

# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation  
ME-3901, A'2010

Lecture 08

22 September 2010



# General information

## Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu  
Everyday from 11:00 to 11:50 am  
or by appointment

Teaching Assistant: Jeffrey Laut & Kazim Naqvi;  
During Lab Sessions



# Comments on: Homework (C and D)

- Homework C:

- ▲ C1.- Make sure to identify level of importance of the uncertainties in the variables involved in the evaluation of the overall uncertainty in  $\sigma_{\text{Hoop}}$
- ▲ C2.- Make sure to identify level of importance of the uncertainties in the variables involved in the evaluation of the overall uncertainty in  $\sigma_{\text{Long}}$
- ▲ C3.- Nominal values for the parameters involved can be found in material for Lab#3

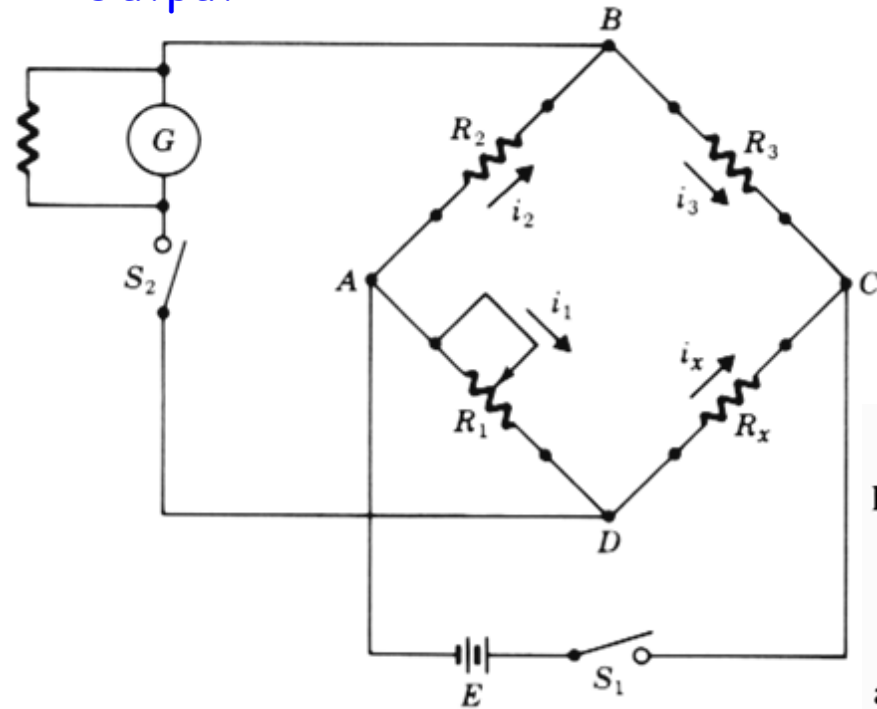
- Homework D:

- ▲ D2.- You need to go to the lab and characterize a 120  $\Omega$  resistor



# Wheatstone bridge

Output



Excitation  
source

- When bridge is balanced: voltage drop across  $R_2$  is equal to voltage drop across  $R_1$ , since voltage difference between  $B$  and  $D$  is equal to zero. Therefore,

