WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 12

06 October 2010

General information

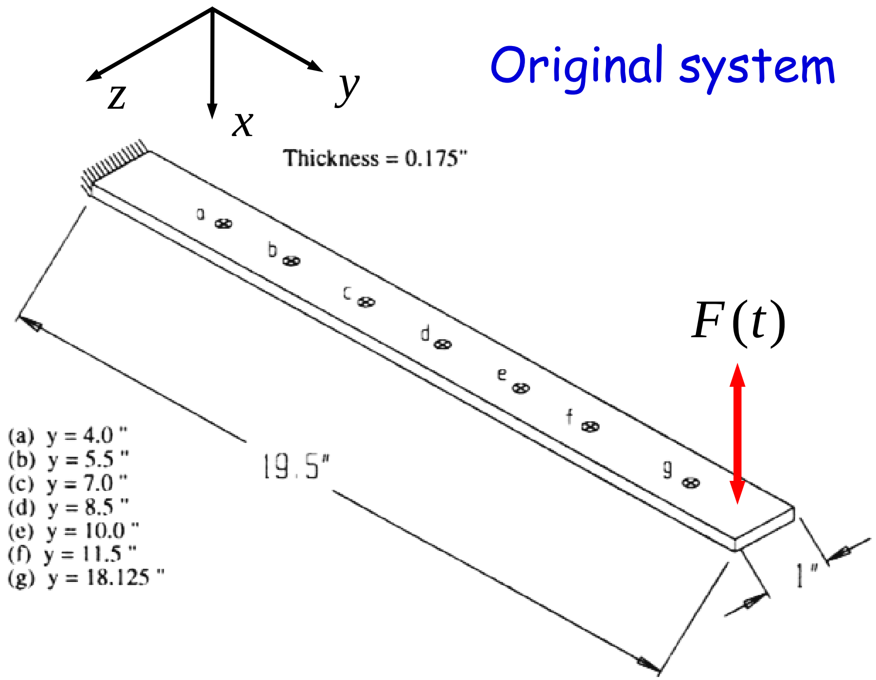
Office hours

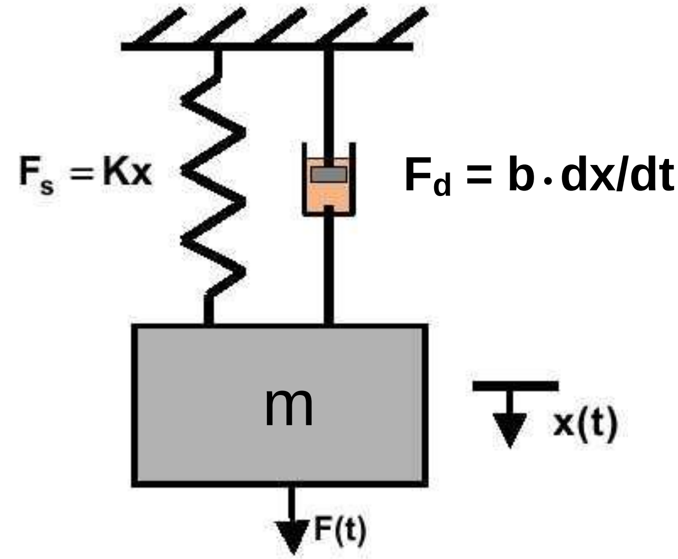
Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

Teaching Assistant: Jeffrey Laut & Kazim Naqvi;
During Lab Sessions

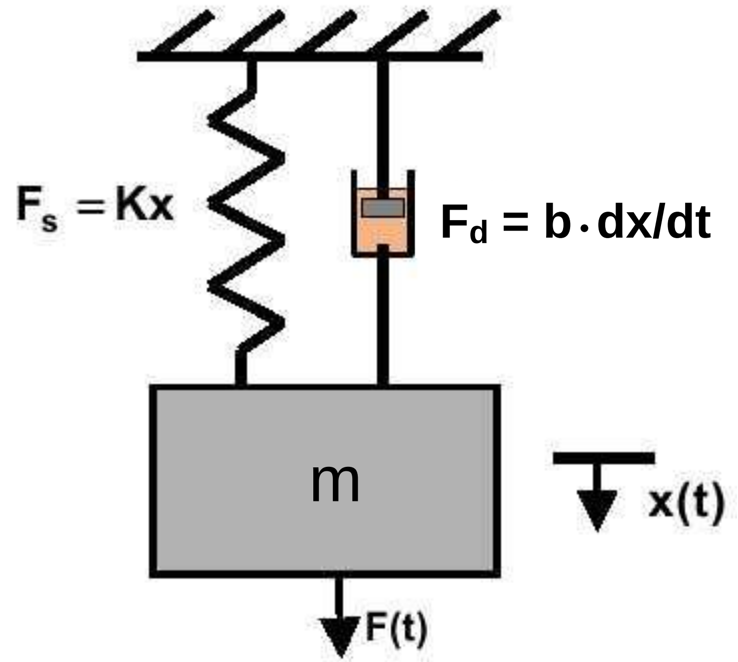
Equivalent systems

Original system ⟶ Equivalent system





Analysis of a single degree of freedom system



Governing equation: $$m\frac{d^2x}{dt^2} = \sum_i F_i$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F(t)$$

External force

Analysis of a single degree of freedom system

First case: $F(t) \neq 0$ – Forced vibrations

Governing equation: $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t)$$

Governing equation can be written as:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F(t)}{m} \longrightarrow \frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = F'(t)$$

Solution has the form:

$$x(t) = x_c(t) + x_p(t)$$

$x_c(t)$ Complementary solution

$x_p(t)$ Particular solution

Solution requires the method of *undetermined coefficients*

Analysis of a single degree of freedom system

First case: $F(t) \neq 0$ – Forced vibrations

Governing equation: → $$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = F_o \sin(\gamma t)$$

Solution has the form: $x(t) = x_c(t) + x_p(t)$

$$x(t) = \underbrace{Ae^{-\lambda t}\sin(\sqrt{\omega^2-\lambda^2}\,t+\phi)}_{\textit{transient}} + \underbrace{\frac{F_o}{\sqrt{(\omega^2-\lambda^2)^2+4\lambda^2\gamma^2}}\sin(\gamma t+\theta)}_{\textit{steady state}}$$

with: $\sin(\phi) = \dfrac{C_1}{A}$; $\sin(\theta) = \dfrac{-2\lambda\gamma}{\sqrt{(\omega^2-\lambda^2)^2+4\lambda^2\gamma^2}}$

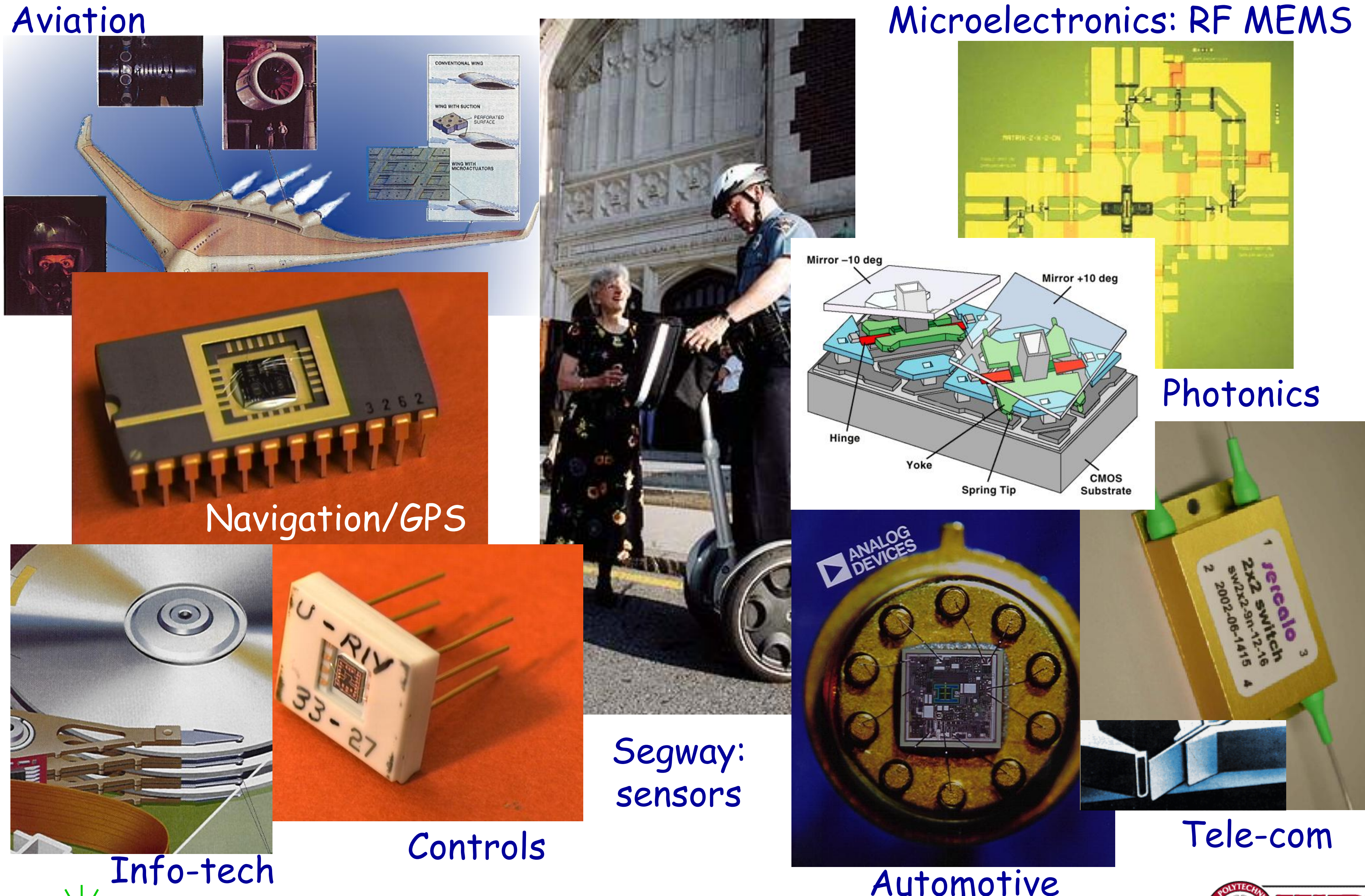
$\cos(\phi) = \dfrac{C_2}{A}$; $\cos(\theta) = \dfrac{\omega^2-\lambda^2}{\sqrt{(\omega^2-\lambda^2)^2+4\lambda^2\gamma^2}}$

