

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 08

22 September 2010



General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

Teaching Assistant: Jeffrey Laut & Kazim Naqvi;
During Lab Sessions



Comments on: Homework (C and D)

- Homework C:

- ▲ C1.- Make sure to identify level of importance of the uncertainties in the variables involved in the evaluation of the overall uncertainty in σ_{Hoop}
- ▲ C2.- Make sure to identify level of importance of the uncertainties in the variables involved in the evaluation of the overall uncertainty in σ_{Long}
- ▲ C3.- Nominal values for the parameters involved can be found in material for Lab#3

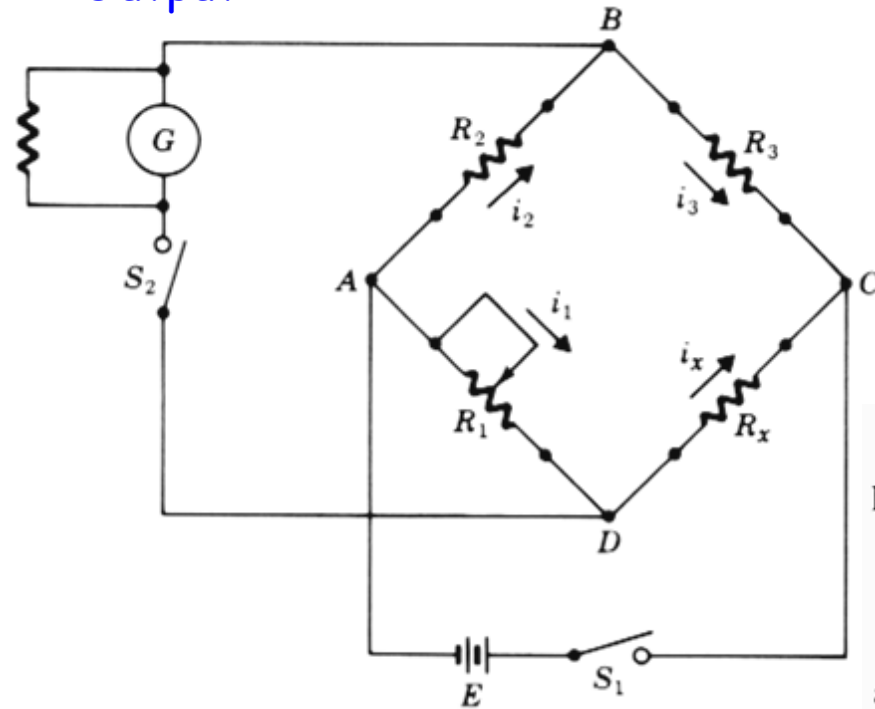
- Homework D:

- ▲ D2.- You need to go to the lab and characterize a 120 Ω resistor



Wheatstone bridge

Output



Excitation
source

- When bridge is balanced: voltage drop across R_2 is equal to voltage drop across R_1 , since voltage difference between B and D is equal to zero. Therefore,

