

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2009

Lecture 07

06 April 2009



General information

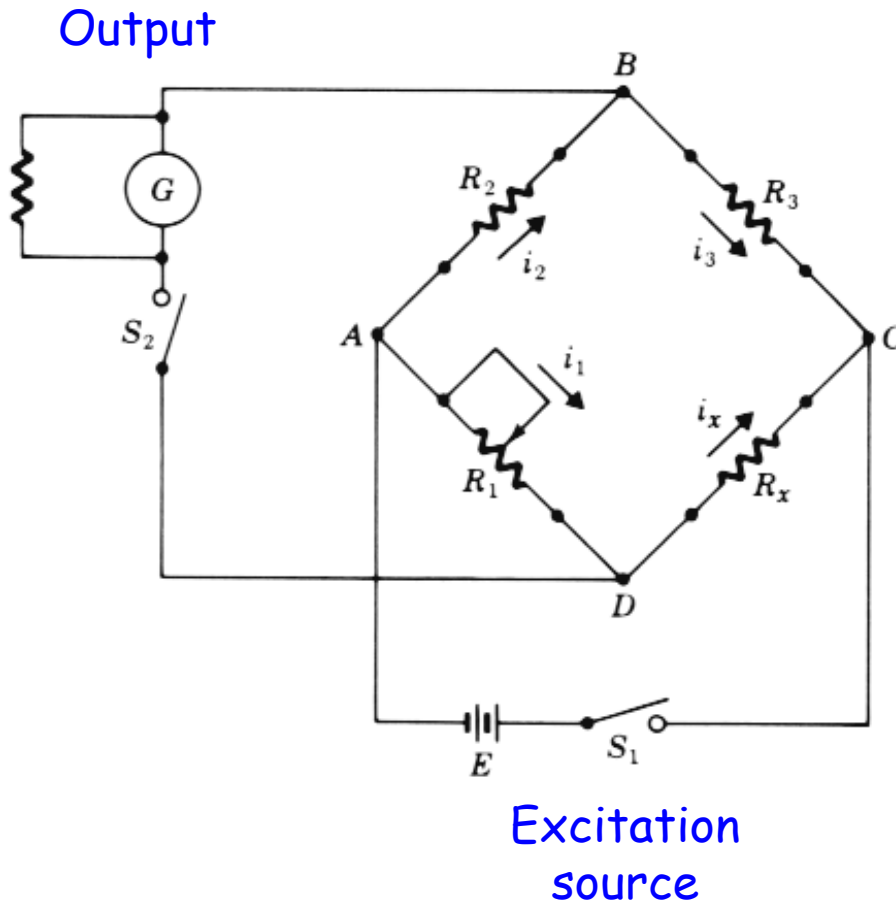
Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 10:00 to 11:00 am
or by appointment

Teaching Assistant: During laboratory sessions



Wheatstone bridge



Use for the comparison and measurement of resistances from 1Ω to $1 \text{ M} \Omega$