Analysis of a single degree of freedom system

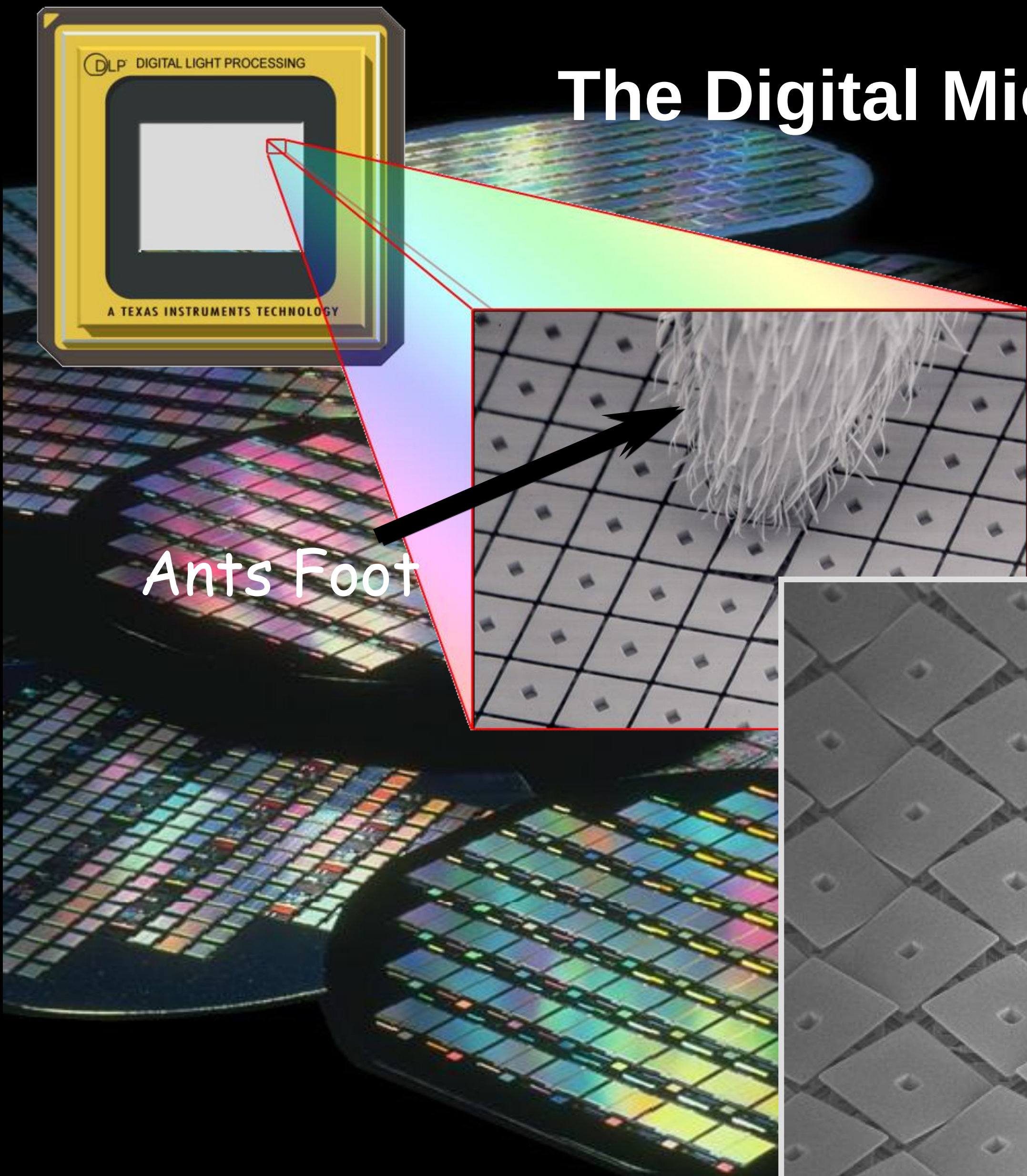
First case: $F(t) \neq 0$ – Forced vibrations

REVIEW - Lecture 10: section on accelerometers

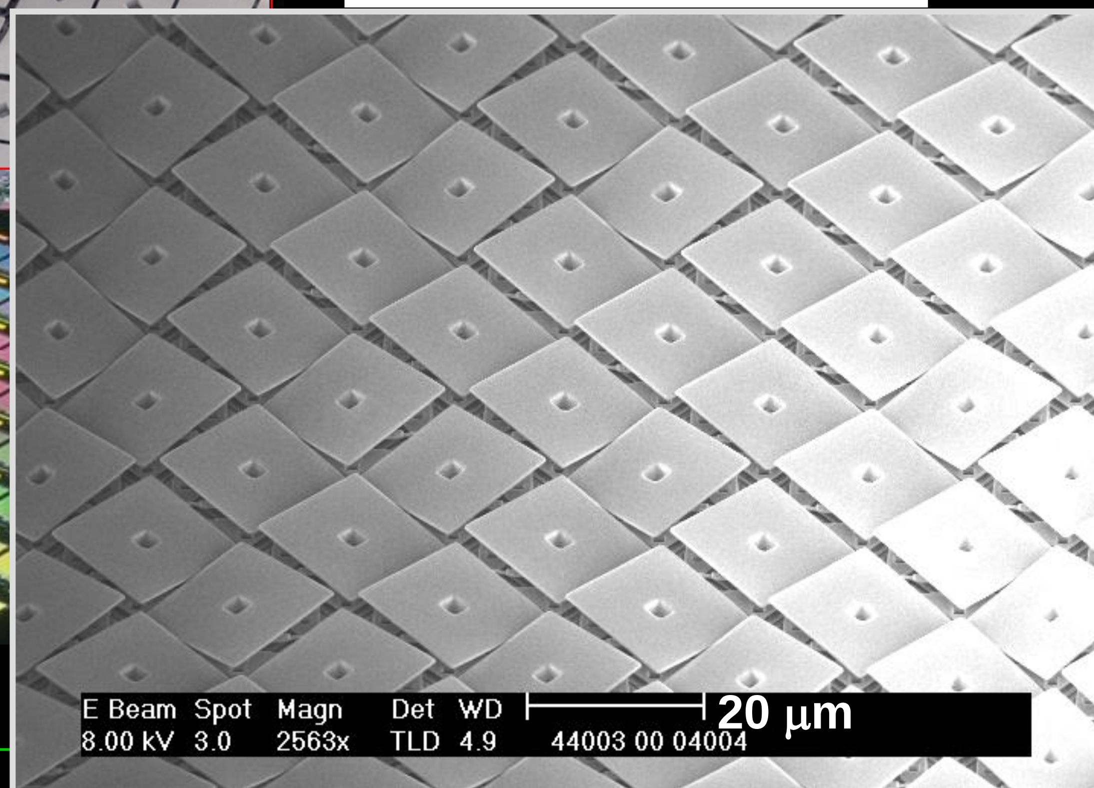
MEMS sensors & actuators: some applications



