WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 09

27 September 2010

General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

Teaching Assistant: Jeffrey Laut & Kazim Naqvi;
During Lab Sessions

Strain gages

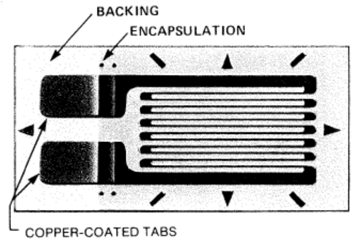
Definition of gage factor: $$F = \frac{dR/R}{\varepsilon_a}$$

(From previous discussion) $$\Rightarrow \quad F = 1 + 2\mu + \frac{1}{\varepsilon_a}\frac{d\rho}{\rho}$$

If resistivity does not change $$\Rightarrow \quad F = 1 + 2\mu$$

And strain with change of resistance is: $$\Rightarrow \quad \varepsilon_a = \frac{1}{F}\frac{\Delta R}{R}$$

2D view of a strain gage



A typical strain gage has a gage factor ≈ 2.095 ± 0.5%.
Why? How is this possible? Open for discussions

Strain gages and a Wheatstone bridge

Recall from previous discussions:

(Changes in resistance & output voltage)

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R} = \frac{\Delta R}{4R}$$

And strain with change of resistance is:

$$\Rightarrow \quad \varepsilon_a = \frac{1}{F}\frac{\Delta R}{R}$$

We want to recover strain from voltage measurements.
Combine previous equations:

$$\Rightarrow \quad \varepsilon_a = \frac{1}{F}\frac{4\Delta E_g}{E}$$

Strain gages and a Wheatstone bridge

We need to amplify output signal: **determine gain**

Re-write previous equation as:

$$\Delta E_g = \frac{F}{4} \cdot E \cdot \varepsilon_a$$

Assume the following values:
(based on an actual setup)

$$E = 10 \pm 0.005\,V$$

$$F = 2.095 \pm 0.5\%$$

Also, assume the measurement of only 1 μstrain (εμ):

$$\varepsilon_a = 1\,\mu\text{strain} = 1 \times 10^{-6}$$

Using these values leads to:

$$\Delta E_g = 5.238 \times 10^{-6}\,V$$

Is it possible to measure this voltage level in HL-031?
Open for discussions

Strain gages and a Wheatstone bridge
We need to amplify output signal: **determine gain**

Assume that measurement resolution of DAQ system is:
(please, update accordingly, while taking into account max./min. voltages allowed in the DAQs input)

$$1 \times 10^{-3}\, V$$

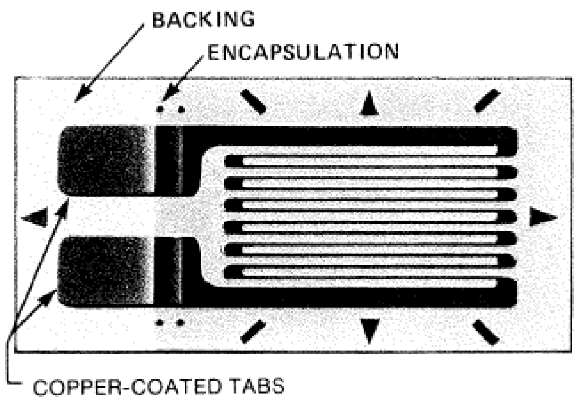
Gain for the <u>output signal</u> should be:

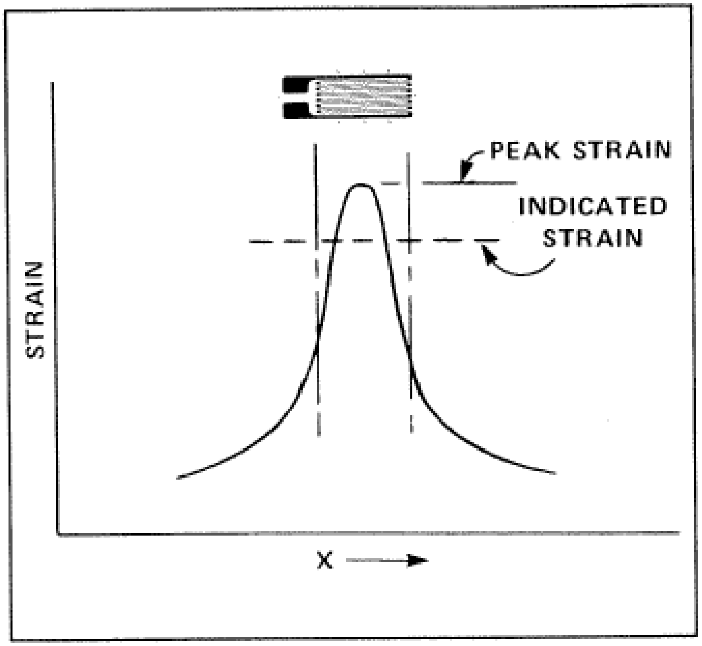
$$Gain = \frac{1 \times 10^{-3}\, V}{5.238 \times 10^{-6}\, V} \approx 191$$

