

Loop abcda:

$$V - \frac{Q}{C} - IR = 0$$

$$I = \frac{dQ}{dt} \Rightarrow V - \frac{Q}{C} - R \frac{dQ}{dt} = 0$$

$$\frac{V}{R} - \frac{Q}{RC} - \frac{dQ}{dt} = 0 \Rightarrow \frac{V}{R} - \frac{Q}{RC} = \frac{dQ}{dt}$$

$$dt = \frac{dQ}{\frac{V}{R} - \frac{Q}{RC}} \Rightarrow \int_0^t dt = \int_{Q_0}^Q \frac{dQ}{\frac{V}{R} - \frac{Q}{RC}} \Rightarrow t = \int_{Q_0}^Q \frac{dQ}{\frac{V}{R} - \frac{Q}{RC}}$$

Let $u = \frac{V}{R} - \frac{Q}{RC}$, then $du = -\frac{1}{RC} dQ \Rightarrow dQ = -RC du$

$$t = \int \frac{-RC du}{u} = -RC \ln u = -RC \ln \left| \frac{V}{R} - \frac{Q}{RC} \right| \Big|_{Q_0}^Q$$

$$-\frac{t}{RC} = \ln \left| \frac{V}{R} - \frac{Q}{RC} \right| - \ln \left| \frac{V}{R} - \frac{Q_0}{RC} \right| = \ln \left| \frac{\frac{V}{R} - \frac{Q}{RC}}{\frac{V}{R} - \frac{Q_0}{RC}} \right|$$

$$\ln a - \ln b = \ln \frac{a}{b}$$

$$\Rightarrow -\frac{t}{RC} = \ln \left| \frac{VC - Q}{VC - Q_0} \right| \quad e^{\ln x} = x \Rightarrow$$

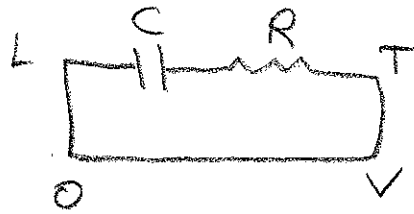
$$e^{-t/RC} = \frac{VC - Q}{VC - Q_0} \Rightarrow (VC - Q_0) e^{-t/RC} = VC - Q$$

$$Q = VC - (VC - Q_0) e^{-t/RC} \quad \text{let } Q_0 = 0$$

$$Q = VC - VC e^{-t/RC} = VC (1 - e^{-t/RC})$$

$$\boxed{Q = VC (1 - e^{-t/RC})} \quad \text{Charging of capacitor.}$$

Capacitor Discharging



Loop VOLTAGE: $-\frac{Q}{C} - IR = 0$ $I = \frac{dQ}{dt} \Rightarrow$

$$-\frac{Q}{C} - R \frac{dQ}{dt} = 0 \Rightarrow -\frac{Q}{RC} = \frac{dQ}{dt}$$

$$\Rightarrow -\frac{1}{RC} dt = \frac{dQ}{Q} \Rightarrow -\frac{1}{RC} \int_0^t dt = \int_{Q_0}^Q \frac{dQ}{Q}$$

$$-\frac{1}{RC} t = \ln Q \Big|_{Q_0}^Q = \ln Q - \ln Q_0 = \ln \left| \frac{Q}{Q_0} \right| \Rightarrow$$

$$e^{-t/RC} = e^{\ln(Q/Q_0)} = \frac{Q}{Q_0} \quad (e^{\ln x} = x)$$

$$\Rightarrow \boxed{Q = Q_0 e^{-t/RC}}$$

Capacitor discharging

$RC = \text{time constant} \quad (1\Omega)(1F) = 1s.$

units are seconds (time)