The Digital Micromirror Device

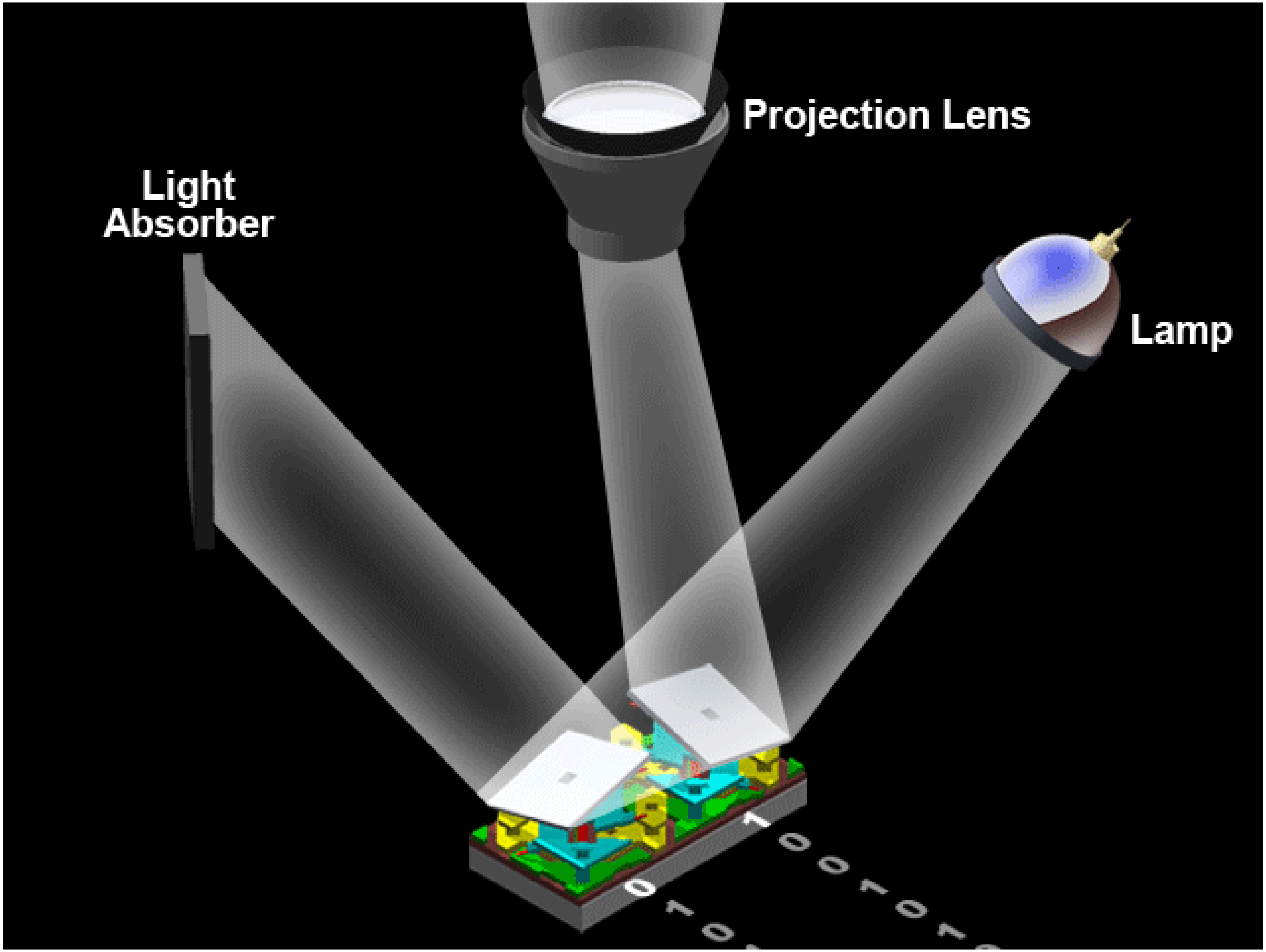


Close-up of Chip Surface



http://www.dlp.com/

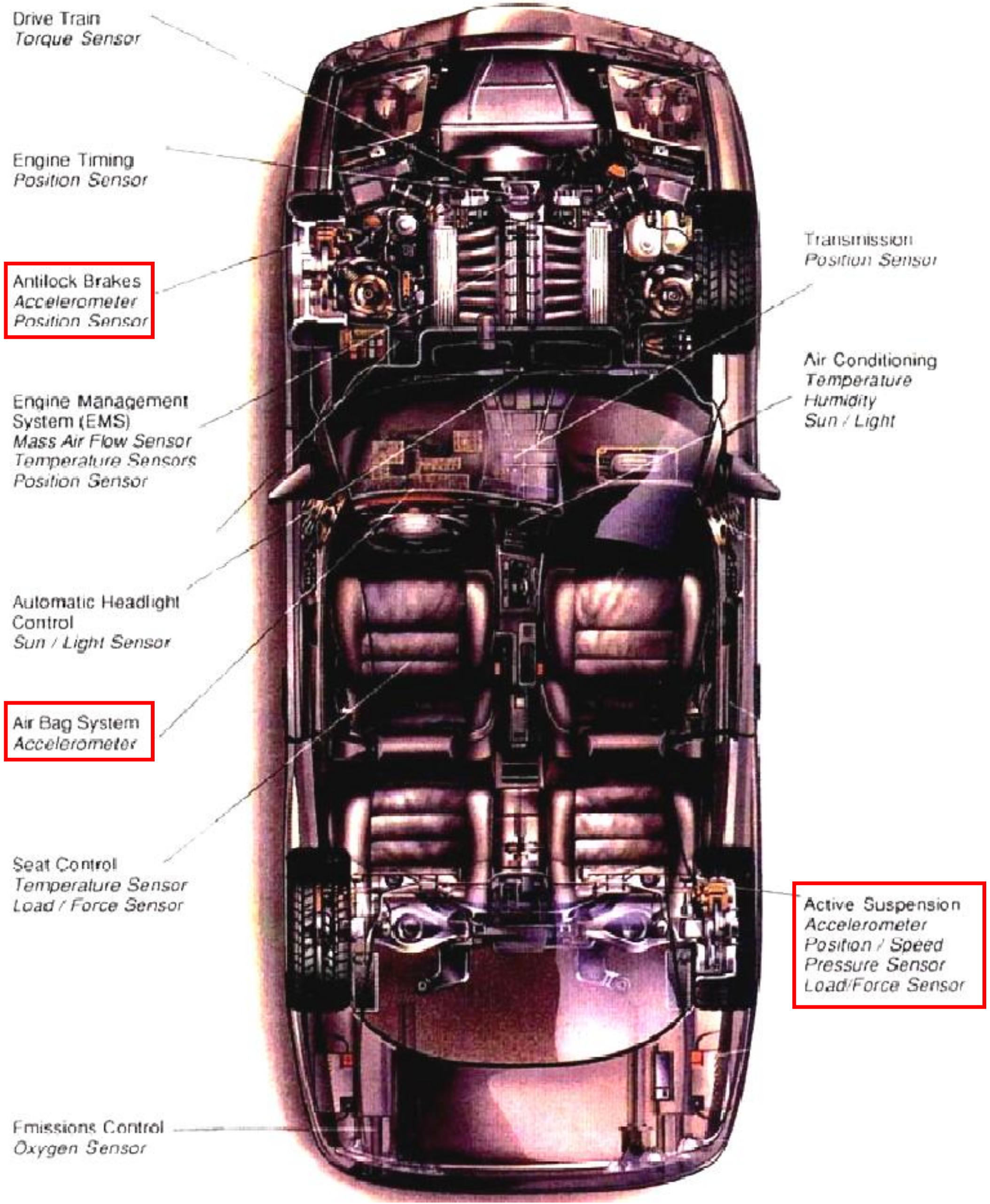
How DMD™ Technology works



http://www.dlp.com/

MEMS sensors: some applications

(MEMS – Microelectromechanical Systems)

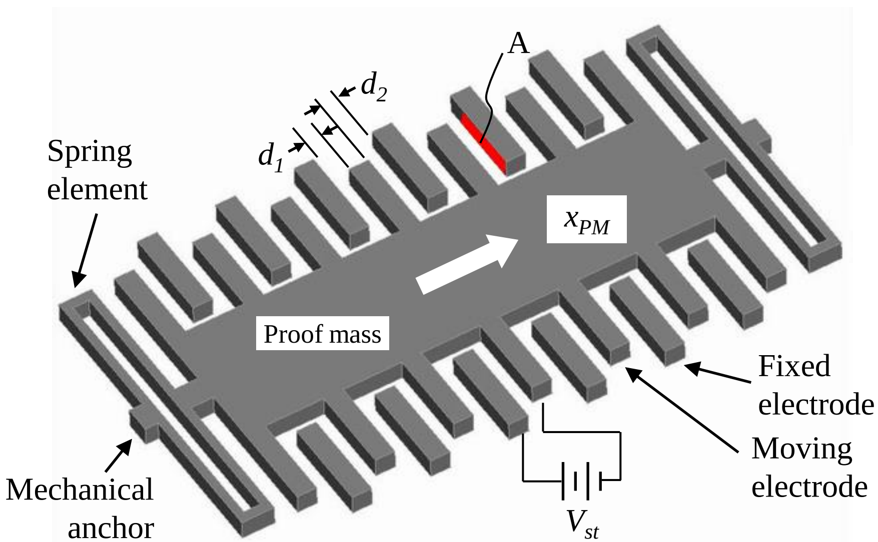


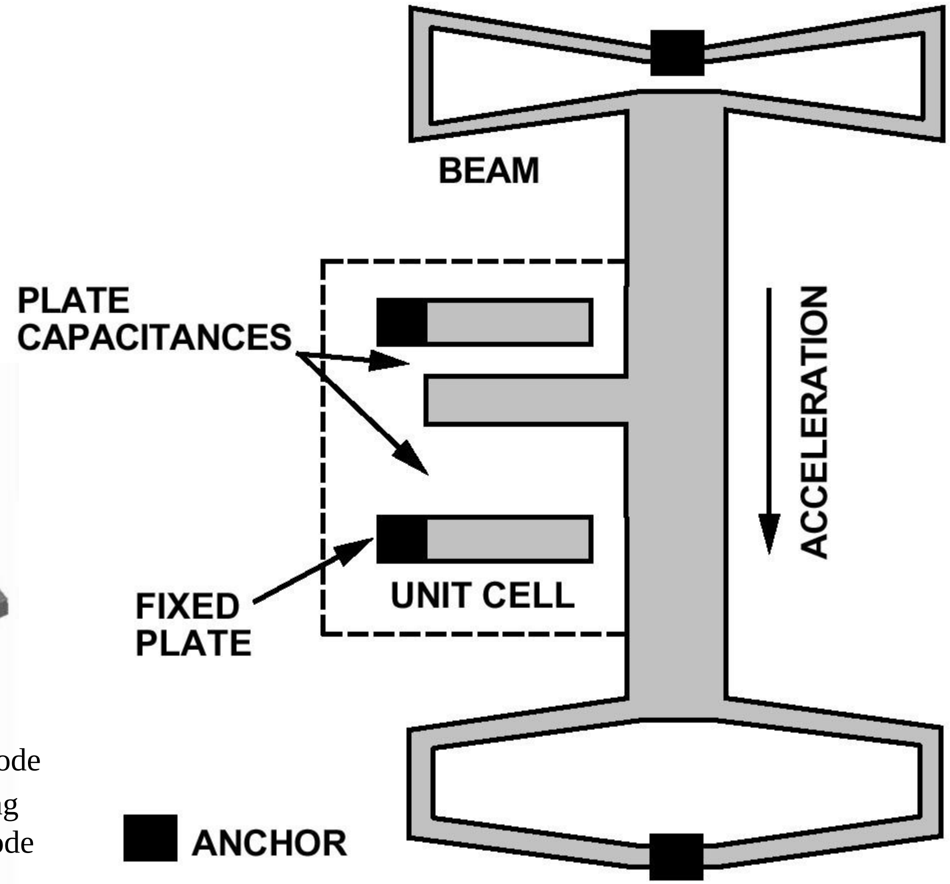
MEMS accelerometers

Simplified view of a sensor subjected to acceleration

Capacitance → $C = \varepsilon_r \varepsilon_o \dfrac{A}{d}$

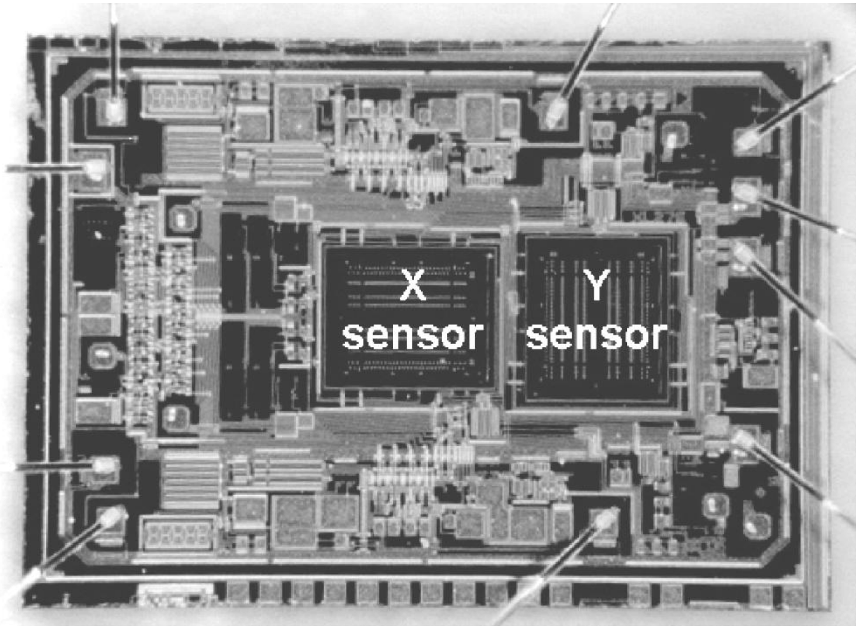
Single axis capacitive MEMS inertial sensor