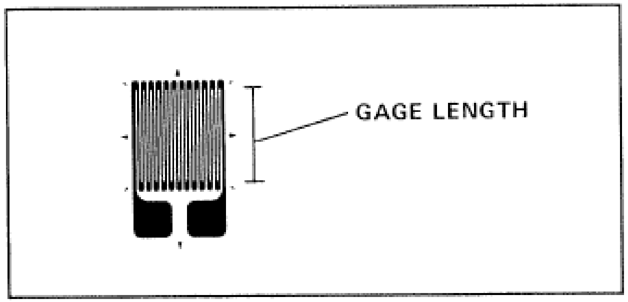
If we use max. meas. resolution of DAQs in HL-031, what is the range of strain values that can be measured?

Open for discussions

Note on strain gages design (typical: 0.001" thick)







$$\text{Gage factor} = F = \frac{dR/R}{\varepsilon_x}$$

$$\text{Resistance} = R = \rho \frac{L}{A}$$

resistivity

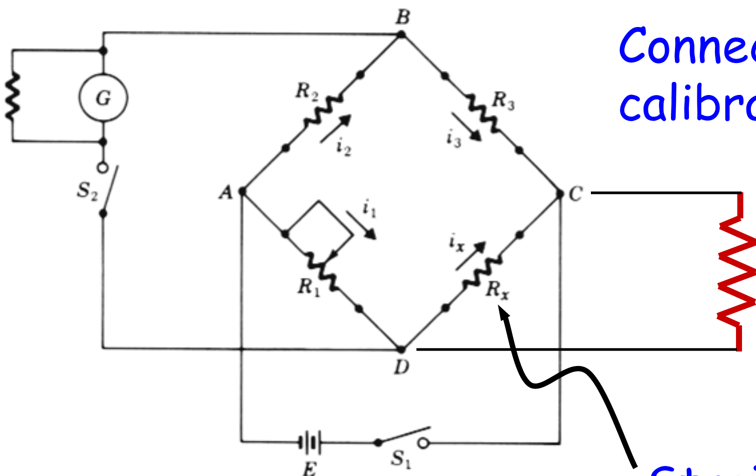
$$\boxed{\varepsilon_x = \frac{1}{F}\frac{\Delta R}{R}}$$

To be measured (use a bridge circuit)

Strain gages and a Wheatstone bridge

Calibration by use of shunt resistors

Output



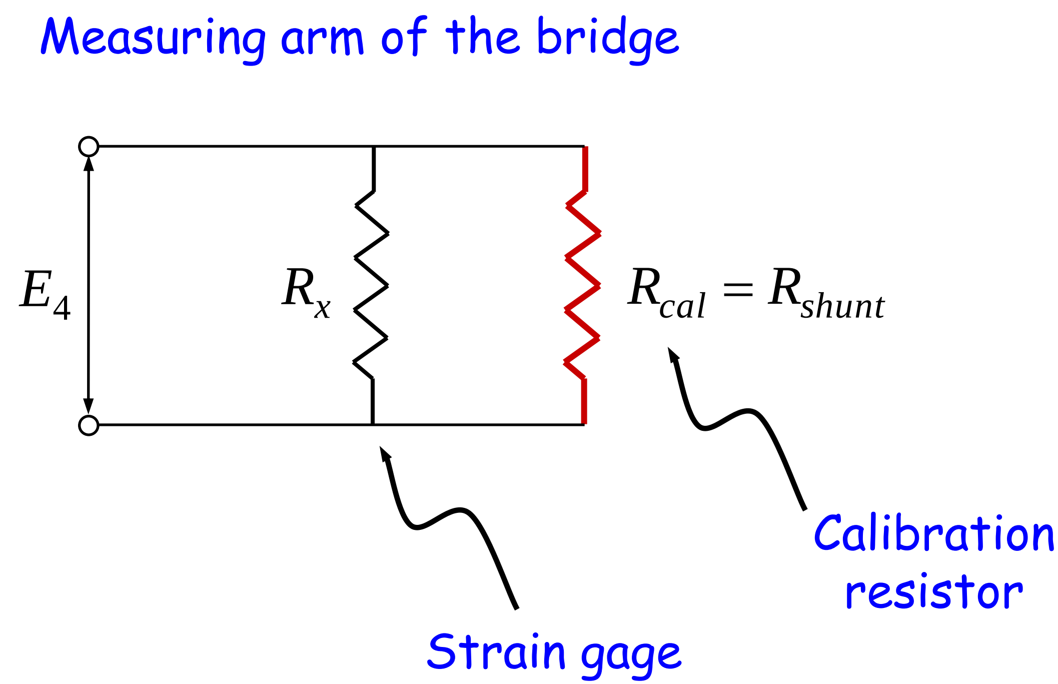
Connect high-precision calibration resistor in "parallel"

