Photoelectric Effect

for a given metal there exists a minimum frequency (threshold frequency), ~~and above~~ below which no electrons are emitted. As frequency increases, energy of electrons that are emitted increases linearly according to Planck's constant

Energy of a photon

$E = hf$ h = Planck's constant

Binding energy; work function

$W = hf_0$ f_0 = threshold frequency - depends on the metal

- If energy of a photon exceeds the threshold frequency, an electron will be knocked off
- some energy will be left as kinetic energy

$$\left.\begin{aligned} hf &= hf_0 + K \\ K &= hf - hf_0 \end{aligned}\right\} \text{ conservation of energy}$$

if $f < f_0$, nothing happens

$I = \frac{J}{\frac{s}{m^2}}$ number of particles per unit time per meters squared angular momentum of an electron is a multiple of $\frac{h}{2\pi}$

Classical Mechanics

$$\frac{m_e v^2}{r} = \frac{ke^2}{r^2} \qquad k = \frac{1}{4\pi\varepsilon_0} \qquad e = \text{charge}$$

$$v = \sqrt{\frac{ke^2}{m_e r}}$$

can ignore relativity - objects moving less than 1/10 speed of light

$$E = \frac{1}{2}m_e v^2 - \frac{ke^2}{r} = -\frac{ke^2}{2r}$$

Quantum Mechanics

$L = rmv$ (moving in a circle)

$m_e r v = n\hbar$ $\quad L$ = integer multiple of $\hbar$ $\left(\frac{h}{2\pi}\right)$

allowed orbit radius at any n is:

$$r_n = \frac{n^2\hbar^2}{ke^2 m_e}$$

smallest possible value

$$r_1 = \frac{\hbar^2}{(ke^2)m_e} = 0.529 \times 10^{-10}\ m$$

Energy of nth level

$$E = \frac{-ke^2}{2r_n} = -\frac{(ke^2)m_e}{2\hbar^2 n^2} = \frac{-13.6\,eV}{n^2}$$

Energy of a certain level is the amount of energy needed to just barely release it - binding energy

Energy needed to jump levels is difference between the two energy levels

Perfect constructive

$$\Delta x = n\lambda$$

Perfect destructive

$$\Delta x = \left(n + \frac{1}{2}\right)\lambda$$

For an electron

$$K = e\Delta V$$

$$\lambda = \frac{h}{mv}$$

$$v = \sqrt{\frac{2K}{m}}$$

$$\lambda = \frac{h}{\sqrt{2Km}}$$

Relativistic Electrons

$$\lambda = \frac{h}{\gamma mv}$$

$$E = K + mc^2$$

$$p = \sqrt{\frac{E^2}{c^2} - (mc)^2}$$

Probability Interpretation

$$\Psi \sim 2\Psi_0 \cos\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$I = 4I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Maxima

$$\sin\theta = 0,\ 1.63\frac{\lambda}{D},\ 2.68\frac{\lambda}{D}, \ldots$$

Minima

$$\sin\theta = 1.22\frac{\lambda}{D},\ 2.23\frac{\lambda}{D},$$

Beat frequency is $\frac{1}{\text{Time from one beat to another}}$

$$\frac{\Delta\omega t}{2} = \pi$$

$$f_{beat} = \frac{1}{\Delta t} = \frac{\Delta\omega}{2\pi}$$

$$f_{beat} = f_2 - f_1$$

$$f_{tone} = \frac{f_2 + f_1}{2}$$

$$V_{wave} = V_{phase} = \frac{\omega}{K} \Big\} \text{individual waves}$$

$$V_{group} = \frac{\Delta\omega}{\Delta K}$$

For light in a vacuum

$$V_{beat} = \frac{\Delta\omega}{\Delta t} = c$$

Exam 1

I. Relativity
- length contraction
- time dilation
- time synchronization
- Galilean transformation
- Lorentz transformation
- Doppler shift
- Gravitational shift (elevator problems)
- Relativistic momentum
- Relativistic Kinetic energy
- Relativistic energy

II. Atomic Physics
- constructive/destructive interference
- particle/wave duality
- photoelectric effect
- intensity of propagated waves
- Compton Scattering
- Hydrogen atom (energy levels, radii)
- Bohr Quantization (angular momentum)
- energy of photon
- absorption/emission

Single Slit Interference

$$\Delta\varphi = ka\sin\theta = \frac{2\pi a\sin\theta}{\lambda}$$

$\Delta\varphi$ = phase difference at screen between rays coming from extreme ends of single slit

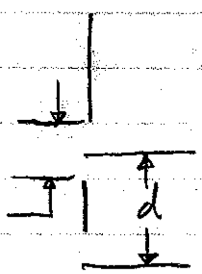
Intensity of wave on screen: $I = I_0 \sin^2\left(\frac{\pi a\sin\theta}{\lambda}\right)$

After central maxima ($\Delta\varphi = 0$), max intensity occurs at $\frac{\Delta\varphi}{2} = \left(n + \frac{1}{2}\right)\pi$ <u>or</u> $a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$

