PH 1121: Supplementary Notes on Inductors and Maxwell's Equations

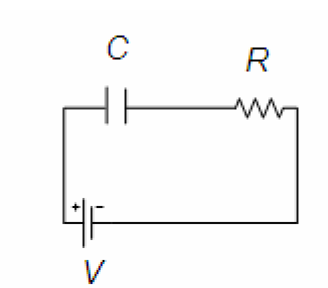
I: Inductors

A. Comparing an Inductor with a Water Wheel*

One way to visualize the action of an inductor is to imagine a narrow channel with water flowing through it, and a heavy water wheel that has its paddles dipping into the channel. Imagine that the water in the channel is not flowing initially. Now you try to start the water flowing. The paddle wheel will tend to prevent the water from flowing until it has come up to speed with the water. If you try to stop the flow of water in the channel, the spinning water wheel will try to keep the water moving until its speed of rotation slows back down to the speed of the water. An inductor is doing the same thing with the flow of electrons in a wire -- an inductor resists a change in the flow of electrons.

B. Comparison between RC Circuits and RL Circuits:

RC Circuit



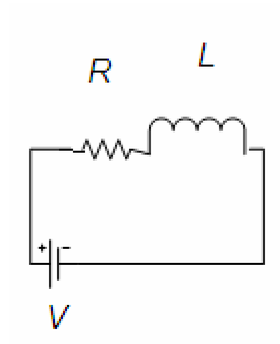
By Loop Rule:
$V - q/C - R\,(dq/dt) = 0$

$(V/R) - (1/RC)q - (dq/dt) = 0$

$q = cV(1 - e^{-t/RC})$

time constant = RC
Stored energy = $\frac{1}{2}(q^2/C)$

RL Circuit



By Loop Rule:
$V - iR - L\,(di/dt) = 0$

$(V/L) - (R/L)i - (di/dt) = 0$

$i = (V/R)[1 - e^{-t(R/L)}]$

time constant = L/R
Stored energy = $\frac{1}{2} Li^2$

*Adopted from: http://www.howstuffworks.com/inductor.htm

II: Maxwell's Equations: The unification of electricity and magnetism

A. Integral Form

1. Gauss' Law for Electricity: $\iint_{\text{closed surface}} (\mathbf{E} \cdot d\mathbf{A}) = q_{enc}/\varepsilon_o$
2. Gauss' Law for Magnetism: $\iint_{\text{closed surface}} (\mathbf{B} \cdot d\mathbf{A}) = 0$ (No Magnetic Monopoles)
3. Ampere's Law: $\int_{\text{closed path}} (\mathbf{B} \cdot d\mathbf{s}) = \mu_o\, i_{enc} + \mu_o\varepsilon_o\, (d\Phi_E/dt)$
4. Faraday's Law: $\int_{\text{closed path}} (\mathbf{E} \cdot d\mathbf{s}) = -(d\Phi_B/dt)$

Notes: Φ_E = electric flux = $\iint (\mathbf{E} \cdot d\mathbf{A})$, and Φ_B = magnetic flux = $\iint (\mathbf{B} \cdot d\mathbf{A})$.
$\varepsilon_o\, (d\Phi_E/dt)$ = displacement current (Maxwell modified Ampere's Law)

B. Differential Form

1. Gauss' Law for Electricity: $\nabla \cdot \mathbf{E} = (1/\varepsilon_o)\, \rho$
2. Gauss' Law for Magnetism: $\nabla \cdot \mathbf{B} = 0$
3. Ampere's Law: $\nabla \times \mathbf{B} = \mu_o\, \mathbf{J} + (\mu_o\varepsilon_o)\, (\partial \mathbf{E}/\partial t)$
4. Faraday's Law: $\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t)$

Notes: ∇ = gradient operator (called 'del')

$\nabla = (\partial/\partial x)\, \mathbf{i} + (\partial/\partial y)\, \mathbf{j} + (\partial/\partial z)\, \mathbf{k}$ (Cartesian Coordinates)

$\nabla \cdot \mathbf{F}$ = divergence of $\mathbf{F} = (\partial F/\partial x) + (\partial F/\partial y) + (\partial F/\partial z)$

curl of F =
$$\vec{G} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Where:

$$G_x = \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \quad G_y = \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \quad G_z = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

In regions of space where there is no charge or current, we may set $\rho = 0$ (equation 1) and $J = 0$ (equation 3).

The above equations, either in differential or integral form, represent a set of coupled, first-order, partial differential equations for **E** and **B**. They can be *de*coupled by applying the curl to equations 3 and 4 (above). When you decouple the equations, you unravel a second-order, partial differential equation called the wave equation (classical). According to Maxwell's equations, empty space supports the propagation of electromagnetic waves at a speed = $(1/\mu_o\varepsilon_o)^{1/2} = 3 \times 10^8$ m/s (the speed of light)!