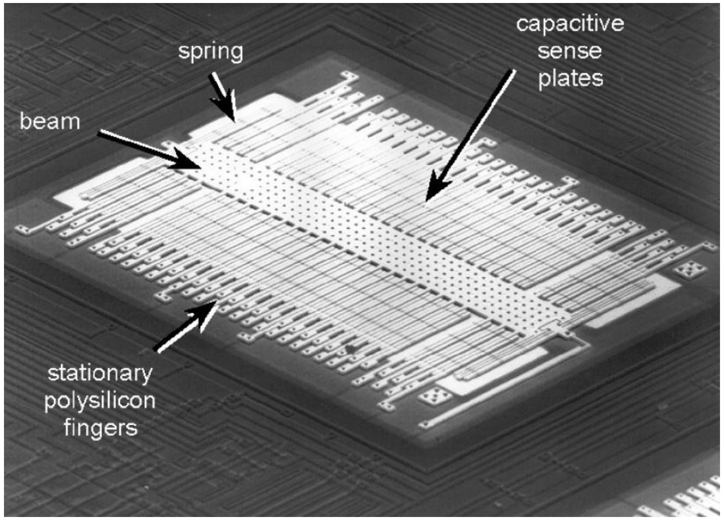




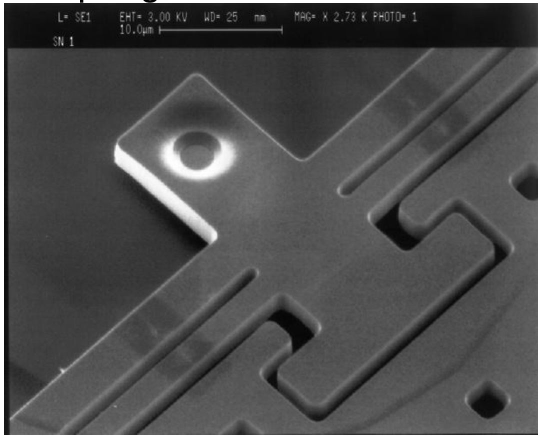
MEMS accelerometers: ADXL 150

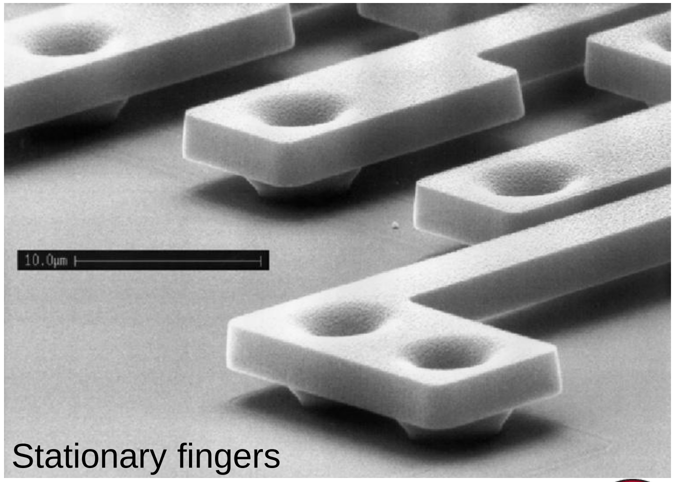
Dual-axes accelerometers





Spring attachment: anchor

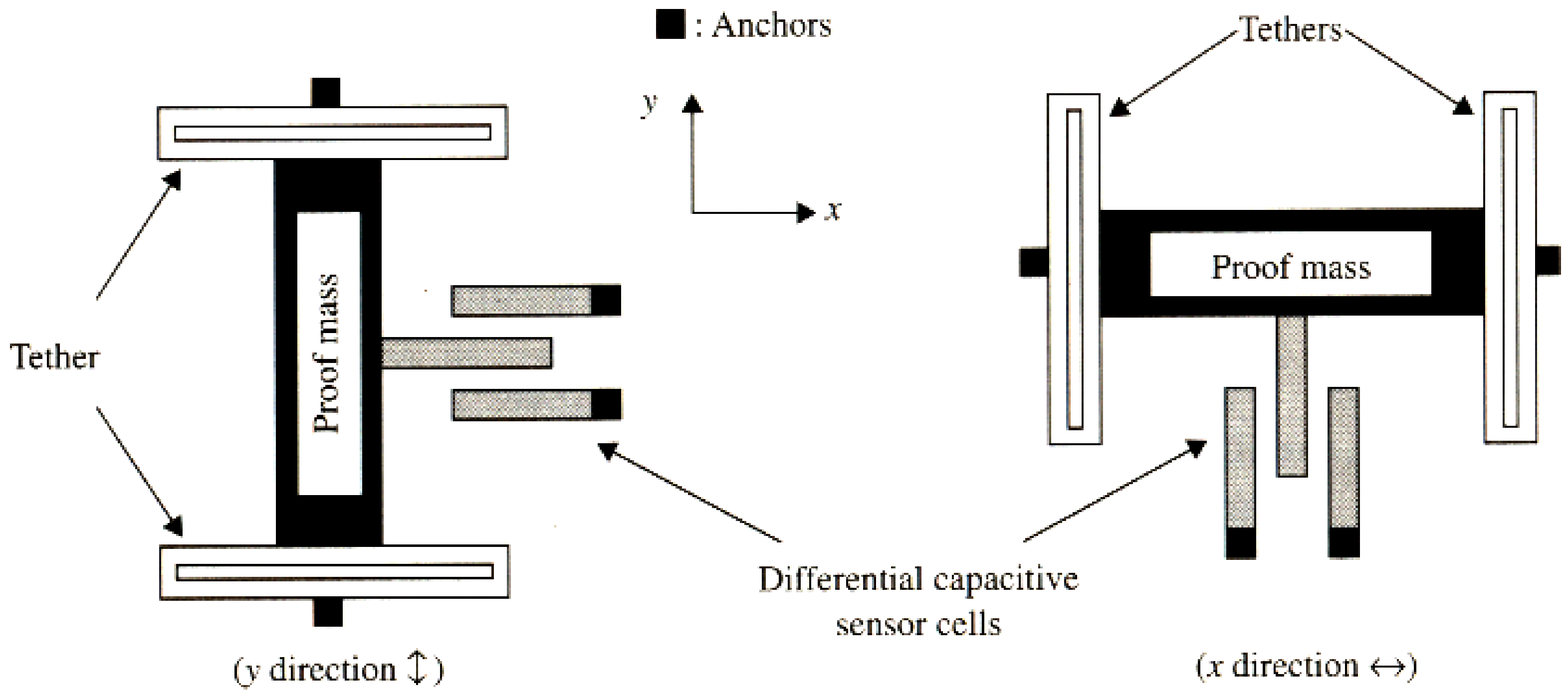




Stationary fingers

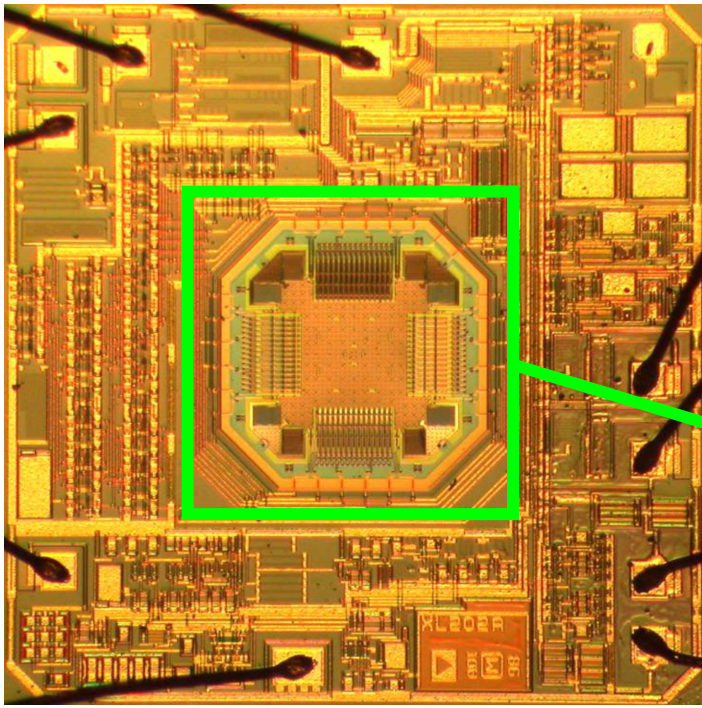
MEMS accelerometers: ADXL 150

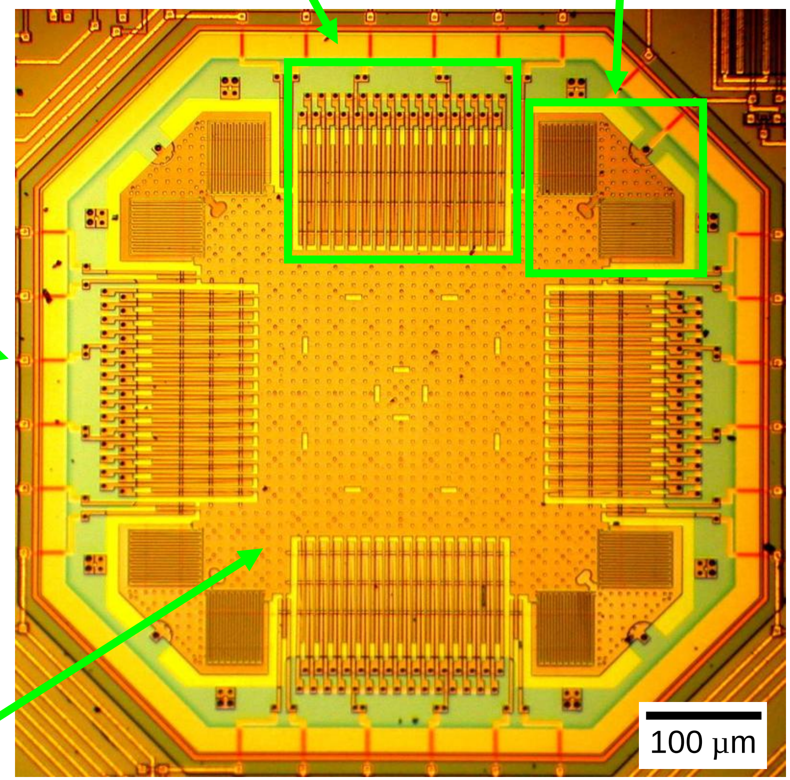
Dual-axes accelerometers



MEMS accelerometers: ADXL 202

Dual-axes accelerometers

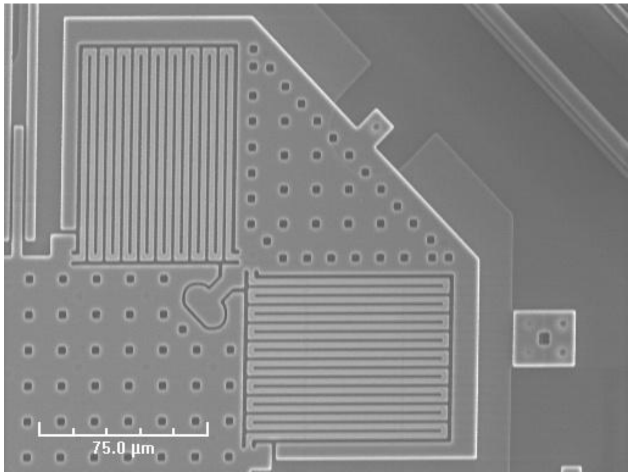


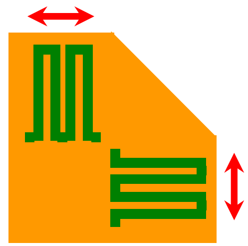


