

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2009

Lecture 13 - Part 1

29 April 2009



General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 10:00 to 11:00 am
or by appointment

Teaching Assistant: During laboratory sessions



Recall: Lab Report #4

(Include uncertainty analyses in your Lab Report #4)

Derive **complete** RSS uncertainty equation for the fundamental natural frequency of vibration of a cantilever. Make sure to:

- (a) Indicate, in order of importance, percentage contribution of all uncertainties to the overall uncertainty.
- (b) Discuss your results.



Include uncertainty analyses in your Lab Report #4

Governing equation:

$$\omega_n = 3.5160 \sqrt{\frac{EI}{m L^4}} = 1.015 \sqrt{\frac{E}{\rho}} \frac{h}{L^2} \Rightarrow \omega_n = \omega_n(E, h, \rho, L)$$

RSS uncertainty of natural frequency

$$\delta\omega_n = \left[\left(\frac{\partial\omega_n}{\partial E} \delta E \right)^2 + \left(\frac{\partial\omega_n}{\partial h} \delta h \right)^2 + \left(\frac{\partial\omega_n}{\partial \rho} \delta \rho \right)^2 + \left(\frac{\partial\omega_n}{\partial L} \delta L \right)^2 \right]^{1/2}$$



ME-3901. Engineering Experimentation

WPI-Mechanical Engineering Department
C. Furlong

Uncertainty on the fundamental frequency of vibration of a cantilever:

Initial variables (Al beam), SI system of units:

$$E := 71.7 \cdot 10^9 \quad \delta E := 0.1 \cdot E \quad \delta E = 7.17 \times 10^9$$

$$\rho := 2766 \quad \delta \rho := 0.05 \cdot \rho \quad \delta \rho = 138.3$$

$$t := 0.003175 \quad \delta t := 0.0001 \quad \delta t = 1 \times 10^{-4}$$

$$L := 0.30 \quad \delta L := 0.05 \cdot L \quad \delta L = 0.015$$

Fundamental frequency:

$$\omega_n := 1.015 \cdot \sqrt{\frac{E}{\rho}} \cdot \frac{t}{L^2} \quad \omega_n = 182.306 \quad [\text{rad/sec}]$$

$$\omega_n \text{Hz} := \frac{\omega_n}{2 \cdot \pi} \quad \omega_n \text{Hz} = 29.015 \quad [\text{Hz}]$$

Partial derivatives:

$$f_{\omega_n E} := \frac{0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^2 \cdot \rho} \quad f_{\omega_n t} := 1.015 \cdot \frac{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}}{L^2}$$

$$f_{\omega_n \rho} := \frac{-0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^2} \cdot \frac{E}{\rho^2} \quad f_{\omega_n L} := -2.030 \cdot \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \cdot \frac{t}{L^3}$$

Evaluation of uncertainty:

$$\delta \omega_n \text{Sum} := (f_{\omega_n E} \cdot \delta E)^2 + (f_{\omega_n t} \cdot \delta t)^2 + (f_{\omega_n \rho} \cdot \delta \rho)^2 + (f_{\omega_n L} \cdot \delta L)^2$$

$$\delta \omega_n := (\delta \omega_n \text{Sum})^{\frac{1}{2}}$$

$$\delta \omega_n = 21.176 \quad [\text{rad/sec}] \quad \delta \omega_n \text{Hz} := \frac{\delta \omega_n}{2 \cdot \pi} \quad \delta \omega_n \text{Hz} = 3.37 \quad [\text{Hz}]$$

Percentage contribution:

$$f_{\omega_n E} \cdot \delta E = 9.115 \quad f_{\omega_n \rho} \cdot \delta \rho = -4.476 \times 10^{-4}$$

$$f_{\omega_n t} \cdot \delta t = 5.742 \quad f_{\omega_n L} \cdot \delta L = -18.231$$

