PH 1111

Office hrs 10^{AM} – Noon

$1 - 3^{PM}$

Tuesday: $3^{30} - 4^{30}$

Wed: 10^{AM} – Noon

Torque revisited: $\vec{\tau} = \vec{r} \times \vec{F}$

$\Downarrow$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$

↑ momentum

Revisit momentum: define: $\vec{p} = m\vec{v}$

What is meant by saying momentum is conserved? $|\vec{p}| = \text{constant} \Rightarrow \boxed{\vec{p}_i^{\,net} = \vec{p}_f^{\,net}}$

$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$

no external forces, then

$\frac{d\vec{p}}{dt} = 0 \Rightarrow |\vec{p}| = \text{Constant} \Rightarrow$ Conservation of momentum!

translational	rotational
$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
momentum is conserved if $\frac{d\vec{p}}{dt} = \vec{0}$	angular momentum is conserved if $\frac{d\vec{L}}{dt} = \vec{0}$
$\Sigma \vec{F} = 0$ $\Rightarrow$ NO external forces	$\Sigma \vec{\tau} = 0$ NO external torques

Conservation of angular momentum:

$$L_i^{net} = L_f^{net}$$

$$\underbrace{I_i}_{\text{outstretched}} \omega_i = \underbrace{I_f}_{\text{arms inward}} \omega_f$$

$$I_i > I_f \Rightarrow \omega_i < \omega_f$$

<u>Example</u>: modified 11-29

$\vec{F} = F_0 \hat{j}$ (N) acts on a pirate of mass $= m$ (kg)

Position vector: $\vec{r} = (r_1 \hat{i} - r_2 \hat{k})$ m

velocity: $\vec{v} = (-v_1 \hat{i} + v_2 \hat{k})$ m/s

a.) Pirate's Angular momentum?

$$\vec{L} = \vec{r} \times \vec{p} \qquad (\vec{L} = \vec{r} \times \vec{p})$$

$$\vec{L} = (r_1 \hat{i} - r_2 \hat{k}) \times m(-v_1 \hat{i} + v_2 \hat{k})$$

$$\vec{L} = m\left\{(r_1 \hat{i} - r_2 \hat{k}) \times (-v_1 \hat{i} + v_2 \hat{k})\right\}$$

$$\vec{L} = m(-r_1 v_2 \hat{j}) - m r_2(-v_1)\hat{j} + 0$$

$$\vec{L} = -m r_1 v_2 \hat{j} + m r_2 v_1 \hat{j} = \underline{-m(r_1 v_2 - r_2 v_1)\hat{j}}$$

b.) Find the torque acting on pirate

$$\vec{\tau} = \vec{r} \times \vec{F} = (r_1 \hat{i} - r_2 \hat{k}) \times F_0 \hat{j}$$

$$\vec{\tau} = r_1 F_0 \hat{k} - r_2 F_0(-\hat{i}) = r_1 F_0 \hat{k} + r_2 F_0 \hat{i}$$

$$\boxed{\vec{\tau} = F_0(r_1 \hat{k} + r_2 \hat{i}) \text{ Nm}}$$

11-30.) $\vec{d} = d_1\hat{i} + d_2\hat{j} - d_3\hat{k}$

$\vec{v} = -v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

$\vec{F} = F_1\hat{i} - F_2\hat{j} + F_3\hat{k}$

mass of pirate $= \alpha m$

a.) Find the acceleration

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{\alpha m}\left[F_1\hat{i} - F_2\hat{j} + F_3\hat{k}\right]$$

b.) Find the angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{d} \times (\alpha m)\vec{v} = \alpha m\,(\vec{d} \times \vec{v})$$

$$\vec{L} = \alpha m\left\{(d_1\hat{i} + d_2\hat{j} - d_3\hat{k}) \times (-v_1\hat{i} + v_2\hat{j} + v_3\hat{k})\right\}$$

$$\vec{L} = \alpha m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d_1 & d_2 & -d_3 \\ -v_1 & v_2 & v_3 \end{vmatrix}$$

$$\vec{L} = \alpha m \left\{ \begin{array}{l} \hat{i}\left[d_2 v_3 + v_2 d_3\right] - \hat{j}\left[d_1 v_3 - v_1 d_3\right] \\ + \hat{k}\left[d_1 v_2 + v_1 d_2\right] \end{array} \right\}$$

c.) Torque = ?

$$\vec{\tau} = \vec{r} \times \vec{F} = (d_1\hat{i} + d_2\hat{j} - d_3\hat{k}) \times (F_1\hat{i} - F_2\hat{j} + F_3\hat{k})$$

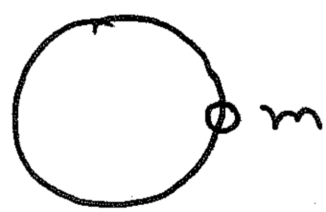
$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d_1 & d_2 & -d_3 \\ F_1 & -F_2 & F_3 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}\,[d_2F_3 - d_3F_2] - \hat{j}\,[d_1F_3 + F_1 d_3] + \hat{k}\,[-d_1F_2 - F_1 d_2]$$

d.) angle between the velocity of pirate and the Force acting on pirate ?

$$\vec{F}\cdot\vec{v} = \boxed{Fv\cos\theta = F_x v_x + v_y F_y + F_z v_z}$$

Slightly modified
11-56



← cockroach
on the edge
of yr CD player
uniform disk

Initially roach + CD rotate with
$\omega = 0.26$ rad/s

Then roach walks halfway to center of disk

CD mass = 4m

Conservation of Ang. Momentum

$$L_i^{net} = L_f^{net}$$

$$\boxed{I_{disk} = \frac{1}{2} m R^2}$$

$$L_i^{net} = I_{roach}\,\omega_i + \underset{\substack{\downarrow \\ \text{disk}}}{I_{CD}}\,\omega_i$$

$$L_i^{net} = \underline{(m \overset{?}{R^2})(0.26) + (\tfrac{1}{2}(4m)\overset{?}{R^2})(0.26)}$$

$$L_f^{net} = I_{roach}\,\omega_f + I_{CD}\,\omega_f$$

$$L_f^{net} = \underline{\left[(m)\left(\tfrac{R}{2}\right)^2 \omega_f + \tfrac{1}{2}(4m)(R^2)\,\omega_f\right]}$$