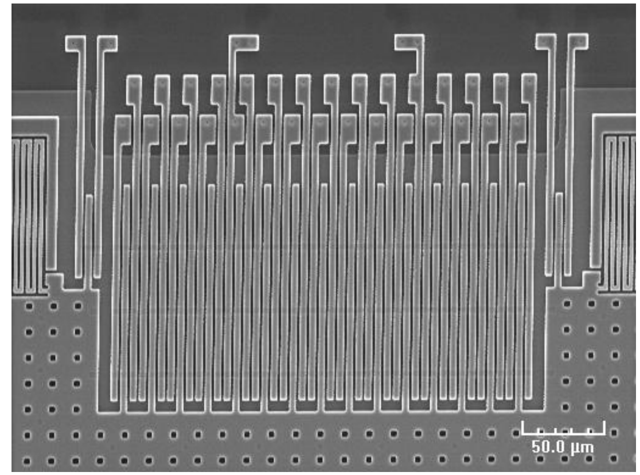
ADXL202 micromechanical sensor

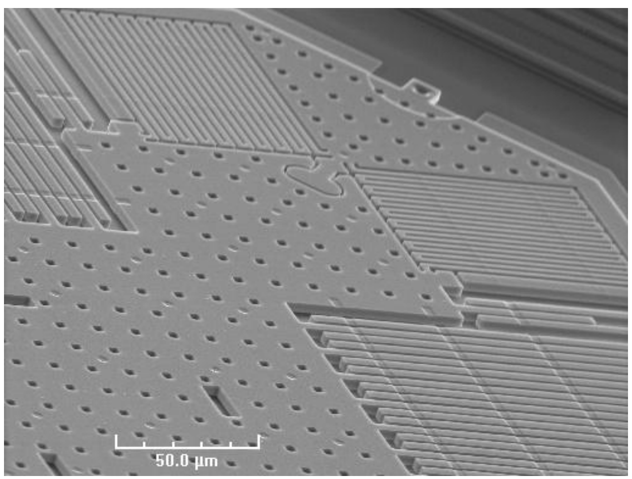
MEMS accelerometers: ADXL 202

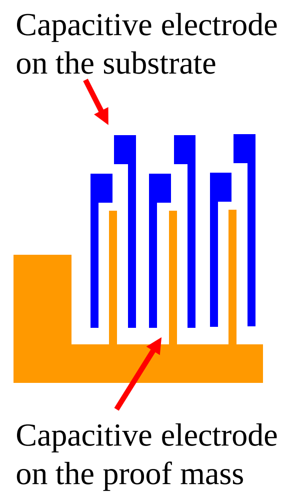
Proof mass has dimensions of $600 \times 600 \times 3\ \mu m^3$

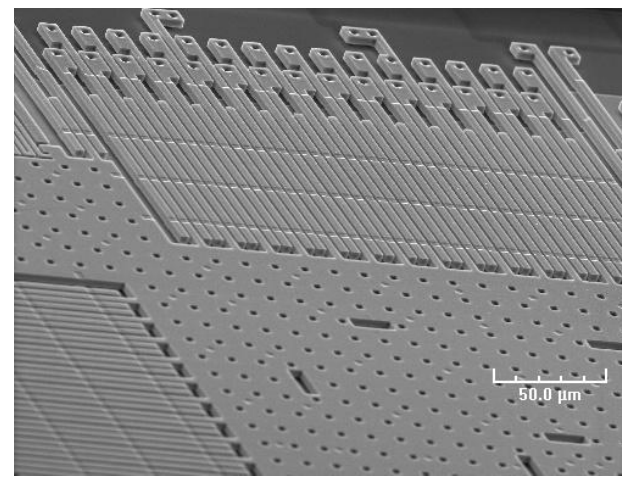










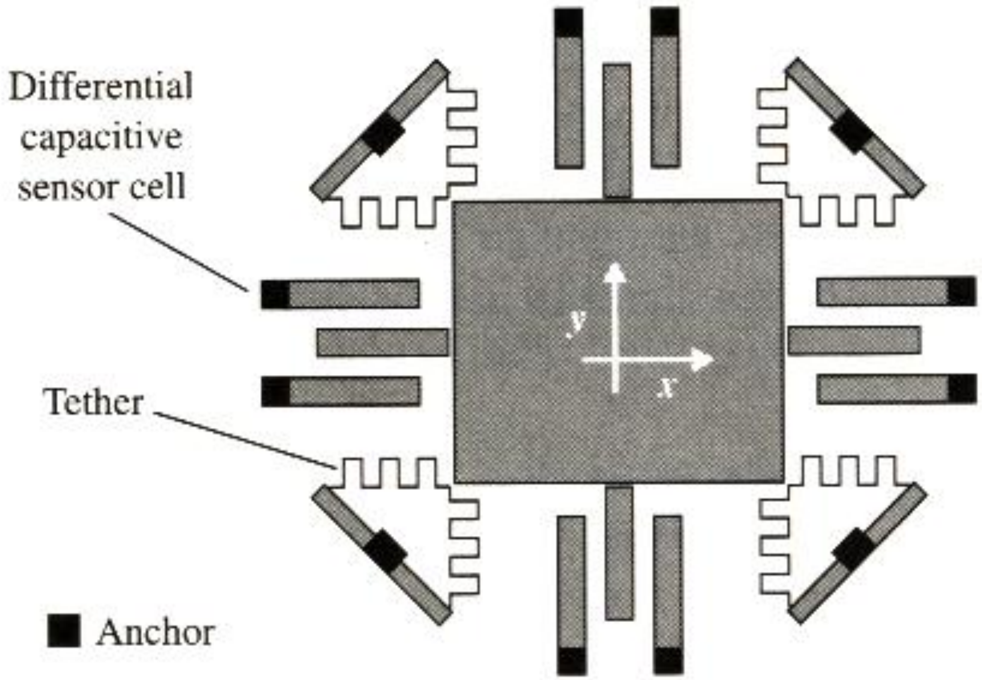


One set of spring elements

One set of capacitive combs

MEMS accelerometers: ADXL 202

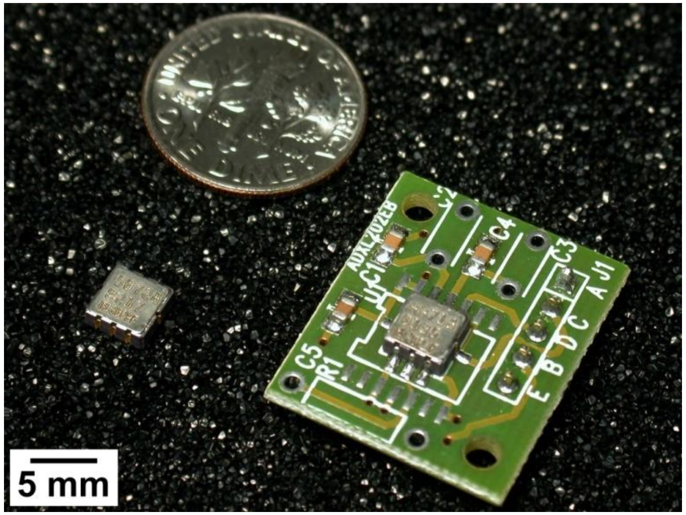
Dual-axes accelerometers



(Courtesy of Analog Devices Inc., Norwood, Massachusetts.)