Uncertainties in order of importance:

$$\%L := \frac{(f_{\omega_n L} \cdot \delta L)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%L = 74.118$$

$$\%t := \frac{(f_{\omega_n t} \cdot \delta t)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%t = 7.353$$

$$\%E := \frac{(f_{\omega_n E} \cdot \delta E)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%E = 18.529$$

$$\%\rho := \frac{(f_{\omega_n \rho} \cdot \delta \rho)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%\rho = 4.468 \times 10^{-8}$$

$$\text{Check} := \%E + \%t + \%L + \%\rho$$

$$\text{Check} = 100$$

Uncertainty on the fundamental frequency of vibration of a cantilever:

Initial variables (Al beam), SI system of units:

$E := 71.7 \cdot 10^9$	$\delta E := 0.05 \cdot E$	$\delta E = 3.585 \times 10^9$
$\rho := 2766$	$\delta \rho := 0.05 \cdot \rho$	$\delta \rho = 138.3$
$t := 0.003175$	$\delta t := 0.0001$	$\delta t = 1 \times 10^{-4}$
$L := 0.30$	$\delta L := 0.05 \cdot L$	$\delta L = 0.015$

Fundamental frequency:

$$\omega_n := 1.015 \cdot \sqrt{\frac{E}{\rho}} \cdot \frac{t}{L^2} \quad \omega_n = 182.306 \quad [\text{rad/sec}]$$

$$\omega_n \text{Hz} := \frac{\omega_n}{2 \cdot \pi} \quad \omega_n \text{Hz} = 29.015 \quad [\text{Hz}]$$

Partial derivatives:

$$f_{\omega n E} := \frac{0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^2 \cdot \rho} \quad f_{\omega n t} := 1.015 \cdot \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \cdot \frac{1}{L^2}$$

$$f_{\omega n \rho} := \frac{-0.5075}{\left(\frac{E}{\rho}\right)^{\frac{1}{2}}} \cdot \frac{t}{L^2} \cdot \frac{E}{\rho^2} \quad f_{\omega n L} := -2.030 \cdot \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \cdot \frac{t}{L^3}$$

Evaluation of uncertainty:

$$\delta \omega_n \text{Sum} := (f_{\omega n E} \cdot \delta E)^2 + (f_{\omega n t} \cdot \delta t)^2 + (f_{\omega n \rho} \cdot \delta \rho)^2 + (f_{\omega n L} \cdot \delta L)^2$$

$$\delta \omega_n := (\delta \omega_n \text{Sum})^{\frac{1}{2}}$$

$$\delta \omega_n = 19.649 \quad [\text{rad/sec}] \quad \delta \omega_n \text{Hz} := \frac{\delta \omega_n}{2 \cdot \pi} \quad \delta \omega_n \text{Hz} = 3.127 \quad [\text{Hz}]$$

Percentage contribution:

$$f_{\omega n E} \cdot \delta E = 4.558 \quad f_{\omega n \rho} \cdot \delta \rho = -4.476 \times 10^{-4}$$

$$f_{\omega n t} \cdot \delta t = 5.742 \quad f_{\omega n L} \cdot \delta L = -18.231$$

Uncertainties in order of importance:

$$\%L := \frac{(f_{\omega n L} \cdot \delta L)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%L = 86.081$$

$$\%t := \frac{(f_{\omega n t} \cdot \delta t)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%t = 8.539$$

$$\%E := \frac{(f_{\omega n E} \cdot \delta E)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%E = 5.38$$

$$\%\rho := \frac{(f_{\omega n \rho} \cdot \delta \rho)^2}{\delta \omega_n \text{Sum}} \cdot 100 \quad \%\rho = 5.189 \times 10^{-8}$$

$$\text{Check} := \%E + \%t + \%L + \%\rho \quad \text{Check} = 100$$

Reading assignment

- Beckwith: Ch. 6, 17
- Bishop:

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, <http://www.gesensing.com/>



Homework assignment: Handout-N

- N1.
- Refer to the equation used in the determination of the “fundamental natural frequency” of a cantilever. Use this equation and solve for the elastic modulus of elasticity, E
 - Do RSS uncertainty analysis on the determined E from measurements of the “fundamental natural frequency” of vibration
 - Determine percentage contribution of all of the parameters involved; check for 100 % contribution

Discuss your observations

(use numerical data obtained in your Lab #4 experiments)