Resistances are arranged in a "diamond" shape

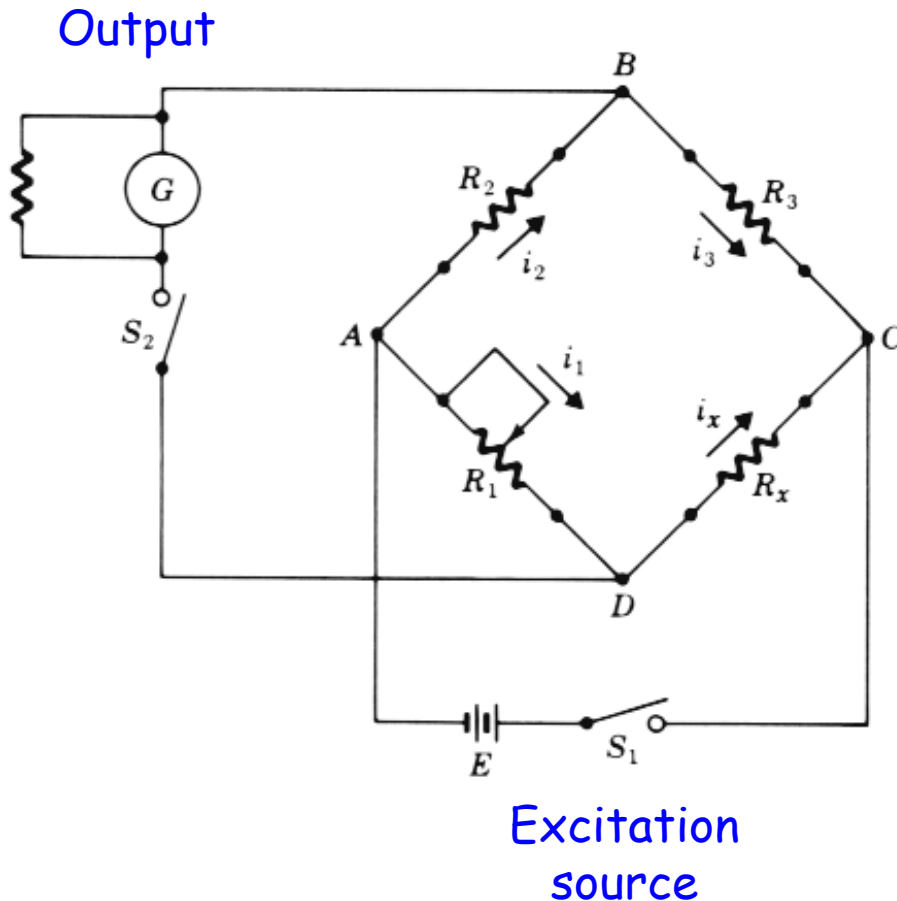
R_2 and R_3 are normally known resistors (of high-quality)

R_1 is a variable resistor

R_x is the unknown resistor



Wheatstone bridge



Voltage E is applied to the bridge (by closing switch S_1)

A "balanced" bridge is one with potential difference between B and D is equal to zero

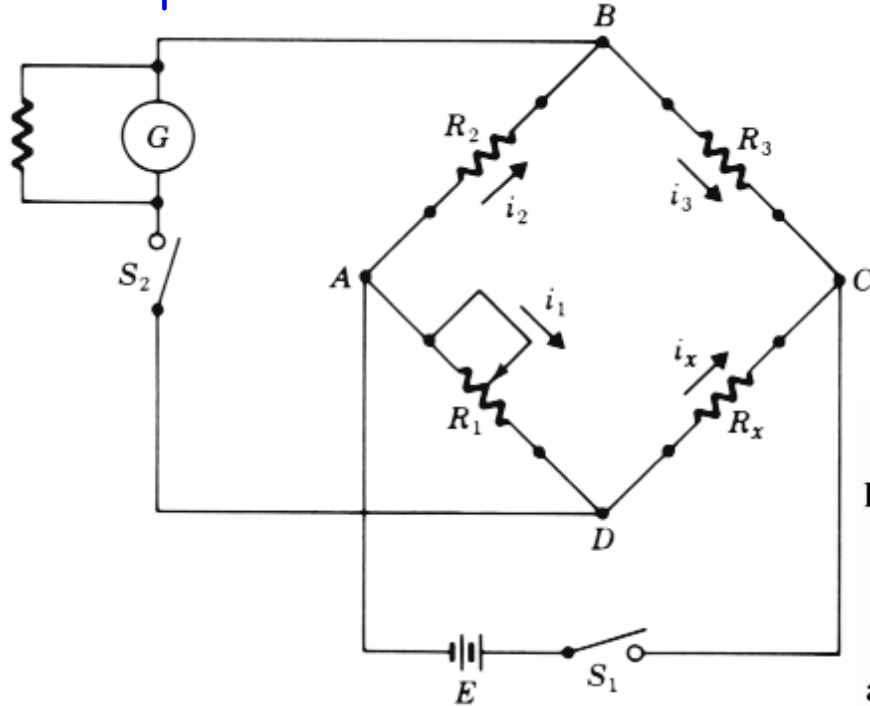
Bridge can be balanced by adjusting resistance R_1

Balance is sensed by closing switch S_2 and measure output current and voltage - to be near zero



