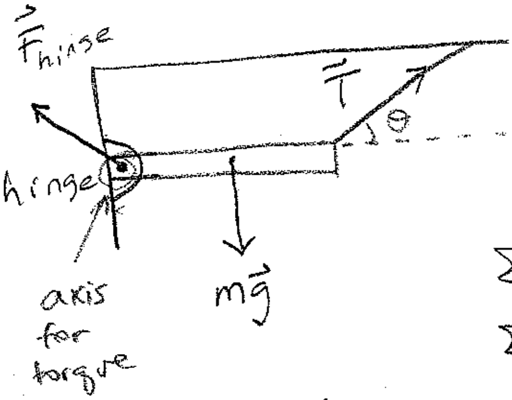
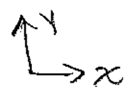
HW 6 Solution

1.)

a)



← Free Body Diagram



$$\Sigma F_y = T\sin\theta + F_{hinge,y} - mg = 0 \quad (1)$$

$$\Sigma F_x = T\cos\theta - F_{hinge,x} = 0 \quad (2)$$

b.) let l = beam length

$$\Sigma \tau_{hinge} = -\frac{l}{2} mg + lT\sin(180-\theta) = 0$$

$\begin{pmatrix} CCW = + \\ CW = - \end{pmatrix}$

$$\sin(180-\theta) = \underbrace{\sin 180}_{0}\cos\theta - \sin\theta\cos 180^\circ$$

$$\sin(180-\theta) = \sin\theta$$

so, $-\frac{\cancel{l}}{2} mg + \cancel{l}\, T\sin\theta = 0 \Rightarrow T = \frac{mg}{2\sin\theta}$

Substitute T into eqn (1):

$$\frac{mg}{2\cancel{\sin}\theta}\cdot \cancel{\sin}\theta + F_{hinge,y} - mg = 0 \Rightarrow F_{hinge,y} = \frac{mg}{2}$$

$\left(\cot\theta = \frac{\cos\theta}{\sin\theta}\right)$

Substitute T into eqn (2):

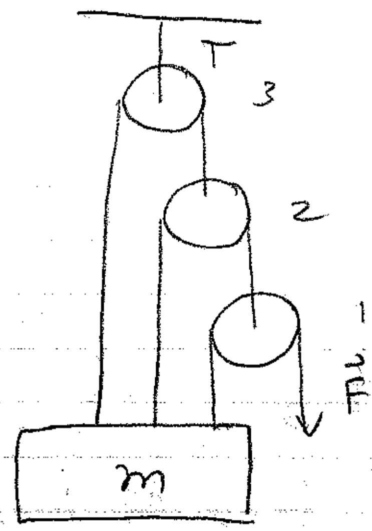
$$\frac{mg}{2\sin\theta}\cos\theta - F_{hinge,x} = 0 \Rightarrow F_{hinge,x} = \frac{mg}{2}\cot\theta$$

In $\hat{i}/\hat{j}$ notation

$$\boxed{\vec{F}_{hinge} = -\left(\frac{mg}{2}\cot\theta\right)\hat{i} + \frac{mg}{2}\hat{j}}$$

2)

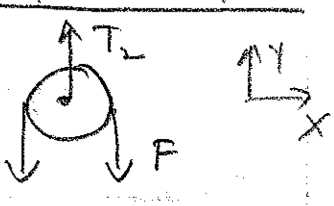
2.)



FBD for pulley 1

CCW = +

CW = −



$$\Sigma F_y = T_2 - T_1 - F = 0$$

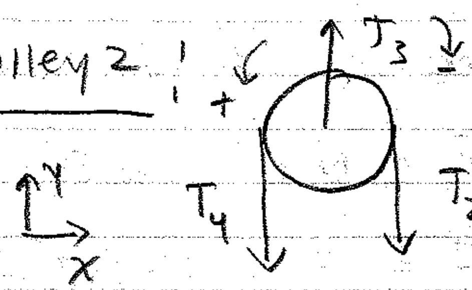
$$\Sigma \tau = rT_1 - rF = 0$$

abt pulley center

$$T_1 = F$$

$$\Rightarrow \quad T_2 = 2F$$

FBD for pulley 2:



$$\Sigma F_y = T_3 - T_2 - T_4 = 0$$

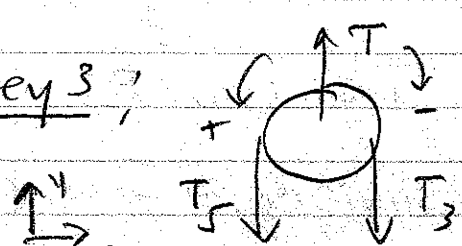
$$\Sigma \tau = T_4 r - T_2 r = 0$$

abt pulley center

$$T_4 = T_2 = 2F$$

$$T_3 = T_2 + T_4 = 4F$$

For Pulley 3:



$$\Sigma F_y = T - T_3 - T_5 = 0$$

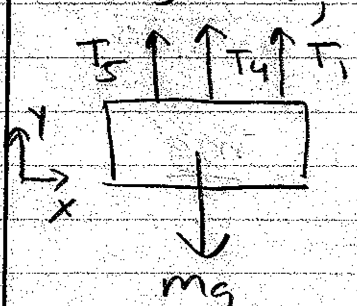
$$\Sigma \tau = T_5 r - T_3 r = 0$$

abt pulley center

$$T_5 = T_3 = 4F$$

$$T = T_3 + T_5 = 8F$$

To get F, do a FBD for block



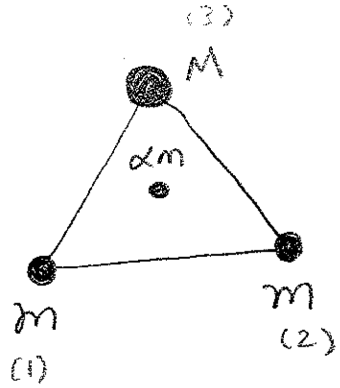
$$\Sigma F_y = T_5 + T_4 + T_1 - mg = 0$$

