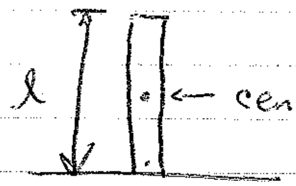
Solution IS8

1.



$\leftarrow$ center of mass @ $\frac{l}{2}$ (potential energy of stick abt. center of mass $= mg\frac{l}{2}$)

$$U_1 + K_1 = U_2 + K_2$$

$$mg\frac{l}{2} + 0 = 0 + \frac{1}{2} I \omega^2$$

From the parallel axis theorem and using the table (10-2) on p. 253, we know

$$I = I_{cm} + \frac{ml^2}{4} = \frac{1}{12}ml^2 + \frac{ml^2}{4} = \frac{1}{3}ml^2$$

$$\Rightarrow \cancel{m}g\frac{\cancel{l}}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{3}\cancel{m}l^{\cancel{2}}\right)\omega^2 \Rightarrow$$

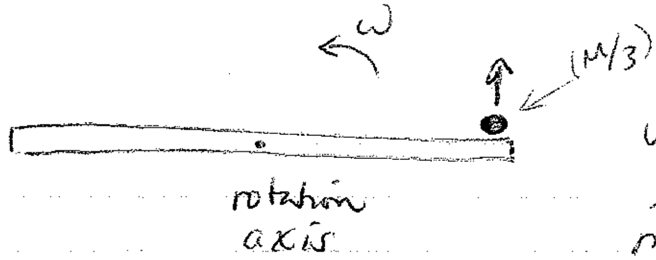
$$\omega^2 = \frac{3g}{l} \Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

The free end of the stick is a distance l from the rotation axis so,

$$v = \omega l = \left(\sqrt{\frac{3g}{l}}\right) l = \sqrt{3gl} \qquad l = 1m \text{ (meter stick)}$$

So, $\boxed{v = \sqrt{3g}}$ OR (5.42 m/s).

2.)



Uniform thin rod
length = l
mass = M
ω

No External torques, Angular Momentum is Conserved.

$$L_0^{net} = L_f^{net}$$

$$L_0^{net} = I_{rod}\,\omega + I_{\substack{point\\mass}}\,\omega = \frac{1}{12}Ml^2\omega + \frac{M}{3}\left(\frac{l}{2}\right)^2\omega$$

$$L_0^{net} = \frac{1}{12}Ml^2\omega + \frac{Ml^2}{(12)}\omega = \frac{1}{\underset{(6)}{6}}Ml^2\omega \qquad \left(\underbrace{L^{point\ mass} = rmv}\right)$$

$$L_f^{net} = L_f^{rod} + L_f^{point\ mass} = \frac{1}{12}Ml^2\omega_f + \frac{l}{2}\left(\frac{M}{3}\right)v_p$$

$$L_f^{net} = \frac{Ml^2}{12}\omega_f + \frac{lM}{6}v_p$$

Given: $v_p = \omega_f r + a \Rightarrow v_p = \omega_f \frac{l}{2} + a$

$$\Rightarrow \omega_f = \frac{v_p - a}{\frac{l}{2}} \qquad \text{so,}$$

$$L_f^{net} = \frac{Ml^2}{12}\left(\frac{v_p - a}{\frac{l}{2}}\right) + \frac{lMv_p}{6} = L_0 = \frac{1}{\underset{(6)}{6}}Ml^2\omega$$

$$\frac{Ml}{6}(v_p - a) + \frac{Mlv_p}{6} = \frac{1}{\underset{(6)}{6}}\cancel{M}l^{\cancel{2}}\omega$$

$$\frac{v_p}{3} = \frac{l\omega}{6} + \frac{a}{6} \Rightarrow v_p = 3\left(\frac{l\omega + a}{6}\right)$$

$$\boxed{v_p = \left(\frac{l\omega + a}{2}\right)}$$