MEMS accelerometers: ADXL 202

- Provide optimum substitution for typical piezoelectric accelerometers because of their
  - high resonance frequency
  - light weight, small size, low power use
- Some properties of the Analog Devices' *ADXL202* dual-axes accelerometer include



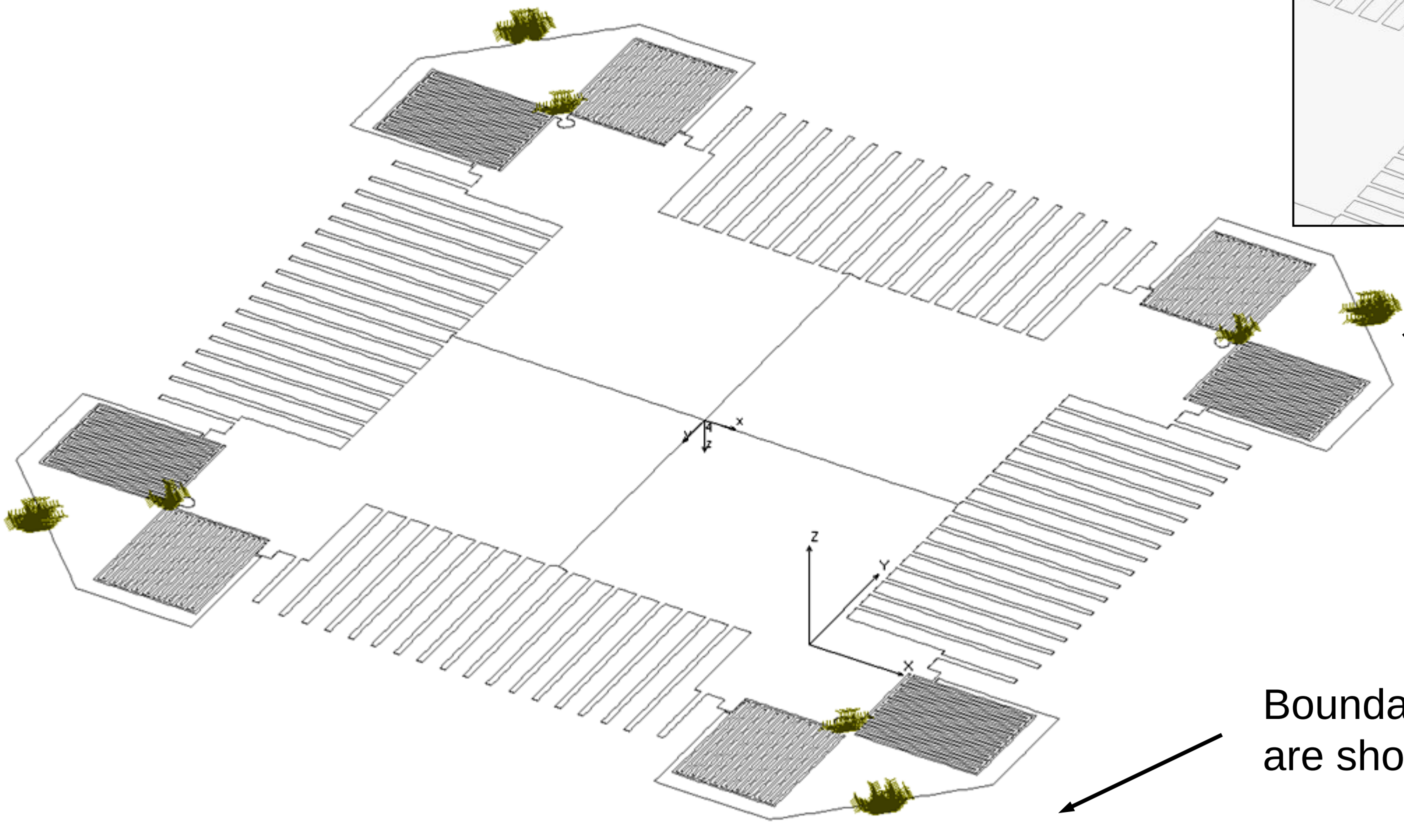
ADXL202 packages

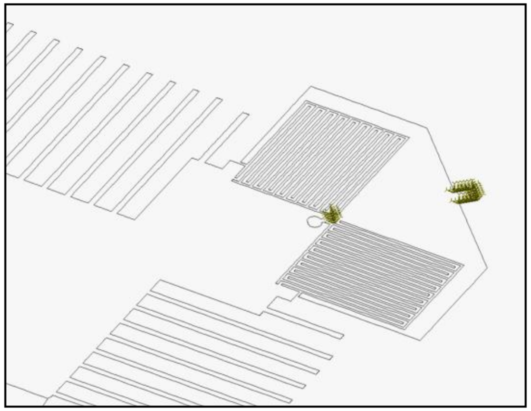
Specifications of Analog Devices ADXL202			
Measurement range	± 2 g	Operating voltage	3.0-5.25 V
Sensitivity	312 mV/ g	Mass	160 ± 10 mgr
Resonance frequency	10 kHz	Operating temperature	[-40,85] °C

MEMS accelerometers: ADXL 202

Modal analysis using FEM (free-vibrations)

Proof mass and suspension mechanism: thickness is 3 μm





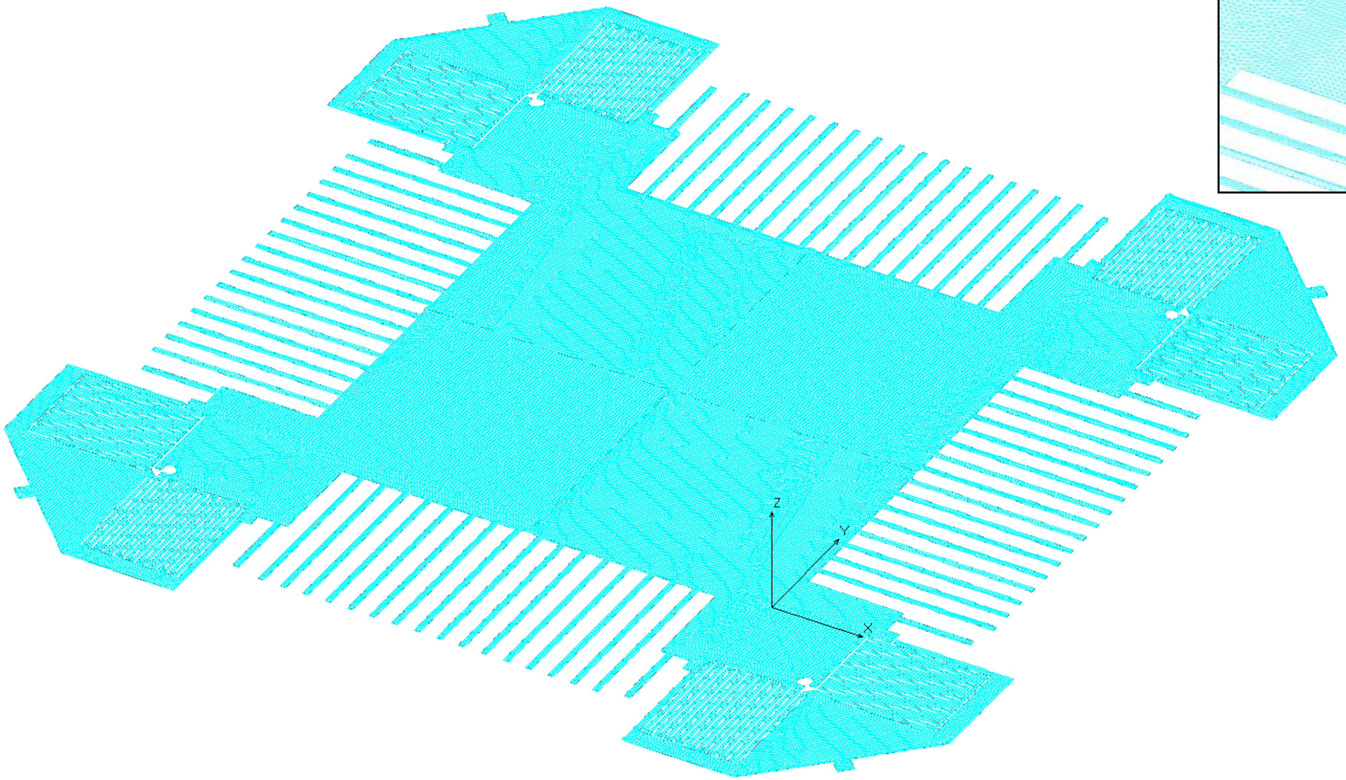
Suspension mechanism

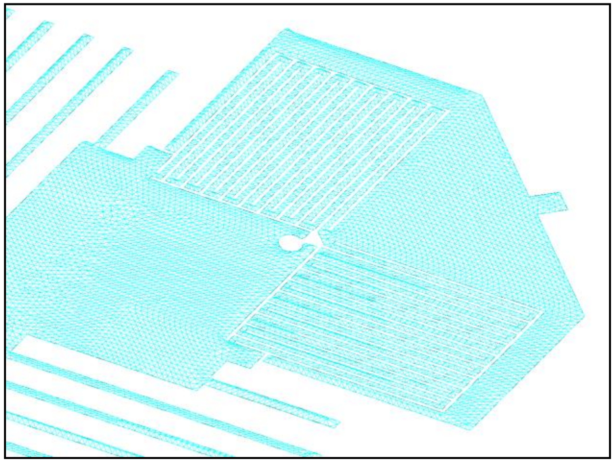
Boundary conditions are shown

MEMS accelerometers: ADXL 202

Modal analysis using FEM (free-vibrations)

Proof mass and suspension mechanism: mesh (domain discretization), 109,772 3-shell elements





