WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 13 – Part 1

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General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

Teaching Assistant: Jeffrey Laut & Kazim Naqvi;
During Lab Sessions

Recall: Lab Report #4

(Include uncertainty analyses in your Lab Report #4)

Example

Derive **complete** RSS uncertainty equation for the fundamental natural frequency of vibration of a cantilever. Make sure to:

(a) Indicate, in order of importance, percentage contribution of all uncertainties to the overall uncertainty.

(b) Discuss your results.

Include uncertainty analyses in your Lab Report #4

Governing equation:

$$\omega_n = 3.5160\sqrt{\frac{EI}{\overline{m}\,L^4}} = 1.015\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad \Rightarrow \quad \omega_n = \omega_n(E, h, \rho, L)$$

RSS uncertainty of natural frequency

$$\delta\omega_n = \left[\left(\frac{\partial\omega_n}{\partial E}\delta E\right)^2 + \left(\frac{\partial\omega_n}{\partial h}\delta h\right)^2 + \left(\frac{\partial\omega_n}{\partial \rho}\delta \rho\right)^2 + \left(\frac{\partial\omega_n}{\partial L}\delta L\right)^2\right]^{1/2}$$

Uncertainty on the fundamental frequency of vibration of a cantilever:

Initial variables (Al beam), SI system of units:

$E := 71.7 \cdot 10^{9}$ $\qquad \delta E := 0.1 \cdot E$ $\qquad \delta E = 7.17 \times 10^{9}$

$\rho := 2766$ $\qquad \delta\rho := 0.05 \cdot \rho$ $\qquad \delta\rho = 138.3$

$t := 0.003175$ $\qquad \delta t := 0.0001$ $\qquad \delta t = 1 \times 10^{-4}$

$L := 0.30$ $\qquad \delta L := 0.05 \cdot L$ $\qquad \delta L = 0.015$

Fundamental frequency:

$\omega n := 1.015 \cdot \sqrt{\frac{E}{\rho}} \cdot \frac{t}{L^{2}}$ $\qquad \omega n = 182.306$ [rad/sec]

$\omega nHz := \frac{\omega n}{2 \cdot \pi}$ $\qquad \omega nHz = 29.015$ [Hz]

Partial derivatives:

$f\omega nE := \frac{0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^{2} \cdot \rho}$ $\qquad f\omega nt := 1.015 \cdot \frac{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}}{L^{2}}$

$f\omega n\rho := \frac{\frac{-0.5075}{\left(\frac{E}{\rho}\right)^{1}}}{2} \cdot \frac{t}{L^{2}} \cdot \frac{E}{\rho^{2}}$ $\qquad f\omega nL := -2.030 \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \cdot \frac{t}{L^{3}}$

Evaluation of uncertainty:

$\delta\omega nSum := \left(f\omega nE \cdot \delta E\right)^{2} + \left(f\omega nt \cdot \delta t\right)^{2} + \left(f\omega n\rho \cdot \delta\rho\right)^{2} + \left(f\omega nL \cdot \delta L\right)^{2}$

$\delta\omega n := \left(\delta\omega nSum\right)^{\frac{1}{2}}$

$\delta\omega n = 21.176$ [rad/sec] $\qquad \delta\omega nHz := \frac{\delta\omega n}{2 \cdot \pi}$ $\qquad \delta\omega nHz = 3.37$ [Hz]

Percentage contribution:

$f\omega nE \cdot \delta E = 9.115$ $\qquad f\omega n\rho \cdot \delta\rho = -4.476 \times 10^{-4}$

$f\omega nt \cdot \delta t = 5.742$ $\qquad f\omega nL \cdot \delta L = -18.231$

Uncertainties in order of importance:

$\%L := \frac{\left(f\omega nL \cdot \delta L\right)^{2}}{\delta\omega nSum} \cdot 100$ $\qquad \%L = 74.118$

$\%t := \frac{\left(f\omega nt \cdot \delta t\right)^{2}}{\delta\omega nSum} \cdot 100$ $\qquad \%t = 7.353$

$\%E := \frac{\left(f\omega nE \cdot \delta E\right)^{2}}{\delta\omega nSum} \cdot 100$ $\qquad \%E = 18.529$

