[redacted]

37.75. **(a)** $f_0 = 4.568110\times10^{14}$ Hz; $f_+ = 4.568910\times10^{14}$ Hz; $f_- = 4.567710\times10^{14}$ Hz

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$$\left.\begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}}\,f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}}\,f_0 \end{aligned}\right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2\,(c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2-1}{(f_+/f_0)^2+1}c \text{ and } (u-v) = \frac{(f_-^2/f_0^2)-1}{(f_-^2/f_0^2)+1}c$$

$$\Rightarrow u+v = 5.25\times10^4 \text{ m/s and } u-v = -2.63\times10^4 \text{ m/s}.$$

This gives $u = +1.31\times10^4$ m/s (moving toward at 13.1 km/s) and $v = 3.94\times10^4$ m/s.

(b) $v = 3.94\times10^4$ m/s; $T = 11.0$ days. $2\pi R = vt \Rightarrow$

$$R = \frac{(3.94\times10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} = 5.96\times10^9 \text{ m}.$$ This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96\times10^9 \text{ m})(3.94\times10^4 \text{ m/s})^2}{6.672\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2} = 5.55\times10^{29} \text{ kg} = 0.279\ m_{\text{sun}}.$$

37.76. For any function $f = f(x,t)$ and $x = x(x',t'), t = t(x',t')$, let $F(x',t') = f(x(x',t'),t(x',t'))$ and use the standard (but mathematically improper) notation $F(x',t') = f(x',t')$. The chain rule is then

$$\frac{\partial f(x',t')}{\partial x} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial x},$$
$$\frac{\partial f(x',t')}{\partial t} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the above relations are then $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x}$, $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial t}$.

(a) $\frac{\partial x'}{\partial x} = 1, \frac{\partial x'}{\partial t} = -v, \frac{\partial t'}{\partial x} = 0$ and $\frac{\partial t'}{\partial t} = 1$. Then, $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}$, and $\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$. For the time derivative, $\frac{\partial E}{\partial t} = -v\frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is,

$$\frac{\partial}{\partial t}\frac{\partial E}{\partial x'} = -v\frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t'\partial x'},$$
$$\frac{\partial}{\partial t}\frac{\partial E}{\partial t'} = -v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}.$$

Using these in $\frac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$\frac{\partial^2 E}{\partial t^2} = v^2\frac{\partial^2 E}{\partial x'^2} - 2v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}$, and using this and the above expression for $\frac{\partial^2 E}{\partial x'^2}$ gives the result.

(b) For the Lorentz transformation, $\frac{\partial x'}{\partial x} = \gamma, \frac{\partial x'}{\partial t} = \gamma v, \frac{\partial t'}{\partial x} = \gamma v/c^2$ and $\frac{\partial t'}{\partial t} = \gamma$.

The first partials are then

$$\frac{\partial E}{\partial x} = \gamma\frac{\partial E}{\partial x'} - \gamma\frac{v}{c^2}\frac{\partial E}{\partial t'}, \frac{\partial E}{\partial t} = -\gamma v\frac{\partial E}{\partial x'} + \gamma\frac{\partial E}{\partial t'}$$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{v^2}{c^4}\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2\frac{v}{c^2}\frac{\partial^2 E}{\partial x'\partial t'}$$
$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v\frac{\partial^2 E}{\partial x'\partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),