$$R_{cal} = R_{shunt}$$

Strain gage

Excitation source

Strain gages and a Wheatstone bridge
Calibration by use of shunt resistors



Strain gages and a Wheatstone bridge
Calibration by use of shunt resistors

Equivalent resistance

$$\frac{1}{R} = \frac{1}{R_x} + \frac{1}{R_{cal}} \quad \Rightarrow \quad R = \frac{R_x \cdot R_{cal}}{R_x + R_{cal}}$$

Strain gages and a Wheatstone bridge
Calibration by use of shunt resistors

Change in resistance is

$$\Delta R = R - R_x = \frac{R_x \cdot R_{cal}}{R_x + R_{cal}} - R_x$$

$$= -\frac{R_x^2}{R_x + R_{cal}}$$

Strain gages and a Wheatstone bridge

Calibration by use of shunt resistors

Using the definition of a gage factor:

$$\varepsilon_{cal} = \frac{\Delta R}{F\ R_x} \quad \Rightarrow \quad \varepsilon_{cal} = -\frac{R_x}{F\,(R_x + R_{cal})}$$

Indicates compression

Strain gages and a Wheatstone bridge
Calibration by use of shunt resistors

Example

If: $R_{cal} = 878{,}000\,\Omega$; $R_x = 120\,\Omega$ with $F = 2.095$

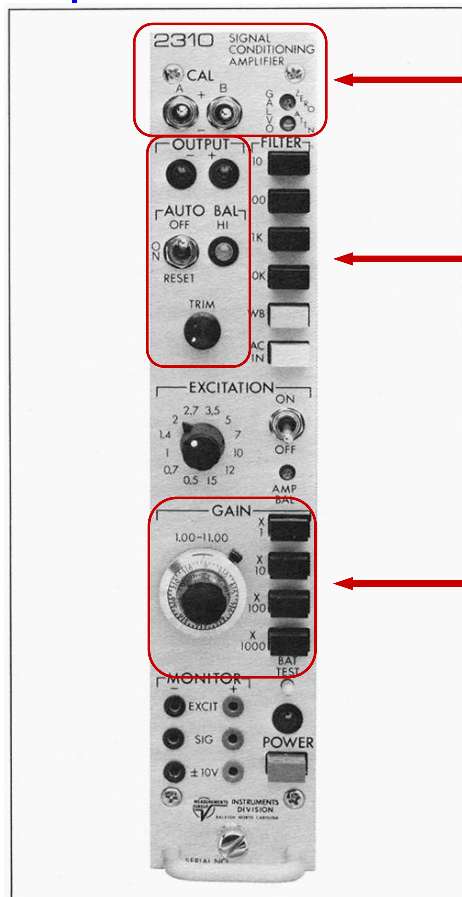
$$\Rightarrow \quad \varepsilon_{cal} = -\frac{R_x}{F\,(R_x + R_{cal})} = -\frac{120}{2.095(120 + 878{,}000)} = -65.2 \times 10^{-6}$$

$$= -65.2\ \mu\text{strain (compression)}$$

Strain gages and a Wheatstone bridge

Calibration by use of shunt resistors

Amplifier model 2310



2310 Front Panel

Internal calibration resistors

Internal variable resistor (bridge calibration)