Wheatstone bridge

Output



Excitation
source

When bridge is balanced: voltage drop across R_2 is equal to voltage drop across R_1 , since voltage difference between B and D is equal to zero. Therefore,

$$i_2 R_2 = i_1 R_1$$

Further,

$$i_2 = i_3 = \frac{E}{R_2 + R_3} \quad \text{if balanced}$$

and

$$i_1 = i_x = \frac{E}{R_1 + R_x} \quad \text{if balanced}$$

If the currents are eliminated from these relations, the result is

$$\frac{R_2}{R_3} = \frac{R_1}{R_x}$$

or

$$R_x = \frac{R_1 R_3}{R_2}$$



Wheatstone bridge: balanced bridge

Example: uncertainty analysis

For a balanced Wheatstone bridge, determine uncertainty in the measured resistance R_x , as a result of an uncertainty of 1% in the known resistances

$$R_x = \frac{R_1 R_3}{R_2} \Rightarrow R_x = R_x(R_1, R_2, R_3)$$

Uncertainty:
$$\delta R_x = \left[\left(\frac{\partial R_x}{\partial R_1} \delta R_1 \right)^2 + \left(\frac{\partial R_x}{\partial R_2} \delta R_2 \right)^2 + \left(\frac{\partial R_x}{\partial R_3} \delta R_3 \right)^2 \right]^{1/2}$$

$$\frac{\partial R_x}{\partial R_1} = \frac{R_3}{R_2}; \quad \frac{\partial R_x}{\partial R_2} = -\frac{R_1 R_3}{R_2^2}; \quad \frac{\partial R_x}{\partial R_3} = \frac{R_1}{R_2};$$

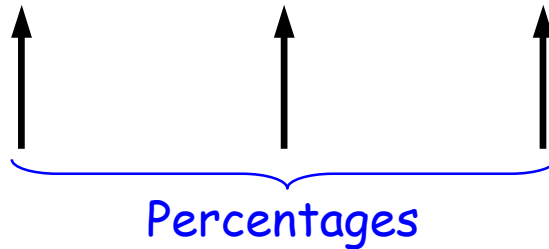


Wheatstone bridge: balanced bridge

Example: uncertainty analysis

Determine percentage:

$$\frac{\delta R_x}{R_x} = \left[\left(\frac{1}{R_1} \delta R_1 \right)^2 + \left(-\frac{1}{R_2} \delta R_2 \right)^2 + \left(\frac{1}{R_3} \delta R_3 \right)^2 \right]^{1/2}$$