$$i_2 R_2 = i_1 R_1$$

Further,

$$i_2 = i_3 = \frac{E}{R_2 + R_3} \quad \text{if balanced}$$

and

$$i_1 = i_x = \frac{E}{R_1 + R_x} \quad \text{if balanced}$$

If the currents are eliminated from these relations, the result is

$$\frac{R_2}{R_3} = \frac{R_1}{R_x}$$

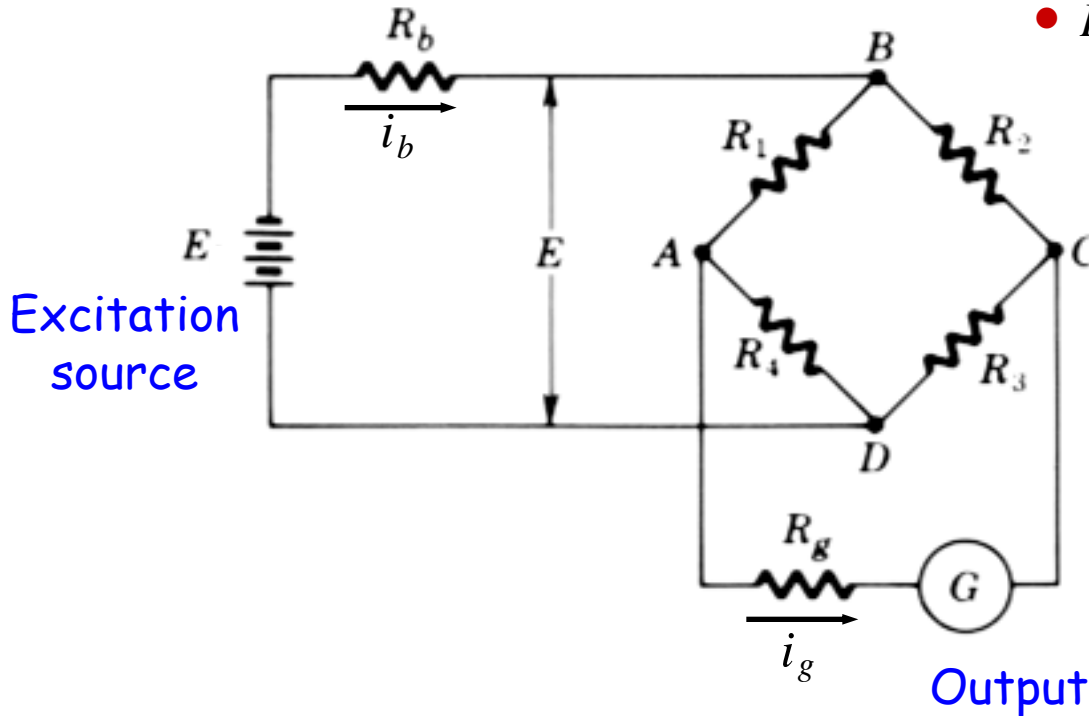
or

$$R_x = \frac{R_1 R_3}{R_2}$$

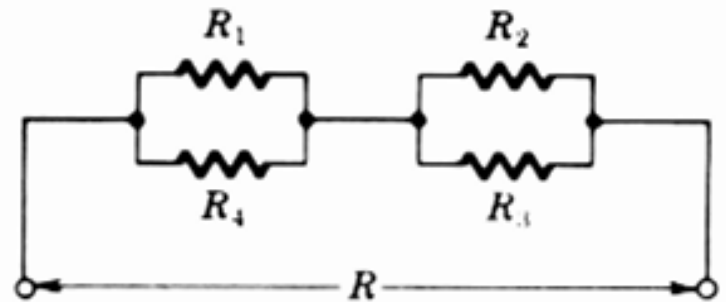


Wheatstone bridge: unbalanced bridge

- R_1, R_2, R_3, R_4 are different



Equivalent circuit of bridge at the output:



Wheatstone bridge: unbalanced bridge

Considering voltage divider
on a bridge:

$$\begin{aligned} E_g &= \left(\frac{E}{R_1 + R_4} \right) R_1 - \left(\frac{E}{R_2 + R_3} \right) R_2 \\ &= E \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \end{aligned}$$

and

$$\begin{aligned} E_g &= \left(\frac{E}{R_1 + R_4} \right) R_4 - \left(\frac{E}{R_2 + R_3} \right) R_3 \\ &= E \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) \end{aligned}$$



Wheatstone bridge: unbalanced bridge

What about if one resistor changes by a small amount?

Use: $\Delta R_4 \Rightarrow \Delta E_g$

Therefore,
$$E_g + \Delta E_g = E \left(\frac{(R_4 + \Delta R_4) R_2 - R_3 R_1}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} \right)$$

Divide numerator and denominator by: $R_2 R_4$

$$E_g + \Delta E_g = E \left(\frac{1 + \Delta R_4 / R_4 - R_3 R_1 / R_4 R_2}{(1 + R_1 / R_4 + \Delta R_4 / R_4)(1 + R_3 / R_2)} \right)$$



Wheatstone bridge: unbalanced bridge

What about if one resistor changes by a small amount?

