^3 - a^3)$$

$$\vec{E} = \frac{\frac{4}{3}\pi \rho (r^3 - a^3)}{4\pi r^2 \epsilon_0} \hat{r} = \frac{\rho (r^3 - a^3)}{3\epsilon_0 r^2} \hat{r}$$

d.)  $r > b$ !  $q_{\text{enc}} = q_{\text{net}} = \rho \frac{4}{3}\pi(b^3 - a^3)$   $E = \frac{q_{\text{enc}}}{A \epsilon_0} = \frac{\rho (b^3 - a^3)}{3\epsilon_0 r^2} \hat{r}$

e.)  $\vec{E}(r > b)|_{r=b} = \frac{\rho (b^3 - a^3)}{3\epsilon_0 b^2} \hat{r}$

**Problem Three: Conceptual Questions:** Please answer the following possibly unrelated questions, using a minimum of mathematics (6 points each, 30 points total).

a. Provide at least two examples illustrating how *electric forces* differ from *gravitational forces*.

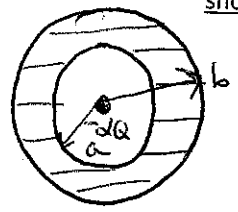
Electric forces can be attractive or repulsive. Gravitational forces only attractive. Electric forces much greater than gravitational forces.

b. A proton and an electron are separated by a distance  $r$ . If the charge is kept constant, for both charges, how, if at all does the electric force change, if the distance between the two given charges is tripled? Will the force be attractive or repulsive?

$$r \rightarrow 3r \Rightarrow F \rightarrow \frac{1}{9} F$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2}$$

c. A negative point charge,  $-\alpha Q$  (where  $\alpha$  is a dimensionless, positive constant so that overall the charge is negative), is placed at the center of a spherical CONDUCTING shell, which has a total charge of  $+2\alpha Q$ , and which consists of an inner radius  $a$  and an outer radius  $b$ . What surface charge density,  $\sigma$ , must reside on the outer radius,  $r = b$ , of the conducting shell? You must show your work to obtain full credit.



$$q_{r=a} + q_{r=b} = 2\alpha Q$$

Since  $E=0$ , for  $a < r < b$  (inside conductor),  
 $q_{enc} = 0$      $q_{enc} = -\alpha Q + q_{r=a} = 0$   
 $\Rightarrow q_{r=a} = \alpha Q$

$$q_{r=a} + q_{r=b} = 2\alpha Q \Rightarrow$$

$$\alpha Q + q_{r=b} = 2\alpha Q \text{ so } q_{r=b} = \alpha Q$$

d. For the information given in part c (above), what is the electric field equal to inside the conducting shell? Please justify your answer.

$$\sigma_b = \frac{q_{r=b}}{4\pi b^2} = \frac{\alpha Q}{4\pi b^2}$$

$E = 0$  inside conductor.

Electrostatic  $\Rightarrow \vec{a} = 0$      $F = m\vec{a} \Rightarrow \vec{F} = 0$

$\vec{E} = \vec{F}/q \Rightarrow \vec{E} = 0$  Gauss  $\leftrightarrow$   $q_{enc} = 0$ .

e. Assessment Questions: A guaranteed six points for answers you provide!

- ❖ Did you write your name and section number on the exam (if No, please do so now!)?
- ❖ What was the most challenging (aka: difficult) topic we studied so far? I hope so!
- ❖ What was the least challenging (aka: easiest) topic we studied so far? —
- ❖ What was most helpful in preparation for this exam? —
- ❖ What was least helpful in preparation for this exam? —
- ❖ What topics in the course are you most excited about? Everything!