$$i_2 R_2 = i_1 R_1$$

Further,

$$i_2 = i_3 = \frac{E}{R_2 + R_3} \quad \text{if balanced}$$

and

$$i_1 = i_x = \frac{E}{R_1 + R_x} \quad \text{if balanced}$$

If the currents are eliminated from these relations, the result is

$$\frac{R_2}{R_3} = \frac{R_1}{R_x}$$

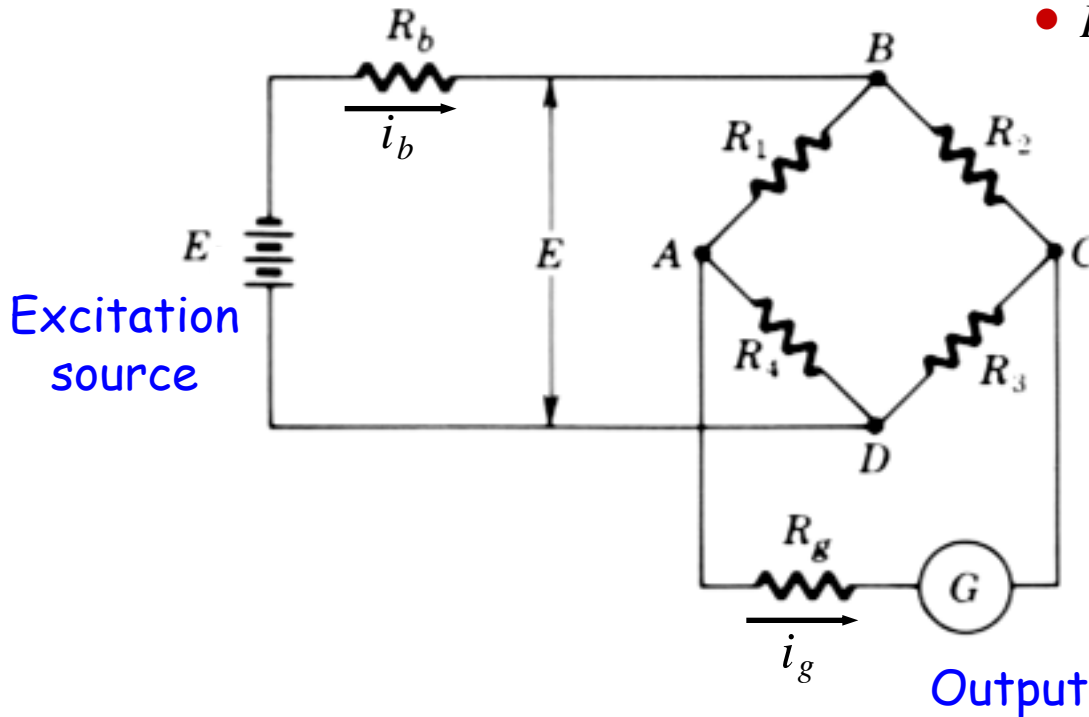
or

$$R_x = \frac{R_1 R_3}{R_2}$$

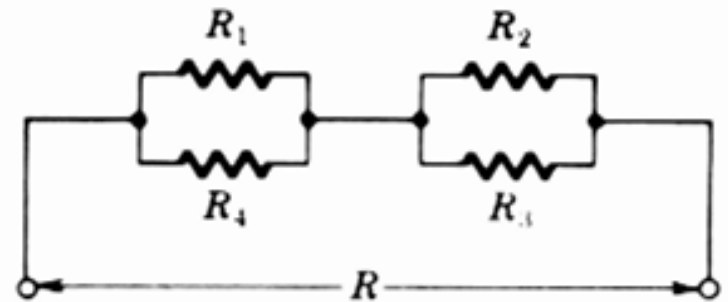


# Wheatstone bridge: unbalanced bridge

- $R_1, R_2, R_3, R_4$  are different



Equivalent circuit of bridge at the output:



# Wheatstone bridge: unbalanced bridge

Considering voltage divider  
on a bridge:

$$\begin{aligned} E_g &= \left( \frac{E}{R_1 + R_4} \right) R_1 - \left( \frac{E}{R_2 + R_3} \right) R_2 \\ &= E \left( \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \end{aligned}$$

and

$$\begin{aligned} E_g &= \left( \frac{E}{R_1 + R_4} \right) R_4 - \left( \frac{E}{R_2 + R_3} \right) R_3 \\ &= E \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) \end{aligned}$$



# Wheatstone bridge: unbalanced bridge

What about if one resistor changes by a small amount?

Use:  $\Delta R_4 \Rightarrow \Delta E_g$

Therefore, 
$$E_g + \Delta E_g = E \left( \frac{(R_4 + \Delta R_4) R_2 - R_3 R_1}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} \right)$$

Divide numerator and denominator by:  $R_2 R_4$

$$E_g + \Delta E_g = E \left( \frac{1 + \Delta R_4 / R_4 - R_3 R_1 / R_4 R_2}{(1 + R_1 / R_4 + \Delta R_4 / R_4)(1 + R_3 / R_2)} \right)$$



# Wheatstone bridge: unbalanced bridge

What about if one resistor changes by a small amount?

