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2	
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NAME (please include last name): _____
 SECTION NUMBER: _____

SAMPLE EXAM III
 FEBRUARY 2008
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 CLOSED BOOK EXAM

PLEASE ANSWER ALL THREE QUESTIONS IN THE SPACE PROVIDED. QUESTIONS ONE AND TWO ARE WORTH 35 POINTS, EACH. QUESTION THREE IS WORTH 30 POINTS. PLEASE BE SURE TO SHOW ALL WORK AND JUSTIFY ALL YOUR ANSWERS. GRADING WILL BE BASED ON EVIDENCE OF YOUR UNDERSTANDING OF THE BASIC CONCEPTS AND PRINCIPLES—PLEASE PROVIDE THOROUGH SOLUTIONS TO THE PROBLEMS. GOOD LUCK!

Possibly Useful Information:

- $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$
- $m_e = \text{mass of electron} = 9.11 \times 10^{-31} \text{ kg}$
- $q_p = \text{charge of proton} = 1.6 \times 10^{-19} \text{ C}$
- $F = (1/4\pi\epsilon_0)(q_1 q_2/r^2) = k(q_1 q_2/r^2)$
- $h = 6.626 \times 10^{-34} \text{ Js}$
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
- $\sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \text{ K}^4)$
- $x = x' + ut, y' = y, z' = z$
- $l = l_0/\gamma = l_0(1 - (u^2/c^2))^{1/2}$
- $t' = \gamma(t - (ux/c^2))$
- $p = \gamma mv$
- $F = \gamma ma$ (F & v perpendicular)
- $E^2 = (mc^2)^2 + (pc)^2$
- $1/2 mv^2 = eV_0 = hf - \phi$
- $R = 1.097 \times 10^7 \text{ m}^{-1}$
- $E_n = (-hcR)/n^2$
- Paschen Series: $(1/\lambda) = R(1/3^2 - 1/n^2)$ $n = 4, 5, 6, \dots$
- Brackett Series: $(1/\lambda) = R(1/4^2 - 1/n^2)$ $n = 5, 6, 7, \dots$
- L_n = $mv_n r_n = nh/(2\pi)$ $r_n = \epsilon_0 [(nh)^2 / (\pi m e^2)]$ $v_n = e^2 / (2nh \epsilon_0)$
- $eV_{AC} = hf_{max} = (hc) / \lambda_{min}$
- $\lambda' - \lambda = [h/(mc)](1 - \cos \phi)$
- $\lambda_m T = 2.90 \times 10^{-3} \text{ m K}$
- $q_e = \text{charge of electron} = 1.6 \times 10^{-19} \text{ C}$
- $m_p = \text{mass of proton} = 1.67 \times 10^{-27} \text{ kg}$
- $c = 3.00 \times 10^8 \text{ m/s}$
- $E = k(q/r^2) = (1/4\pi\epsilon_0)(q/r^2)$
- $E = F/q$
- $k = 1.381 \times 10^{-23} \text{ J/K}$
- $\gamma = (1 - (u^2/c^2))^{-1/2}$
- $\Delta t = \gamma(\Delta t_0) = (\Delta t_0) / (1 - (u^2/c^2))^{1/2}$
- $x' = \gamma(x - ut), y' = y, z' = z$
- $v_x' = (v_x + u) / (1 + (u v_x / c^2))$
- $F = \gamma^3 ma$ (F & v along same line)
- $E = K + mc^2 = \gamma(mc^2)$ $K = (\gamma - 1)(mc^2)$
- $(pc)^2 = K^2 + 2K(mc^2)$ $E = hf = h(c/\lambda)$
- $P = E/c = (hf)/c = h/\lambda$ $hf = E_i - E_f$
- Balmer Series: $(1/\lambda) = R(1/2^2 - 1/n^2)$ $n = 3, 4, 5, \dots$
- Lyman Series: $(1/\lambda) = R(1/1^2 - 1/n^2)$ $n = 2, 3, 4, \dots$
- $I = \sigma T^4$
- $I(\lambda) = (2\pi^5 hc^2) / (\lambda^5) [e^{hc/\lambda kT} - 1]^{-1}$

$d(\sin \theta) = m\lambda$ $\Delta x \Delta p \geq \hbar$ $\Delta E \Delta t \geq \hbar$
 $\Psi(x, y, z) = \psi(x, y, z)e^{-iEt/\hbar}$ $e^{i\theta} = \cos \theta + i \sin \theta$
 $-\hbar^2/2m(d^2 \psi(x)/dx^2) + U(x)\psi(x) = E\psi(x)$ $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$
 $\psi(x) = Ae^{ikx}$ $E = (\hbar k)^2/2m$ $E_n = (n\hbar)^2/(2mL^2)$
 $\psi(x) = \sqrt{2/L} \sin[(n\pi x)/L]$ $E_\infty = (\pi\hbar)^2/(2mL^2)$ $\hbar \omega = \hbar \sqrt{k^2/m}$
 $E_n = (n + 1/2)\hbar \omega$ $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$
 $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ $\int \sin u du = -\cos u + C$
 $\int \cos u du = \sin u + C$ $i = \sqrt{-1}$ $C = \text{integration constant}$
 $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