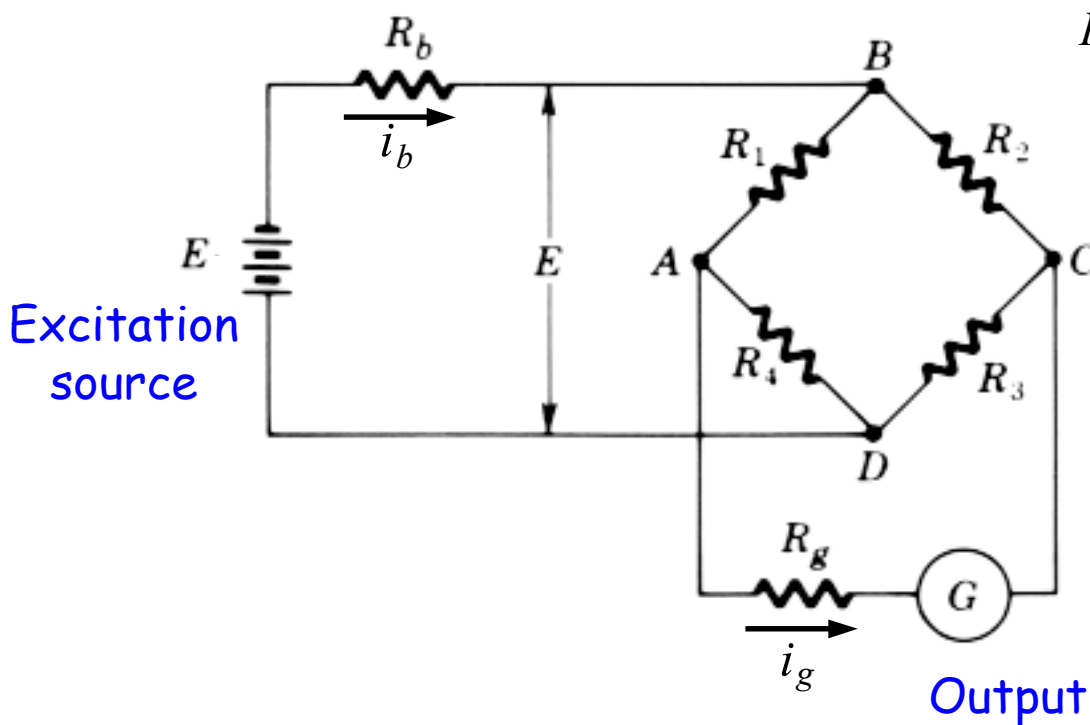
Recall: $R_x = \frac{R_1 R_3}{R_2}$

Determine percentage (numerical value):

$$\frac{\delta R_x}{R_x} = \left[(0.01)^2 + (-0.01)^2 + (0.01)^2 \right]^{1/2} = 0.01732 \Rightarrow 1.732\%$$



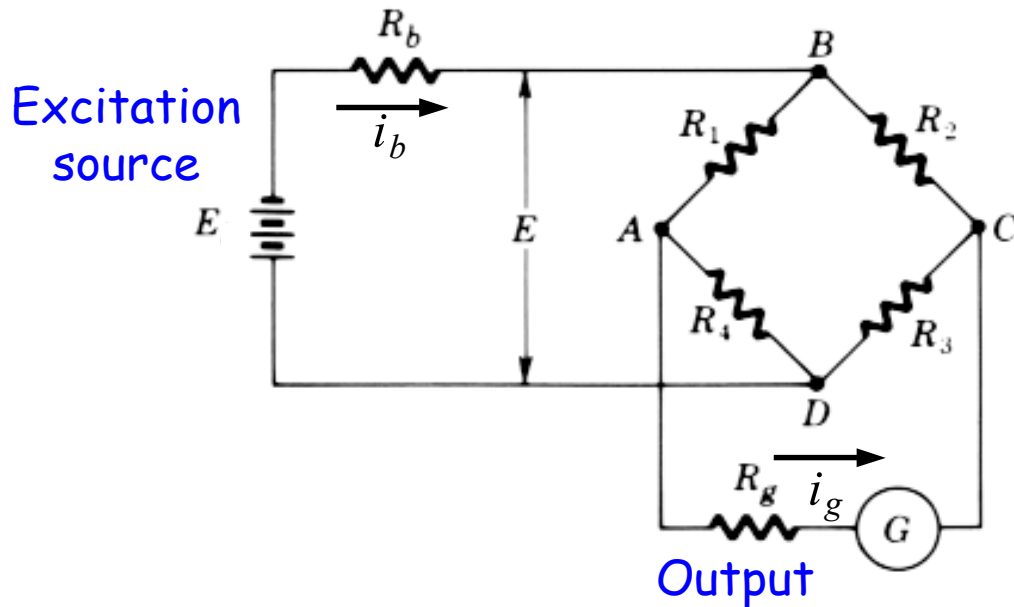
Wheatstone bridge: unbalanced bridge



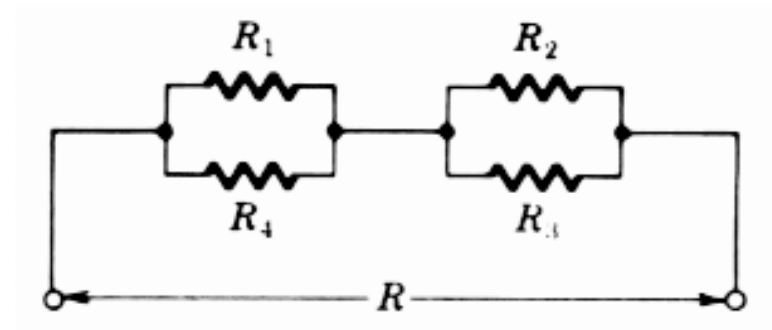
R_1, R_2, R_3, R_4 are different



Wheatstone bridge: unbalanced bridge



Equivalent circuit of bridge at the output:



Wheatstone bridge: unbalanced bridge

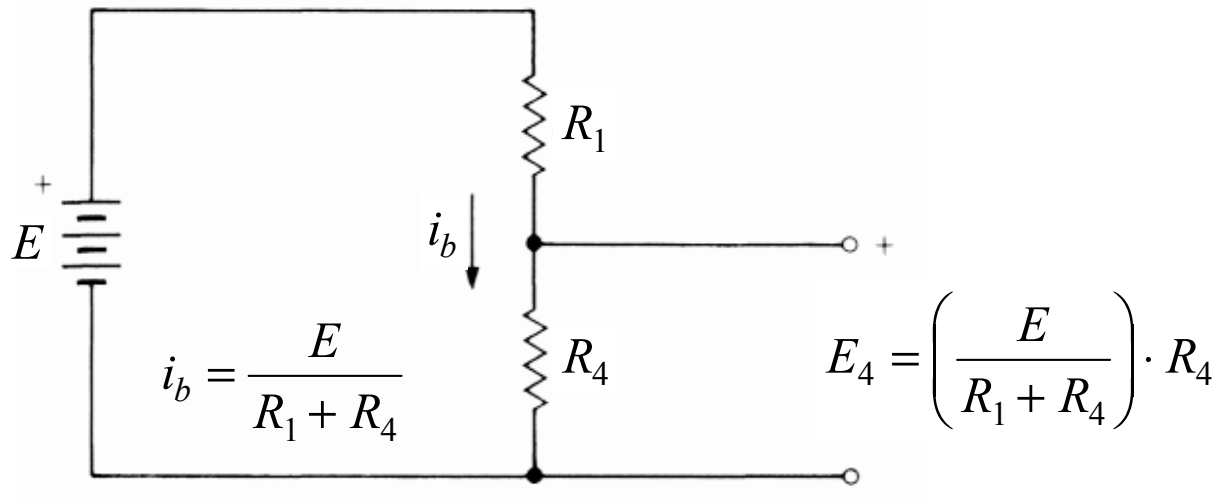
Equivalent resistance:

$$R = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3}$$

Current at the output is:

$$i_g = \frac{E_g}{R + R_g}$$

Recall a voltage divider:



Wheatstone bridge: unbalanced bridge

Considering voltage divider
on a bridge:

$$\begin{aligned} E_g &= \left(\frac{E}{R_1 + R_4} \right) R_1 - \left(\frac{E}{R_2 + R_3} \right) R_2 \\ &= E \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \end{aligned}$$

and

$$\begin{aligned} E_g &= \left(\frac{E}{R_1 + R_4} \right) R_4 - \left(\frac{E}{R_2 + R_3} \right) R_3 \\ &= E \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) \end{aligned}$$



Wheatstone bridge: unbalanced bridge

What about if one resistance changes by a small amount?

Use: $\Delta R_4 \Rightarrow \Delta E_g$

Therefore,
$$E_g + \Delta E_g = E \left(\frac{(R_4 + \Delta R_4) R_2 - R_3 R_1}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} \right)$$

Divide numerator and denominator by: $R_2 R_4$

$$E_g + \Delta E_g = E \left(\frac{1 + \Delta R_4 / R_4 - R_3 R_1 / R_4 R_2}{(1 + R_1 / R_4 + \Delta R_4 / R_4)(1 + R_3 / R_2)} \right)$$



Wheatstone bridge: unbalanced bridge

What about if one resistance changes by a small amount?

If all resistors are initially the same:
($E_g = 0$; $R_i = R$)