If all resistors are initially the same:

$$(E_g = 0; \quad R_i = R)$$

$$\frac{\Delta E_g}{E} = \frac{\Delta R_4 / R}{4 + 2(\Delta R_4 / R)}$$

But because changes in resistance are small, i.e.,  $\Delta R_4 \ll 1$

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R}$$



(Remember this equation)



# Strain gages and a Wheatstone bridge

Recall from previous discussions that:  
(Changes in resistance & output voltage)

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R} = \frac{\Delta R}{4R}$$

And strain with change of resistance is:

$$\Rightarrow \varepsilon_a = \frac{1}{F} \frac{\Delta R}{R}$$

We want to recover strain from voltage measurements.  
Combine previous equations:

$$\Rightarrow \varepsilon_a = \frac{1}{F} \frac{4\Delta E_g}{E}$$



# Strain gages and a Wheatstone bridge

We need to amplify output signal: **determine gain**

Re-write previous equation as:

$$\Delta E_g = \frac{F}{4} \cdot E \cdot \varepsilon_a$$

Assume the following values:  
(based on an actual setup)

$$E = 10 \pm 0.005 \text{ V}$$

$$F = 2.095 \pm 0.5\%$$

Also, assume the measurement of  
only 1  $\mu$ strain ( $\varepsilon_\mu$ ):

$$\varepsilon_a = 1 \mu\text{strain} = 1 \times 10^{-6}$$

Using these values leads to:

$$\Delta E_g = 5.238 \times 10^{-6} \text{ V}$$

Is it possible to measure this voltage level in HL-031?

**Open for discussions**



# Strain gages and a Wheatstone bridge

We need to amplify output signal: **determine gain**

Assume that measurement resolution of DAQ system is:  
(please, update accordingly, while taking into account max./min. voltages allowed in the DAQs input)

$$1 \times 10^{-3} \text{ V}$$

Gain for the output signal should be:

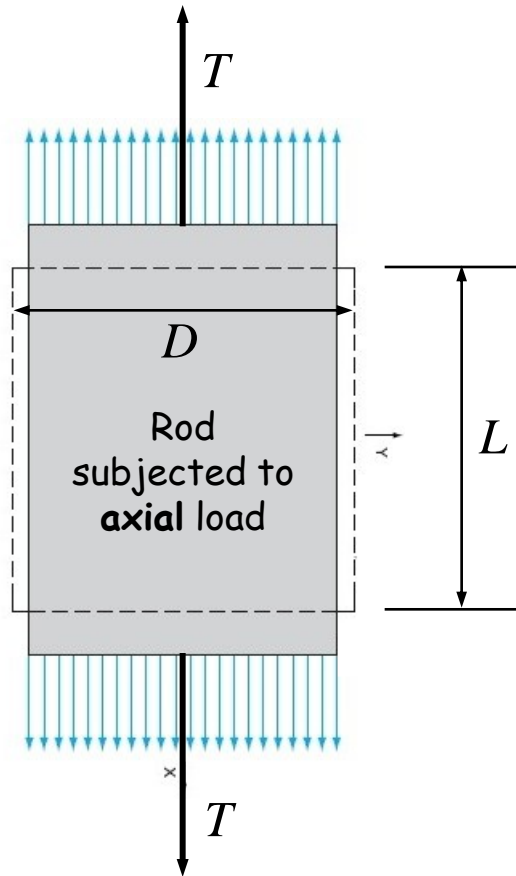
$$\text{Gain} = \frac{1 \times 10^{-3} \text{ V}}{5.238 \times 10^{-6} \text{ V}} \approx 191$$

If we use max. meas. resolution of DAQs in HL-031, what is the range of strain values that can be measured?