Suspension mechanism

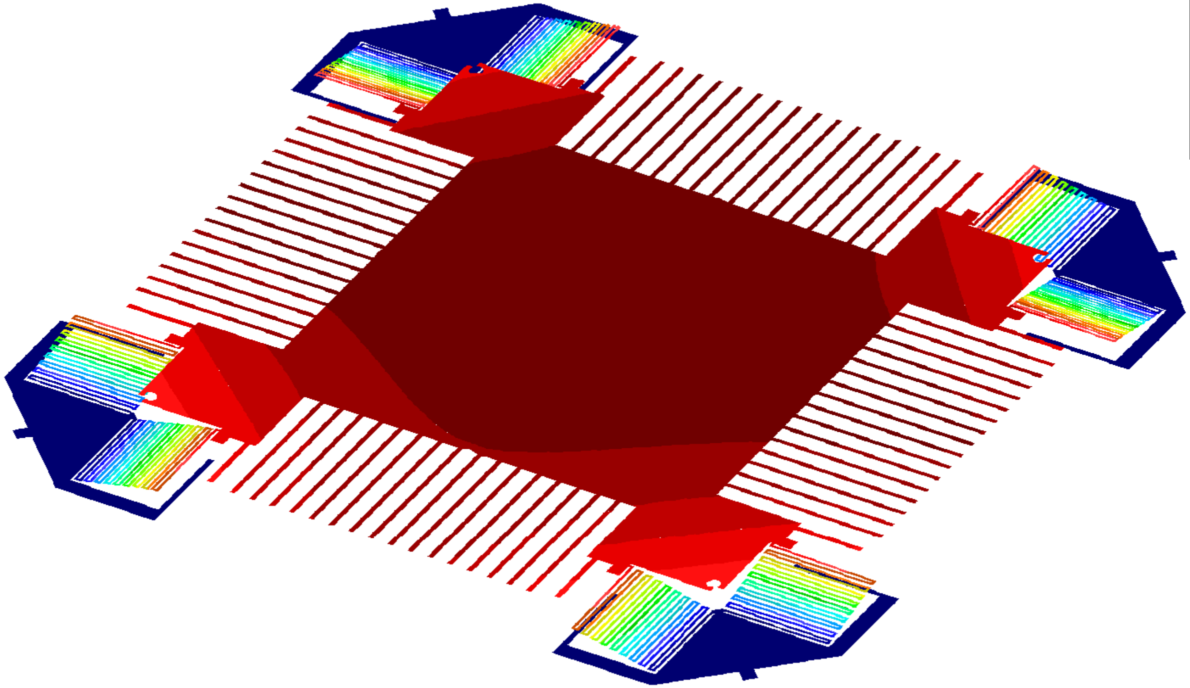
MEMS accelerometers: ADXL 202

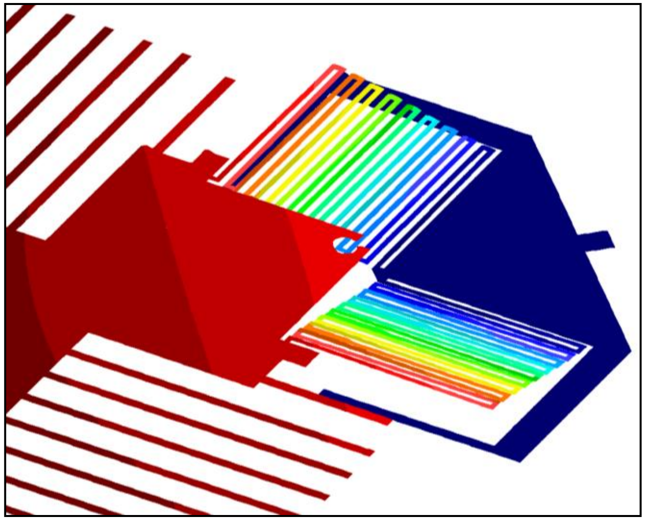
Modal analysis using FEM (free-vibrations)

First mode of vibration: predicted at 10.8 kHz

Material properties utilized:

- 160 Gpa
- 2.33 gr/cm³



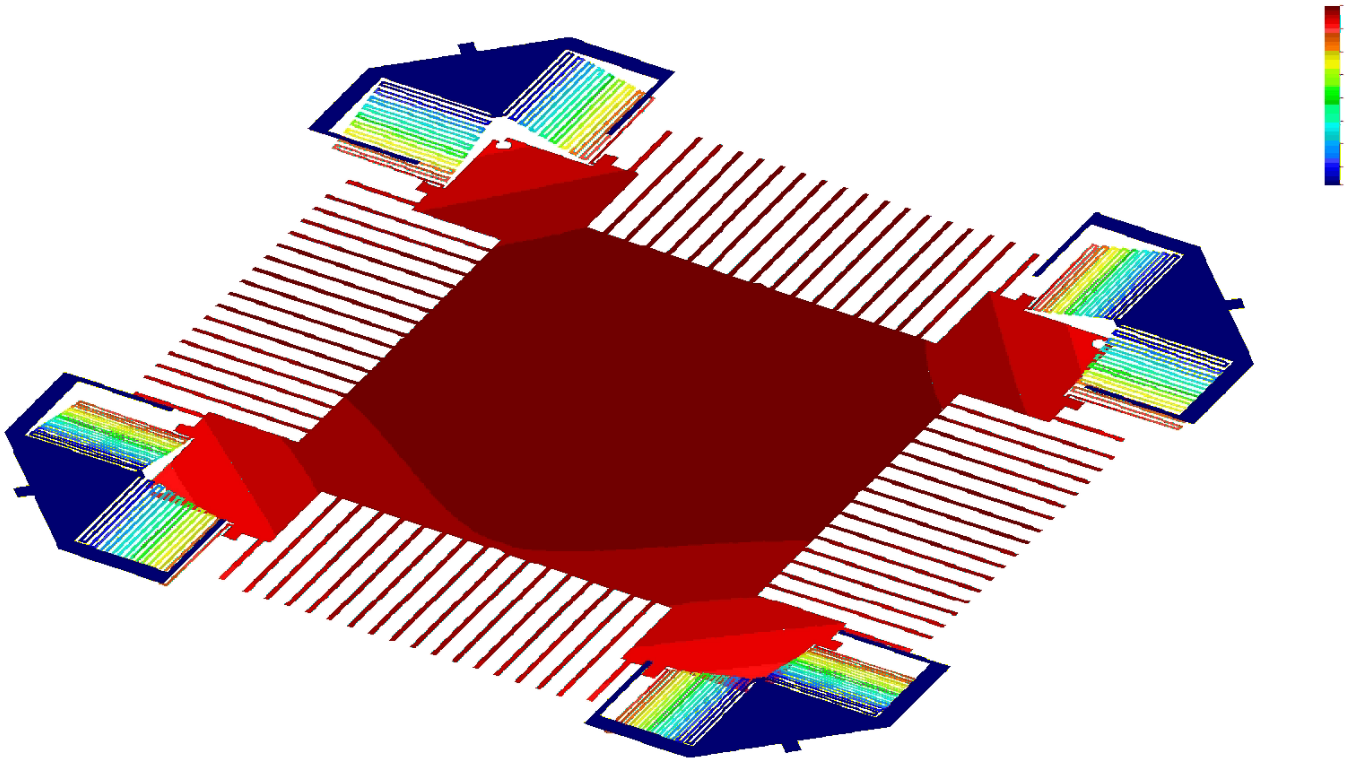


Suspension mechanism

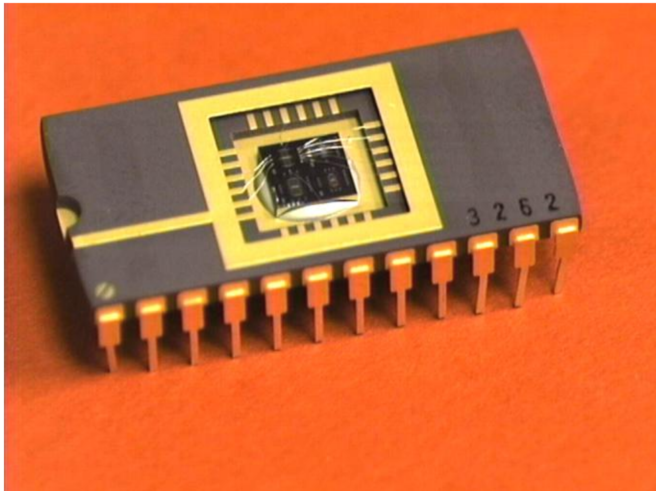
MEMS accelerometers: ADXL 202

Modal analysis using FEM (free-vibrations)

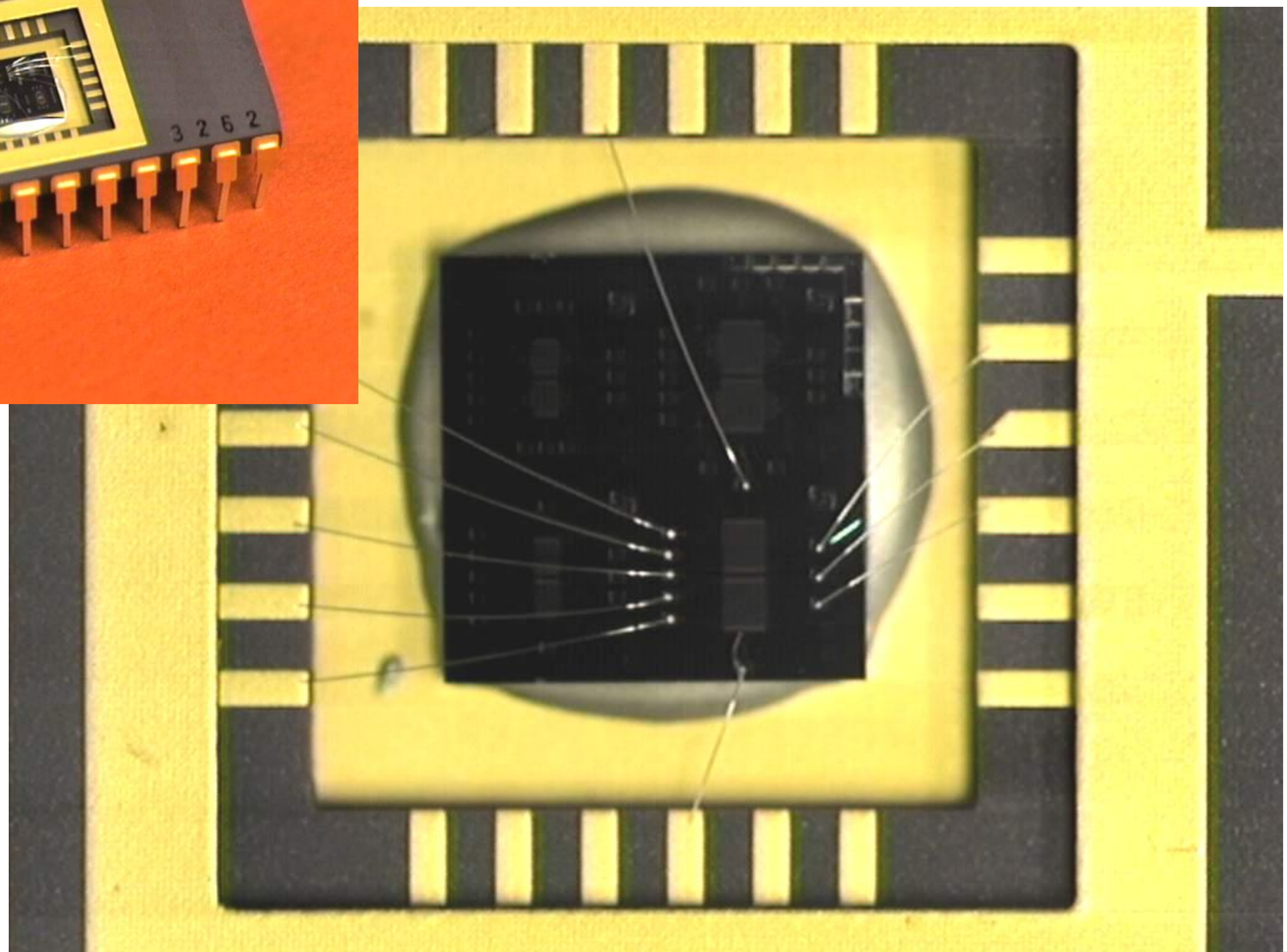
Animation of the first mode of vibration: predicted at 10.8 kHz