$$L = \sqrt{l(l+1)} \hbar$$

$$L_z = m_l \hbar$$

$$P(r) dr = |\psi|^2 dV = |\psi|^2 4\pi r^2 dr$$

$$a = \frac{4\pi\epsilon_0 \hbar^2}{m_r e^2}$$

$$\vec{\mu} = I \vec{A}$$

$$\vec{\mu} \cdot \vec{B} = U$$

$$\mu_B = \frac{e \hbar}{2m}$$

$$U = -\mu_z B = m_l \frac{e \hbar}{2m} B$$

$$S_z = \pm \frac{1}{2} \hbar$$

$$S = \sqrt{\frac{1}{2}(\frac{1}{2}+1)} \hbar = \sqrt{\frac{3}{4}} \hbar$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$J = \sqrt{j(j+1)} \hbar$$

$$E_n = -\frac{Z_{\text{eff}}^2 (13.6 \text{ eV})}{n^2}$$

$$E_l = \frac{l(l+1) \hbar^2}{2I}$$

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

$$I = m_r r_0^2$$

$$E_n = (n + \frac{1}{2}) \hbar \omega = (n + \frac{1}{2}) \hbar \sqrt{\frac{k'}{m_r}}$$

$$g(E) = \frac{(2m)^{3/2} \sqrt{E}^{1/2}}{2\pi^2 \hbar^3}$$

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$E_{F0} = \frac{3^{2/3} \pi^{4/3} \hbar^2 n^{2/3}}{2m}$$

$$E_{F0} = \frac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3}$$

$$n \geq 1$$

$$|m_l| \leq l$$

$$0 \leq l \leq n-1$$

$$m_s = \pm \frac{1}{2}$$

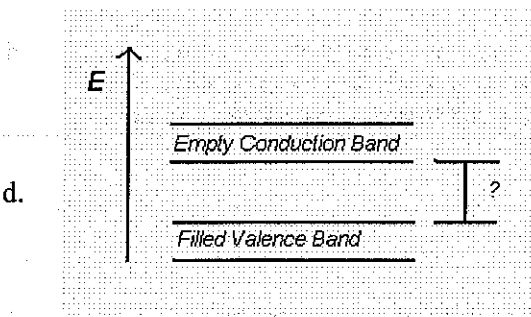
$$j = |l \pm \frac{1}{2}|$$

Problem One (35 points): An electron is in a state with $n = 3$. (a) What are the number of possible values of ℓ ? (b) What are the number of possible values of m_ℓ ? (c) What are the number of possible values of m_s ? (d) What are the number of states in the $n = 3$ shell? (e) What are the number of subshells in the $n = 3$ shell? (7 points each)

Problem Two (35 points): (a) In the Free Electron Model of Metals, write down the equation that describes the probability that an energy state of energy E is occupied. (b) For a solid having a Fermi energy of E_F eV, what is the probability, at temperature τ (in Kelvin), that a state having an energy of E_F eV is occupied by an electron? (c) For an energy state having energy above the Fermi Energy, at temperature τ (in Kelvin), explain how the probability that an energy state of $E > E_F$ differs (if at all) from your result of part (b). (d) For an energy state having energy below the Fermi Energy, at temperature τ (in Kelvin), explain how the probability that an energy state of $E < E_F$ differs (if at all) from your result of part (b). (e) As temperature increases, explain what happens to the probability of finding electrons occupying states $E > E_F$. (7 points each)

Problem Three: Conceptual Questions: Please answer the following possibly unrelated questions, using a minimum of mathematics (6 points each, 30 points total).

- a. Briefly describe the Zeeman Effect.
- b. An atom in a state with $\ell = 1$ emits a photon with wavelength λ nm as it decays to a state with $\ell = 0$. If the atom is placed in a magnetic field with magnitude B T, determine the shifts in energy levels resulting from the interaction of the magnetic field and the atom's orbital magnetic moment.
- c. A Hydrogen atom in a particular orbital angular momentum state is found to have j quantum numbers $7/2$ and $9/2$. What is the letter that labels the value of ℓ for that state?



For the figure shown to the left, what is the difference in energy between the conduction band and valence band called? What type of material has such an energy structure?

- e. **Assessment Questions:** A guaranteed six points for answers you provide! Please be sure that your name and section number appear on the first page!
- ❖ What was the most challenging (aka: *difficult*) topic we studied?
 - ❖ What was the least challenging (aka: *difficult*) topic we studied?
 - ❖ What was most helpful, in preparation for this exam?
 - ❖ What was least helpful, in preparation for this exam?
 - ❖ On a scale of 1-5, 5 = identical, 3 = neutral, and 1 = very different, how would you rate the sample exam with the actual exam?
 - ❖ What did you like most about our course? Why?