If all resistors are initially the same:

$$(E_g = 0; \quad R_i = R)$$

$$\frac{\Delta E_g}{E} = \frac{\Delta R_4 / R}{4 + 2(\Delta R_4 / R)}$$

But because changes in resistance are small, i.e., $\Delta R_4 \ll 1$

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R}$$



(Remember this equation)



Strain gages and a Wheatstone bridge

Recall from previous discussions that:
(Changes in resistance & output voltage)

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R} = \frac{\Delta R}{4R}$$

And strain with change of resistance is:

$$\Rightarrow \varepsilon_a = \frac{1}{F} \frac{\Delta R}{R}$$

We want to recover strain from voltage measurements.
Combine previous equations:

$$\Rightarrow \varepsilon_a = \frac{1}{F} \frac{4\Delta E_g}{E}$$



Strain gages and a Wheatstone bridge

We need to amplify output signal: **determine gain**

Re-write previous equation as:

$$\Delta E_g = \frac{F}{4} \cdot E \cdot \varepsilon_a$$

Assume the following values:
(based on an actual setup)

$$E = 10 \pm 0.005 \text{ V}$$

$$F = 2.095 \pm 0.5\%$$

Also, assume the measurement of
only 1 μ strain (ε_μ):

$$\varepsilon_a = 1 \mu\text{strain} = 1 \times 10^{-6}$$

Using these values leads to:

$$\Delta E_g = 5.238 \times 10^{-6} \text{ V}$$

Is it possible to measure this voltage level in HL-031?

Open for discussions



Strain gages and a Wheatstone bridge

We need to amplify output signal: **determine gain**

Assume that measurement resolution of DAQ system is:
(please, update accordingly, while taking into account max./min. voltages allowed in the DAQs input)

$$1 \times 10^{-3} \text{ V}$$

Gain for the output signal should be:

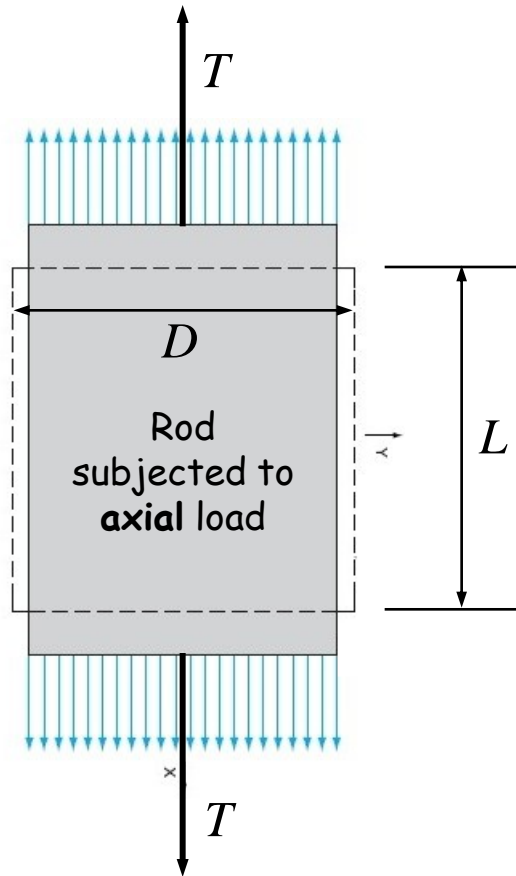
$$\text{Gain} = \frac{1 \times 10^{-3} \text{ V}}{5.238 \times 10^{-6} \text{ V}} \approx 191$$

If we use max. meas. resolution of DAQs in HL-031, what is the range of strain values that can be measured?