MEMS gyroscopes: packages
SNL – Sandia National Laboratories

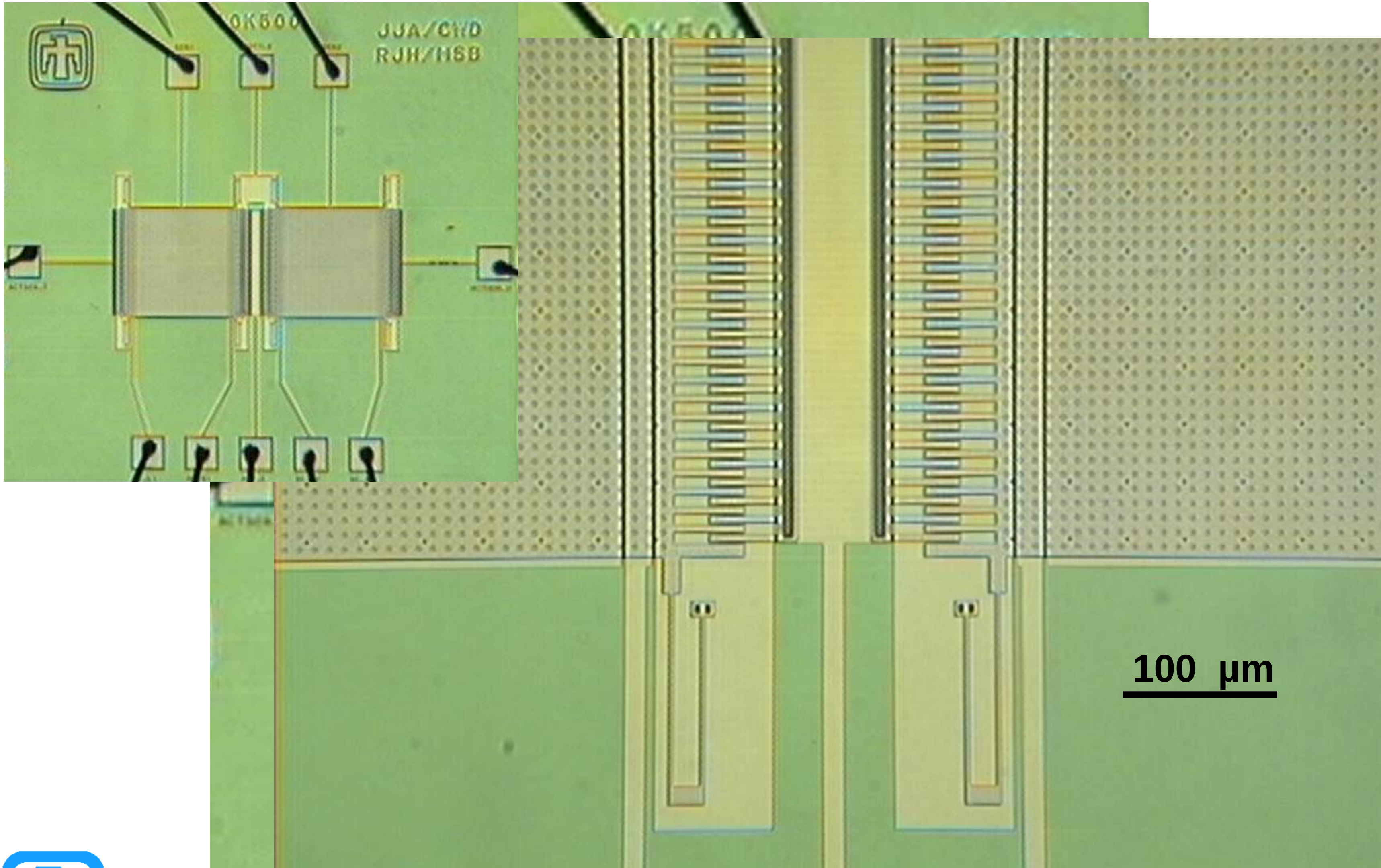


Electronic packages



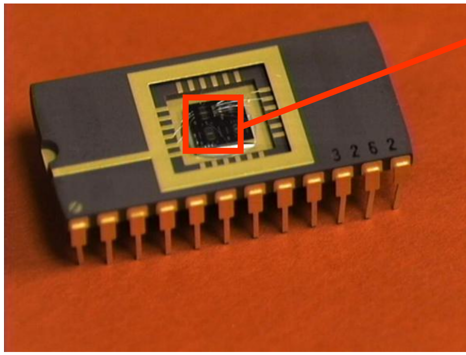
MEMS gyroscopes: packages

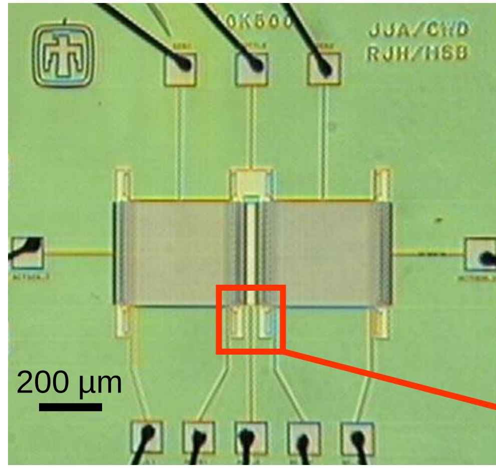
SNL – Sandia National Laboratories



MEMS gyroscopes: SNL

Microgyro package





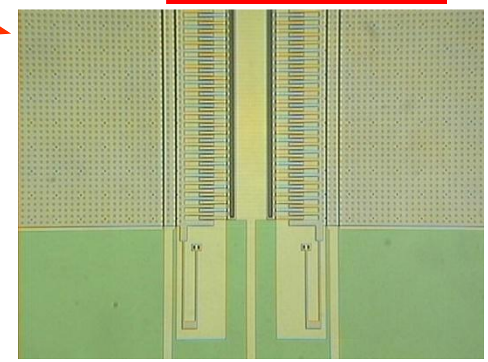
10 kHz microgyro: 300 µm × 300 µm shuttles, configured as a differential pair

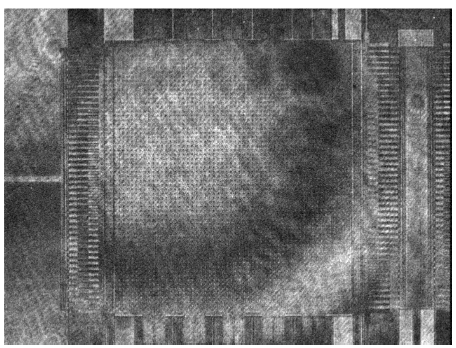
Relative motion analysis:

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

Acceleration:

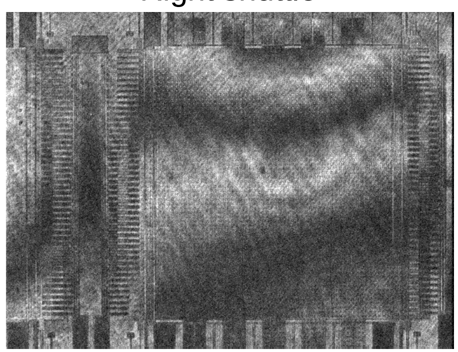
$$\bar{a}_B = \bar{a}_A + \dot{\bar{\Omega}} \times \bar{r}_{B/A} + \bar{\Omega} \times (\bar{\Omega} \times \bar{r}_{B/A}) + 2\bar{\Omega} \times (\dot{\bar{r}}_{B/A})_{xyz} + (\bar{a}_{B/A})_{xyz}$$

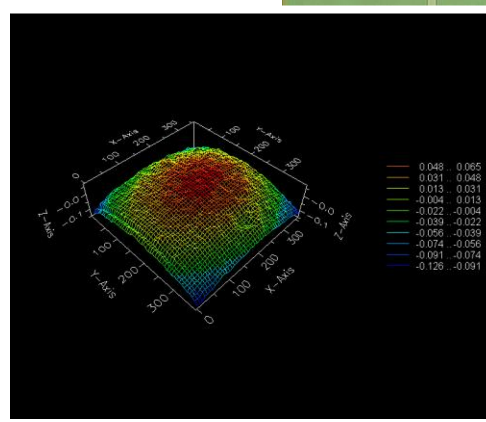




Left shuttle

Right shuttle





OEHM images and deformations of the microgyro operating at 10 kHz: 217 nm max

Reading assignment

- **Beckwith**: Ch. 6, 17, Appendix B
- **Bishop:** Ch. 11

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- Sandia National Laboratories. http://www.sandia.gov

Homework assignment: Handout-M

- **Beckwith**: 17.1
- **Bishop:** Section 11.7.1