$\%\rho := \frac{\left(f\omega n\rho \cdot \delta\rho\right)^{2}}{\delta\omega nSum} \cdot 100$ $\qquad \%\rho = 4.468 \times 10^{-8}$

$Check := \%E + \%t + \%L + \%\rho$ $\qquad Check = 100$

Uncertainty on the fundamental frequency of vibration of a cantilever:

Initial variables (Al beam), SI system of units:

$E := 71.7 \cdot 10^{9}$ $\quad \delta E := 0.05 \cdot E$ $\quad \delta E = 3.585 \times 10^{9}$

$\rho := 2766$ $\quad \delta\rho := 0.05 \cdot \rho$ $\quad \delta\rho = 138.3$

$t := 0.003175$ $\quad \delta t := 0.0001$ $\quad \delta t = 1 \times 10^{-4}$

$L := 0.30$ $\quad \delta L := 0.05 \cdot L$ $\quad \delta L = 0.015$

Fundamental frequency:

$$\omega n := 1.015 \cdot \sqrt{\frac{E}{\rho}} \cdot \frac{t}{L^2} \qquad \omega n = 182.306 \qquad \text{[rad/sec]}$$

$$\omega nHz := \frac{\omega n}{2 \cdot \pi} \qquad \omega nHz = 29.015 \qquad \text{[Hz]}$$

Partial derivatives:

$$f\omega nE := \frac{0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^2 \cdot \rho} \qquad f\omega nt := 1.015 \cdot \frac{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}}{L^2}$$

$$f\omega n\rho := \frac{\frac{-0.5075}{\left(\frac{E}{\rho}\right)^{1}}}{2} \cdot \frac{t}{L^2} \cdot \frac{E}{\rho^2} \qquad f\omega nL := -2.030 \cdot \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \cdot \frac{t}{L^3}$$

Evaluation of uncertainty:

$$\delta\omega nSum := (f\omega nE \cdot \delta E)^2 + (f\omega nt \cdot \delta t)^2 + (f\omega n\rho \cdot \delta\rho)^2 + (f\omega nL \cdot \delta L)^2$$

$$\delta\omega n := (\delta\omega nSum)^{\frac{1}{2}}$$

$$\delta\omega n = 19.649 \quad \text{[rad/sec]} \qquad \delta\omega nHz := \frac{\delta\omega n}{2 \cdot \pi} \qquad \delta\omega nHz = 3.127 \quad \text{[Hz]}$$

Percentage contribution:

$f\omega nE \cdot \delta E = 4.558$ $\quad f\omega n\rho \cdot \delta\rho = -4.476 \times 10^{-4}$

$f\omega nt \cdot \delta t = 5.742$ $\quad f\omega nL \cdot \delta L = -18.231$

Uncertainties in order of importance:

$$\%L := \frac{(f\omega nL \cdot \delta L)^2}{\delta\omega nSum} \cdot 100 \qquad \%L = 86.081$$

$$\%t := \frac{(f\omega nt \cdot \delta t)^2}{\delta\omega nSum} \cdot 100 \qquad \%t = 8.539$$

$$\%E := \frac{(f\omega nE \cdot \delta E)^2}{\delta\omega nSum} \cdot 100 \qquad \%E = 5.38$$

$$\%\rho := \frac{(f\omega n\rho \cdot \delta\rho)^2}{\delta\omega nSum} \cdot 100 \qquad \%\rho = 5.189 \times 10^{-8}$$

$$Check := \%E + \%t + \%L + \%\rho \qquad Check = 100$$

Reading assignment

- **Beckwith**: Ch. 6, 17
- **Bishop:**

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, http://www.gesensing.com/

Homework assignment: Handout-O, part 1

O1.
- **Refer to the equation used in the determination of the "fundamental natural frequency" of a cantilever. Use this equation and solve for the elastic modulus of elasticity, E**
- **Do RSS uncertainty analysis on the determined E from measurements of the "fundamental natural frequency" of vibration**
- **Determine percentage contribution of all of the parameters involved; check for 100 % contribution**
  - Discuss your observations
  - (use numerical data obtained in your Lab #4 experiments)