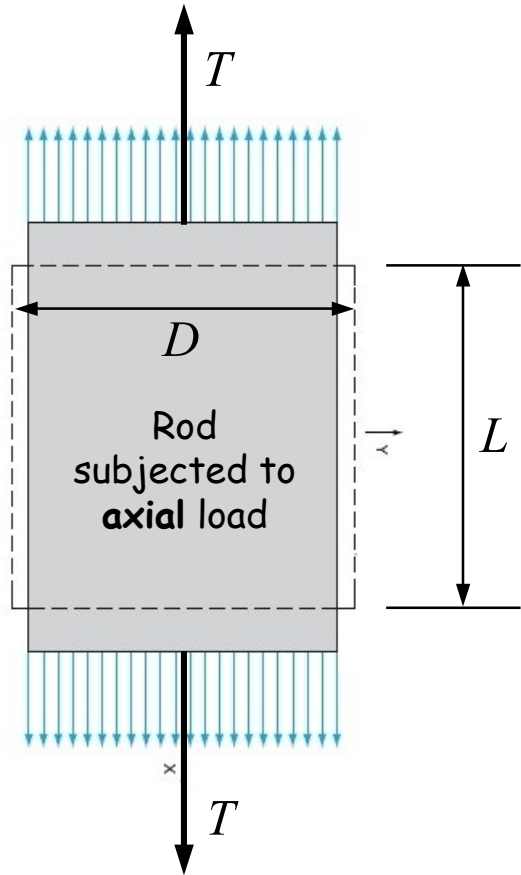
$$\frac{\Delta E_g}{E} = \frac{\Delta R_4 / R}{4 + 2(\Delta R_4 / R)}$$

But because changes in resistance are small, i.e., $\Delta R_4 \ll 1$

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R}$$



Stress and strain



Axial strain: $\epsilon = \frac{T / A}{E} = \frac{\sigma_a}{E} = \frac{dL}{L} = \epsilon_a$

Poisson's ratio: $\mu = -\frac{\epsilon_t}{\epsilon_a} = -\frac{dD / D}{dL / L}$

($\mu \approx 0.3$ for most metals)

Volume of rod is: $V = L \cdot A = L \cdot \frac{\pi}{4} D^2$

Volume is constant, therefore

$$dV = 0 = L dA + A dL$$

$$\Rightarrow \frac{dA}{A} = -\frac{dL}{L} \Rightarrow 2 \frac{dD}{D} = -\frac{dL}{L}$$

$$dV = 0 = D dL + 2L dD$$

(i.e., $\mu = 0.5$, in this condition)



Strain gages

Electrical resistance: $R = \rho \frac{L}{A}$

← length
← cross-sectional area
↑ resistivity

Differentiate resistance: $dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \frac{dD}{D} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left(-\mu \frac{dL}{L} \right)$$

$$\Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left(-\mu \frac{dL}{L} \right) = \frac{d\rho}{\rho} + \varepsilon_a (1 + 2\mu)$$



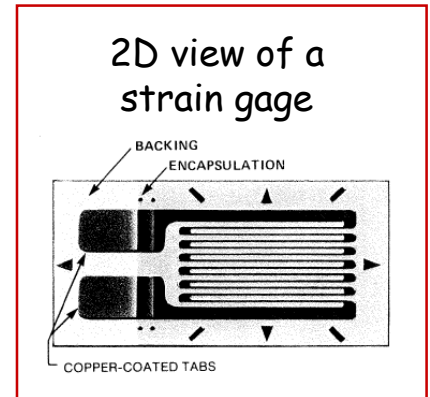
Strain gages

Definition of gage factor: $F = \frac{dR/R}{\epsilon_a}$

(From previous page) $\Rightarrow F = 1 + 2\mu + \frac{1}{\epsilon_a} \frac{d\rho}{\rho}$

If resistivity does not change $\Rightarrow F = 1 + 2\mu$

And strain with change of resistance is: $\Rightarrow \epsilon_a = \frac{1}{F} \frac{\Delta R}{R}$



A typical strain gage has a gage factor $\approx 2.095 \pm 0.5\%$.
Why? How is this possible? Open for discussions



Reading assignment

- Beckwith: Ch. 7, 12
- Bishop: Ch. 11

References

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, 2004, Worcester, MA
- GE NovaSensors, <http://www.gesensing.com/>



Homework assignment

- Bishop: Section 11.2.1

Handout-D

- D1.- Derive **complete** RSS uncertainty equation for the output voltage of an **unbalanced** Wheatstone bridge. Discuss your observations
- D2.- Derive **complete** RSS uncertainty equation for the electrical resistance of a resistor. Consider electrical resistivity, active length, and cross-sectional area. Apply your uncertainty evaluations to a 120 Ω resistor (available in the lab)

Make sure to:

- (a) Indicate, in order of importance, percentage contribution of all uncertainties to the overall uncertainty

Discuss your observations