$$4F + 2F + F = mg$$

$$7F = mg \Rightarrow F = mg/7$$

So. $T = 8F = \boxed{(8mg)/7}$

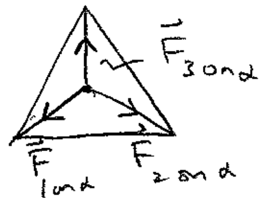
3



$$\vec{F}_{net,\alpha} = \vec{F}_{1\,on\,\alpha} + \vec{F}_{2\,on\,\alpha} + \vec{F}_{3\,on\,\alpha}$$

FBD for αM

$$|F_G| = G\frac{m_1 m_2}{r^2}$$



notice, from the symmetry, r is the same for $F_{1\,on\,\alpha}$, $F_{2\,on\,\alpha}$, $F_{3\,on\,\alpha}$

Also, from the FBD we see that the net direction of F must be downward ($F_{1\,on\,\alpha,x} = -F_{2\,on\,\alpha,x}$)

$$\vec{F}_{net,\alpha} = \left(-F_{1\,on\,\alpha,y} - F_{2\,on\,\alpha,y} + F_{3\,on\,\alpha}\right)\hat{j}$$

to set y-component

$$\vec{F}_{net,\alpha} = \left(-\frac{Gm(\alpha m)}{r^2}\cos 60^\circ - \frac{Gm(\alpha m)}{r^2}\cos 60^\circ + \frac{G\alpha m M}{r^2}\right)\hat{j} \quad (1)$$

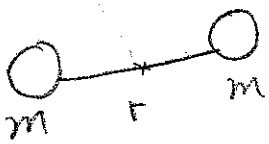
Given, $\vec{F}_{net,\alpha} = 0$, so

$$-\frac{\cancel{G}\cancel{\alpha}m^{\cancel{2}}}{\cancel{r^2}}\left(\frac{1}{2}\right) - \frac{\cancel{G}\cancel{\alpha}m^{\cancel{2}}\left(\frac{1}{2}\right)}{\cancel{r^2}} + \frac{\cancel{G\alpha m}M}{\cancel{r^2}} = 0$$

$$\Rightarrow \boxed{m = M}$$

b.) As we can see from Equation 1, if $\vec{F}_{net,\alpha} = 0$, doubling αm will have no effect on $F_{net,\alpha}$. The αm is contained within all three terms and cancels. Thus, $\boxed{\vec{F}_{net,\alpha} = \vec{0}}$

4.



$m = 1.99 \times 10^{30}\,\mathrm{kg}$ $\quad r = 1.50 \times 10^{11}\,\mathrm{m}$

$$|F_G| = \frac{G m \cdot m}{r^2} = \frac{G m^2}{r^2}$$

$$r_{cm} = \frac{m(0) + m(r)}{2m} = \frac{r}{2}$$

$$v = \frac{2\pi r_{cm}}{T} = \frac{\cancel{2}\pi (r/\cancel{2})}{T} =$$

$$\sum F = ma = \frac{mv^2}{r_{cm}} = \frac{2mv^2}{r} \qquad v = \frac{2\pi r_{cm}}{T} \Rightarrow \frac{v^2}{r} = \frac{\pi^2 r^{\cancel{2}}}{\cancel{r} T^2}$$

period (time of one rev.)

$$r_{cm} = \frac{r}{2} = 0.75 \times 10^{11}\,\mathrm{m}$$

$$\Rightarrow \frac{G m^{\cancel{2}}}{r^2} = \frac{2 \cancel{m} \pi^2 r}{T^2} \Rightarrow T^2 = \frac{2\pi^2 r^3}{Gm}$$

$$T = \left(\frac{2\pi^2 r^3}{Gm}\right)^{1/2} = \pi \left(\frac{2r^3}{Gm}\right)^{1/2} = \pi \left(\frac{2(1.5 \times 10^{11}\,\mathrm{m})^3}{6.67 \times 10^{-11}\,\frac{\mathrm{Nm^2}}{\mathrm{kg^2}} \times 1.99 \times 10^{30}\,\mathrm{kg}}\right)^{1/2}$$

$$T = 2.2403 \times 10^7\,\mathrm{s} \times \frac{1\,\mathrm{min}}{60\,\mathrm{s}} \times \frac{1\,\mathrm{hr}}{60\,\mathrm{min}} \times \frac{1\,\mathrm{day}}{24\,\mathrm{hr}} \times \frac{1\,\mathrm{yr}}{365\,\mathrm{days}}$$

$$\boxed{T = 0.710\,\mathrm{yr}}$$

BONUS: On the side of the Earth facing the moon there is an oceanic bulge. On the side opposite from the moon there is another oceanic bulge. These oceanic bulges move in relation to an orbiting satellite which causes (in most places) two high tides a day as Earth rotates on its axis. For the near side oceanic bulge, the moon's gravity pulls the ocean toward itself.