Gain
(note resolution in gain settings)

Strain gages and a Wheatstone bridge
Calibration by use of shunt resistors

Amplifier model 2310 in ¼ bridge configuration

+ A: 59.94 kΩ ⇒ ≈ 1000 μstrain

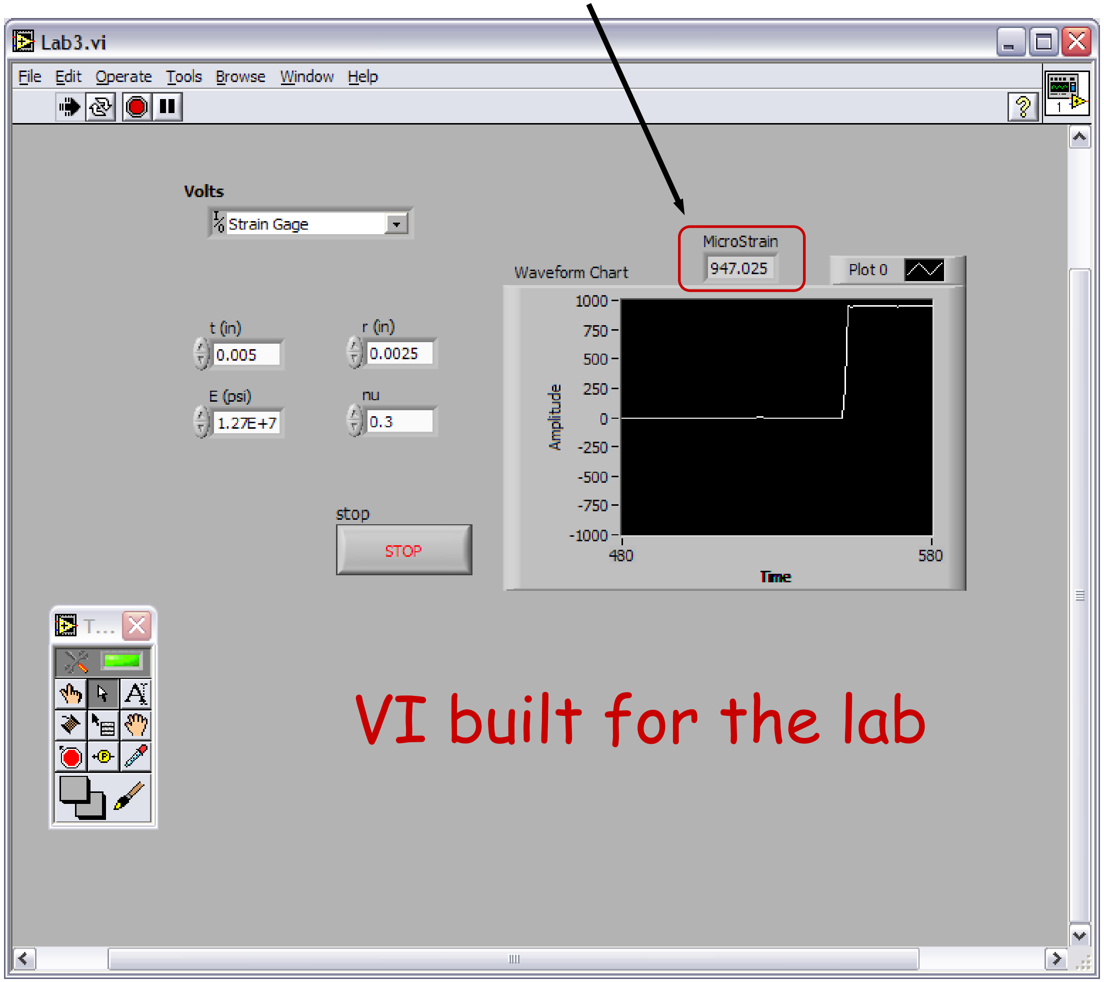
+ B: 174.8 kΩ ⇒ ≈ 340 μstrain

Check + + - and ε_{cal}

Strain gages and a Wheatstone bridge

Calibration by use of shunt resistors

+ A resistor

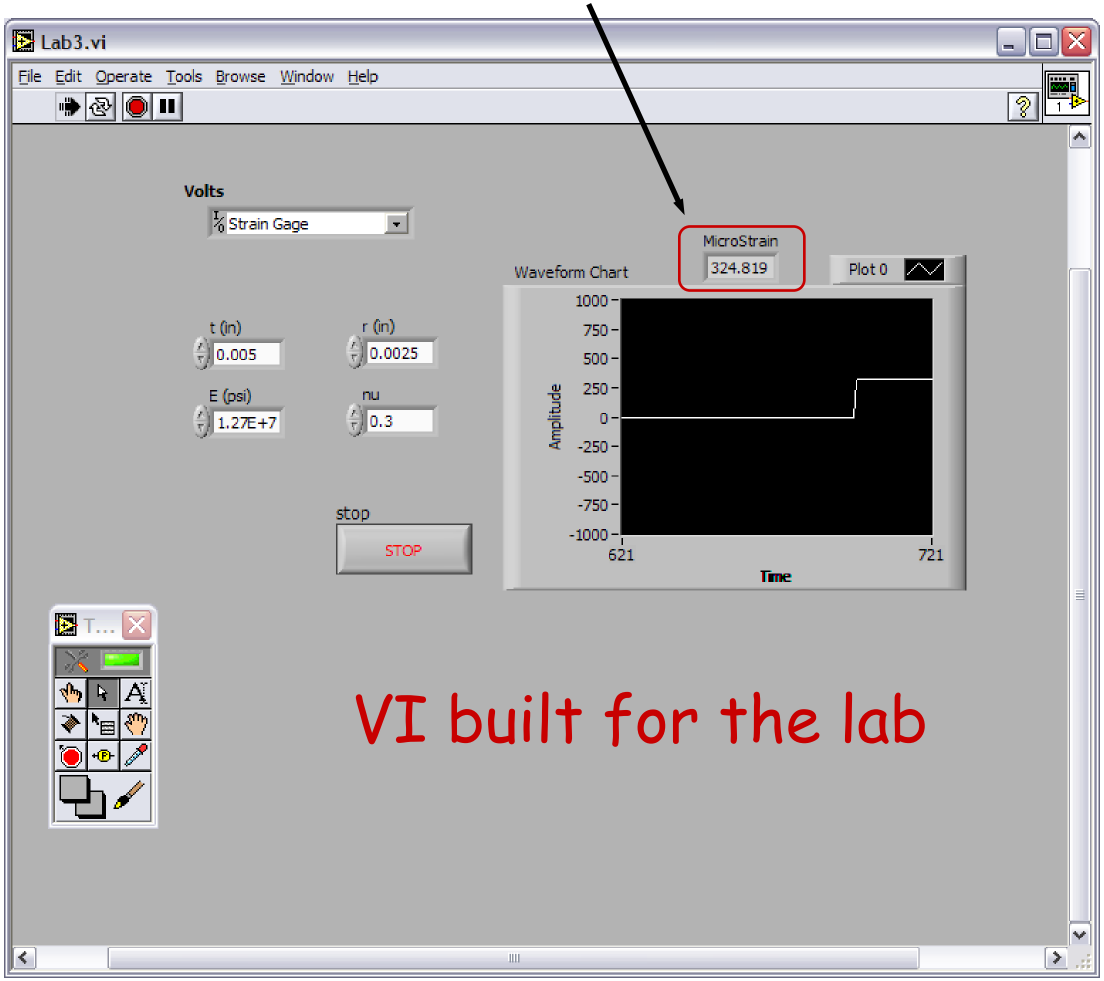


VI built for the lab

Strain gages and a Wheatstone bridge

Calibration by use of shunt resistors

+ B resistor



VI built for the lab

Use of strains to compute pressure and stresses

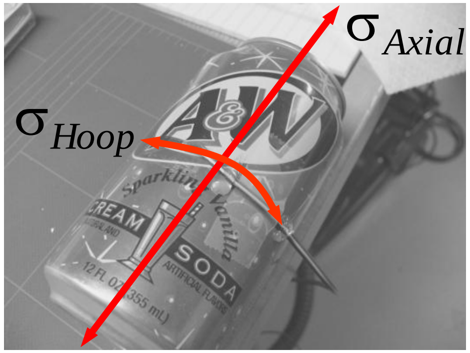
Governing equation (based on tangential - Hoop - strain):

$$P = \frac{E\,t\,\varepsilon}{r\,(1-\nu/2)} \quad \Rightarrow \quad P = P(E, t, \varepsilon, r, \nu)$$