Open for discussions



Stress and strain



Axial strain:
$$\epsilon = \frac{T/A}{E} = \frac{\sigma_a}{E} = \frac{dL}{L} = \epsilon_a$$

Poisson's ratio:
$$\mu = -\frac{\epsilon_t}{\epsilon_a} = -\frac{dD/D}{dL/L}$$

($\mu \approx 0.3$ for most metals)

Volume of rod is:
$$V = L \cdot A = L \cdot \frac{\pi}{4} D^2$$

Volume is constant, therefore

$$dV = 0 = L dA + A dL$$

$$\Rightarrow \frac{dA}{A} = -\frac{dL}{L} \Rightarrow 2 \frac{dD}{D} = -\frac{dL}{L}$$

$$dV = 0 = D dL + 2L dD$$

(i.e., $\mu = 0.5$, in this condition)



Strain gages

Electrical resistance: $R = \rho \frac{L}{A}$

← length
← cross-sectional area
↑ resistivity

Differentiate resistance: $dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \frac{dD}{D} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left(-\mu \frac{dL}{L} \right)$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left(-\mu \frac{dL}{L} \right) = \frac{d\rho}{\rho} + \varepsilon_a (1 + 2\mu)$$



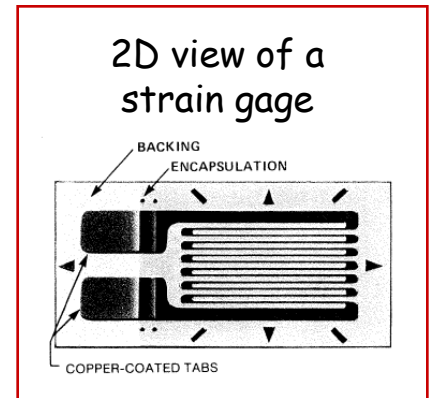
Strain gages

Definition of gage factor: $F = \frac{dR/R}{\varepsilon_a}$

(From previous page) $\Rightarrow F = 1 + 2\mu + \frac{1}{\varepsilon_a} \frac{d\rho}{\rho}$

If resistivity does not change $\Rightarrow F = 1 + 2\mu$

And strain with change of resistance is: $\Rightarrow \varepsilon_a = \frac{1}{F} \frac{\Delta R}{R}$



A typical strain gage has a gage factor $\approx 2.095 \pm 0.5\%$.
Why? How is this possible? Open for discussions



Strain gages

Consider these typical strain gage parameters:

$$\Rightarrow F = 2.0 \quad R = 120 \Omega$$

If interested in measuring strains at the level of 1 ppm (i.e., 1-microstrain), the expected change in resistance is

$$dR = F R \varepsilon_a = (2)(120 \Omega)(1 \times 10^{-6}) = 0.00024 \Omega \Rightarrow 240 \mu\Omega$$

or a change in resistance of $100 \times 0.00024/120 = 0.0002\%$!!

We need a measuring device more
sensitive than an Ohmmeter !

(A Wheatstone bridge + Amplification)



Reading assignment

- **Beckwith:** Ch. 7, 12, Appendix E
- **Bishop:** Ch. 11

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, <http://www.gesensing.com/>



Homework assignment: Handout-F

- Bishop: Section 11.2.3

F1.- A typical strain gage has a gage factor, F , of $2.095 \pm 0.5\%$. Strain gages are made with alloys having typical Poisson's ratios of 0.35. How is it possible to (hint) obtain such value for F ? Discuss in detail.

F2.- Derive **complete** RSS uncertainty equation for the "amplified" output voltage of a Wheatstone bridge configured to measure one strain component, and

(a) Plot uncertainty of the "amplified" output voltage as a function of strain; use values for F and E given in these notes (assume that uncertainty in strain is constant- why?); use a gain value (and uncertainty) suitable to the maximum resolution that can be obtain with the DAQs in HL-031;

(b) Plot "amplified" output voltage as a function of measured strain together with corresponding uncertainty limits (add and subtract uncertainties);

(c) Plot, in a same graph, individual uncertainties as a function of strain (make sure to check that sum of all uncertainties is 100 %);

(d) For the meas. resolution of the DAQs in HL-031, what is the maximum (tension) and minimum (compression) level of strain that can be measured?



Discuss your observations.

Reminder

Please, bring at least three "prepared" soda cans (aluminum) to the lab