Minimum intensity $\Rightarrow \frac{\Delta\varphi}{2} = n\pi$ <u>or</u> $a\sin\theta = n\lambda$

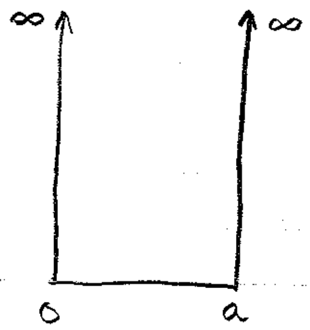
Double Slit with Finite width Slits

Slit width = a
Center to center distance = d



$$\text{Intensity} = I = I_0 \frac{\sin^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\left(\frac{\pi a\sin\theta}{\lambda}\right)^2} \cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

1D Particle in a box



Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$K = \frac{n\pi}{a}$$

$$\psi(x) = c\sin\left(\frac{n\pi}{a}x\right)$$

$$E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

Ground state: $n=1$
1st excited level: $n=2$
2nd excited level: $n=3$

$C = \sqrt{\frac{2}{a}}$ since $\int_0^a \psi^2\, dx = 1$

Bosons: Particles that mediate forces between other particles
Photons, gluons, W and Z particles, graviton

Fermions - massive constituents of matter
quarks, leptons

Fermions have wave (distinguishable) functions which obey the following $\psi(x_1, x_2) = -\psi(x_2, x_1)$

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\psi_a(x_1)\psi_b(x_2) - \psi_a(x_2)\psi_b(x_1)\right)$$

*Bosons have wave functions (indistinguishable) which obey the following: $\psi(x_1, x_2) = +\psi(x_2, x_1)$

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\psi_a(x_1)\psi_b(x_2) + \psi_a(x_2)\psi_b(x_1)\right)$$

Space	Time
very small $\Rightarrow$ quantum mechanics	Very fast $\Rightarrow$ relativity

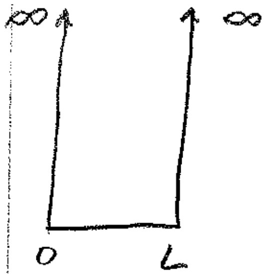
What makes Quantum Mechanics Different?

- Particle - wave duality
- Size of the material you're measuring
- Heisenberg Uncertainty Principle: $\Delta x \Delta p \geq \frac{\hbar}{2}$
- Relative strengths of forces

Classical	Quantum
- conservation of energy - $E = PE + KE$	- $E = PE + KE$ - $KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mv \cdot mv}{m}$ $KE = \frac{p^2}{2m} = \frac{1}{2m}\left(-\hbar^2 \frac{d^2}{dx^2}\right)$ $(PE + KE)\psi = E\psi$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}}_{\text{Kinetic Energy}} + \underbrace{V_{(x)}\psi}_{\text{Potential Energy}} = \underbrace{E}_{\text{Total Energy}}\psi$$

Example: 1-D Infinite Square Well



$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow E\psi \Rightarrow \underbrace{\boxed{\frac{2mE}{\hbar^2}}}_{=K^2}\psi$$

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0$$

Much like block spring, electron oscillates in the square well

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \Rightarrow \psi = A\sin kx$$

Get $A \Rightarrow \int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$$\int_0^L A^2 \sin^2 kx \, dx = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Probability of finding a particle in the region from x_1 to x_2

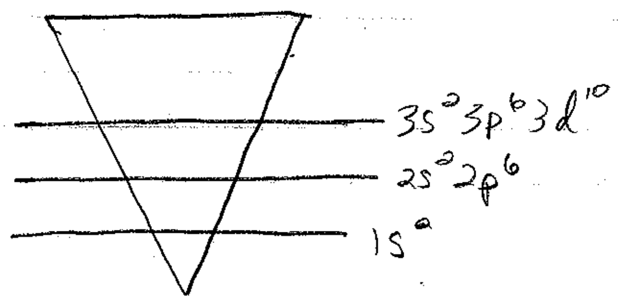
$$\int_{x_1}^{x_2} |\psi|^2 dx$$

Fermions: obey Pauli Exclusion principle - no 2 electrons can occupy the same state (have same 4 quantum #s)

Bosons: Very social - can have as many as you want (photons...)

examples: electrons, protons

Where atoms Live



Quantum #s

$n \rightarrow$ energy level
$l \rightarrow$ orbital quantum # (azimuthal number)
$m_s \rightarrow$ spin $(\frac{1}{2}$ or $-\frac{1}{2})$

Hydrogen ~~Atom~~

$$\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V\Psi = E\Psi$$

$$V = \frac{-e^2}{4\pi \varepsilon_0 r}$$

$$E = \frac{-13.6\,eV}{n^2} \longrightarrow \text{ground level } -13.6\,eV$$

for other atoms $E = \frac{(-13.6\,eV)}{n^2} \cdot Z^2$

$Z \Rightarrow$ protons

3 Bosons

$E_0 + E_0 + E_0 = 3E_0$

3 Fermions (all same spin)

$E_0 + 4E_0 + 9E_0 = 14E_0$

3 Fermions (2 up, 1 down)

$E_0 + E_0 + 4E_0 = 6E_0$