Pressure is obtained from these equations:

$$\varepsilon_{Hoop} = \varepsilon = \frac{1}{E}(\sigma_{Hoop} - \nu\sigma_{Axial});$$

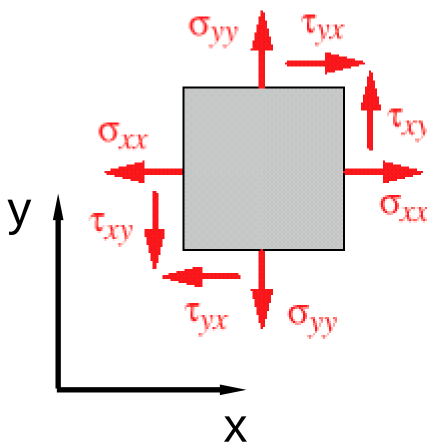
$$\sigma_{Hoop} = \frac{P\,r}{t}; \quad \sigma_{Axial} = \frac{P\,r}{2\,t}$$



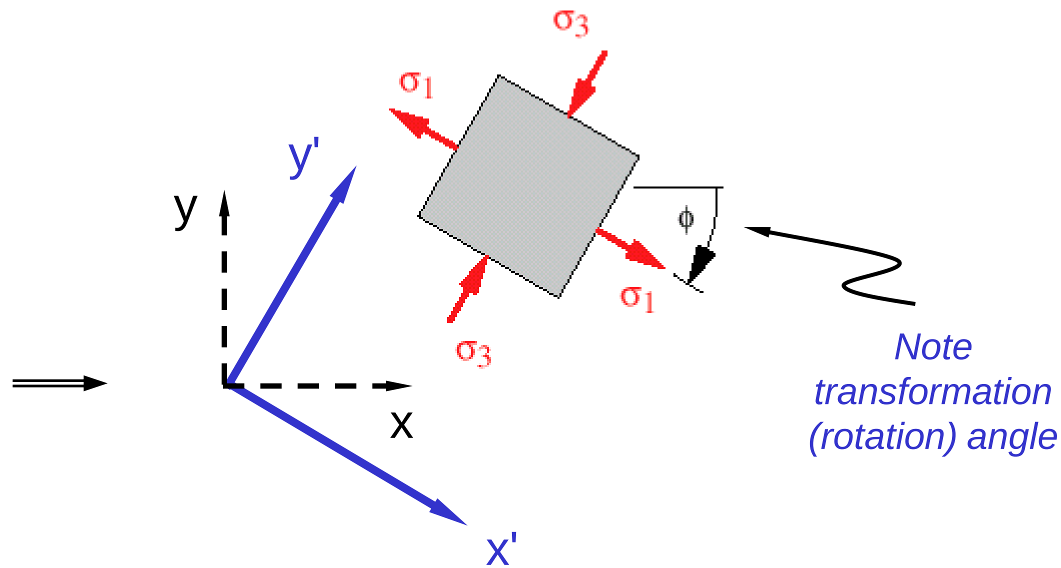
Determination of principal stresses

Principal ***normal*** stresses

- This problem involves performing coordinate transformation, which can provide a stress tensor that does NOT contain shear stresses
- In 2D, this can be illustrated as:



Stress cube in original coordinate system (x,y)



Stress cube in transformed coordinate system (x',y') -- only normal stresses exist: σ_1 and σ_3, in this 2D case

Determination of principal stresses

Principal ***normal*** stresses

This problem involves performing coordinate transformation, which can provide a stress tensor that does NOT contain shear stresses, that is

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \hat{\mathbf{n}} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} \hat{\mathbf{n}}$$

Initial stress tensor

Unit vector, normal to principal plane

Transformed stress tensor

Unit vector, normal to principal plane

Same vectors

Determination of principal stresses

Principal ***normal*** stresses

Previous equation can be written as

$$\begin{bmatrix} \sigma_{xx}-\sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz}-\sigma \end{bmatrix}\hat{\mathbf{n}} = \begin{bmatrix} \sigma_{xx}-\sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz}-\sigma \end{bmatrix}\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

implying that the determinant

$$\begin{vmatrix} \sigma_{xx}-\sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz}-\sigma \end{vmatrix} = 0$$

Determination of principal stresses

Principal ***normal*** stresses

Expanding determinant and setting it to zero yields

$$\sigma^3 - C_2\sigma^2 + C_1\sigma - C_0 = 0$$

in which

$$C_2 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$C_1 = \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$C_0 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2$$

are stress invariants (have the same magnitudes for all choices of coordinate axes (x,y,z) in which the applied stresses are measured or calculated.)