Open for discussions



# Stress and strain



Axial strain: 
$$\varepsilon = \frac{T/A}{E} = \frac{\sigma_a}{E} = \frac{dL}{L} = \varepsilon_a$$

Poisson's ratio: 
$$\mu = -\frac{\varepsilon_t}{\varepsilon_a} = -\frac{dD/D}{dL/L}$$

( $\mu \approx 0.3$  for most metals)

Volume of rod is: 
$$V = L \cdot A = L \cdot \frac{\pi}{4} D^2$$

Volume is constant, therefore

$$dV = 0 = L dA + A dL$$

$$\Rightarrow \frac{dA}{A} = -\frac{dL}{L} \Rightarrow 2 \frac{dD}{D} = -\frac{dL}{L}$$

$$dV = 0 = D dL + 2L dD$$

(i.e.,  $\mu = 0.5$ , in this condition)



# Strain gages

Electrical resistance:  $R = \rho \frac{L}{A}$

resistivity  $\rho$  (indicated by a blue arrow pointing to  $\rho$ )

length  $L$  (indicated by a blue arrow pointing to  $L$ )

cross-sectional area  $A$  (indicated by a blue arrow pointing to  $A$ )

Differentiate resistance:  $dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \frac{dD}{D} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left( -\mu \frac{dL}{L} \right)$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left( -\mu \frac{dL}{L} \right) = \frac{d\rho}{\rho} + \varepsilon_a (1 + 2\mu)$$



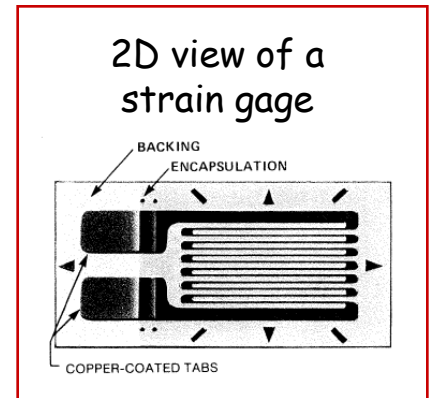
# Strain gages

Definition of gage factor:  $F = \frac{dR/R}{\varepsilon_a}$

(From previous page)  $\Rightarrow F = 1 + 2\mu + \frac{1}{\varepsilon_a} \frac{d\rho}{\rho}$

If resistivity does not change  $\Rightarrow F = 1 + 2\mu$

And strain with change of resistance is:  $\Rightarrow \varepsilon_a = \frac{1}{F} \frac{\Delta R}{R}$



A typical strain gage has a gage factor  $\approx 2.095 \pm 0.5\%$ .  
Why? How is this possible? Open for discussions



# Strain gages

Consider these typical strain gage parameters:

$$\Rightarrow F = 2.0 \quad R = 120 \Omega$$

If interested in measuring strains at the level of 1 ppm (i.e., 1-microstrain), the expected change in resistance is

$$dR = F R \varepsilon_a = (2)(120 \Omega)(1 \times 10^{-6}) = 0.00024 \Omega \Rightarrow 240 \mu\Omega$$

or a change in resistance of  $100 \times 0.00024/120 = 0.0002\%$  !!

We need a measuring device more  
sensitive than an Ohmmeter !

(A Wheatstone bridge + Amplification)



# Reading assignment

- **Beckwith:** Ch. 7, 12, Appendix E
- **Bishop:** Ch. 11

## References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, <http://www.gesensing.com/>



# Homework assignment: Handout-F

- Bishop: Section 11.2.3

F1.- A typical strain gage has a gage factor,  $F$ , of  $2.095 \pm 0.5\%$ . Strain gages are made with alloys having typical Poisson's ratios of 0.35. How is it possible to (hint) obtain such value for  $F$ ? Discuss in detail.

F2.- Derive **complete** RSS uncertainty equation for the "amplified" output voltage of a Wheatstone bridge configured to measure one strain component, and

(a) Plot uncertainty of the "amplified" output voltage as a function of strain; use values for  $F$  and  $E$  given in these notes (assume that uncertainty in strain is constant- why?); use a gain value (and uncertainty) suitable to the maximum resolution that can be obtain with the DAQs in HL-031;

(b) Plot "amplified" output voltage as a function of measured strain together with corresponding uncertainty limits (add and subtract uncertainties);

(c) Plot, in a same graph, individual uncertainties as a function of strain (make sure to check that sum of all uncertainties is 100 %);

(d) For the meas. resolution of the DAQs in HL-031, what is the maximum (tension) and minimum (compression) level of strain that can be measured?



Discuss your observations.

# Reminder

Please, bring at least three "prepared" soda cans (aluminum) to the lab