The *principal normal stresses*, σ_1, σ_2, σ_3, are the three roots of the cubic polynomial -- always real and typically ordered as: $\sigma_1 > \sigma_2 > \sigma_3$

Determination of principal stresses

Principal ***shear*** stresses

Principal ***shear stresses can be found from values*** of the principal normal stresses as

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{21} = \frac{|\sigma_2 - \sigma_1|}{2}$$

$$\tau_{32} = \frac{|\sigma_3 - \sigma_2|}{2}$$

Determination of principal stresses

Principal ***normal and shear*** stresses: **2D case**

These equations are used extensively

Principal normal stresses:

$$\sigma_1, \sigma_3 = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

Maximum shear stress:

$$\tau_{Max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

Mohr's circle: principal normal and shear stresses

Graphical representation of previous equations: **3D**

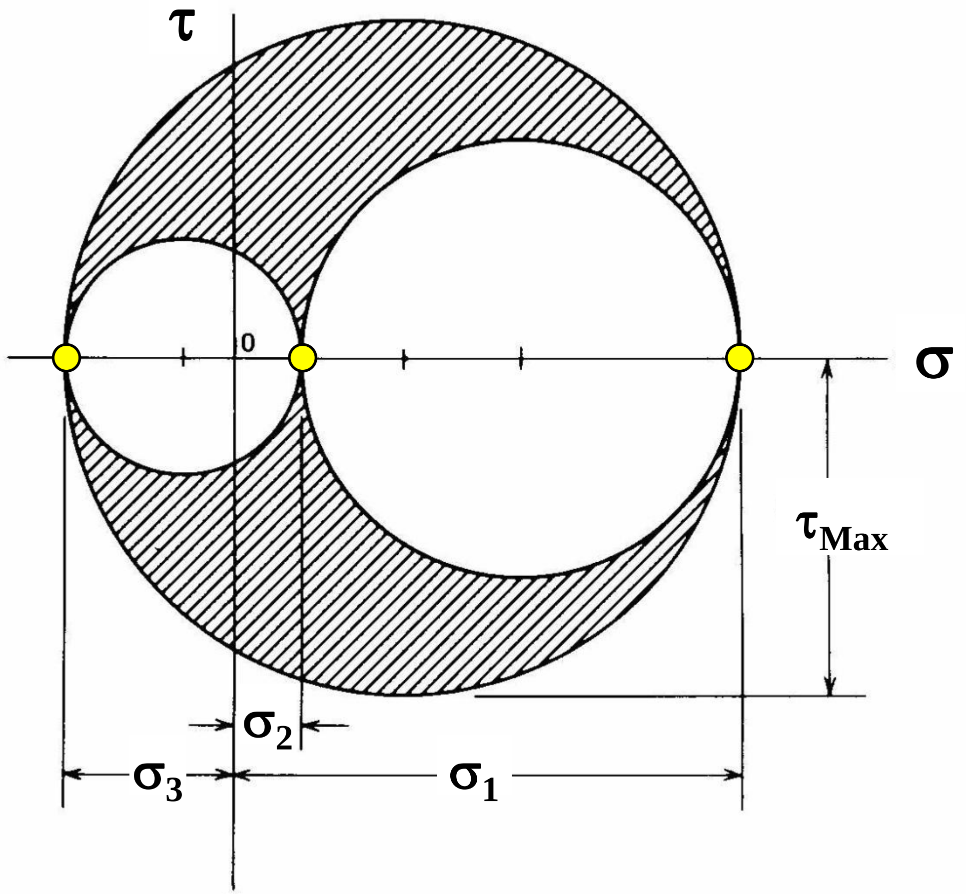


Fig. 1-4.5/Mohr's Circle in three dimensions.
From Boresi: Mechanics of materials

Reading assignment

- **Beckwith**: Ch. 7, 12, Appendix E
- **Bishop:** Ch. 11

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 6th ed., Prentice-Hall, 2007
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, http://www.gesensing.com/

Homework assignment: Handout-H

- **Beckwith**: 12.7, 12.10, 12.11
- **Bishop:** Section 11.3