

# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation  
ME-3901, D'2010

Lecture 06

31 March 2010



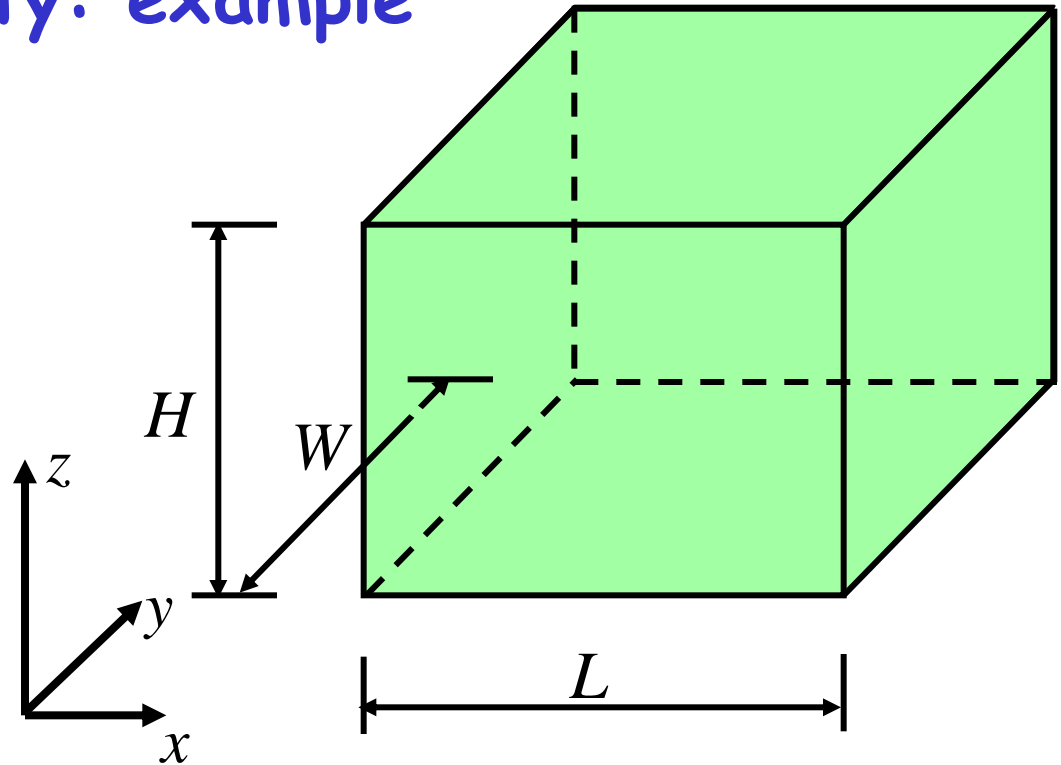
# Error versus uncertainty

- Errors are based on knowledge of actual values (or standard values)
- Uncertainties are evaluated analytically
  - there are a number of procedures for determination of the overall uncertainty
  - the most popular and most widely used relationship for determination of uncertainty is known as the RSS-type
  - the relationship defining the RSS-type overall uncertainty is based on a partial differential representation



# RSS-type uncertainty: example

Consider a solid block with dimensions of  $L$ ,  $W$ , and  $H$ , as shown in the figure. The block is placed in the environment at temperature of  $20^\circ\text{C}$  where the convective heat transfer coefficient is  $15 \pm 3 \text{ W/m}^2\text{-}^\circ\text{C}$ . The block is at the temperature of  $300 \pm 5^\circ\text{C}$ . Determine the magnitude of the heat transfer by convection,  $Q_c$ , *from the top surface of the block and the corresponding overall uncertainty in this magnitude*. Also, list in descending order percentage contributions to the overall uncertainty in  $Q_c$  due to the individual uncertainties.



$$L = 140 \pm 3 \text{ cm} \longrightarrow \delta L = \pm 3 \text{ cm}$$

$$W = 25 \pm 1 \text{ cm} \longrightarrow \delta W = \pm 1 \text{ cm}$$

$$H = 9 \text{ cm} \xrightarrow{\frac{1}{2} \text{LSD}} \delta H = \pm 0.5 \text{ cm}$$

LSD = Least Significant Digit



# RSS-type uncertainty

Consider *explicit equation* for convective heat transfer:

$$Q_c = hA_c(T_s - T_e)$$

This equation can be represented, in the most general way, by the *fundamental equation*, also known as *phenomenological equation*, as follows:

$$Q_c = Q_c(h, A_c, T_s, T_e)$$

The *RSS-type uncertainty*, based on the above equation, can be expressed as

$$\delta Q_c = \left[ \left( \frac{\partial Q_c}{\partial h} \delta h \right)^2 + \left( \frac{\partial Q_c}{\partial A_c} \delta A_c \right)^2 + \left( \frac{\partial Q_c}{\partial T_s} \delta T_s \right)^2 + \left( \frac{\partial Q_c}{\partial T_e} \delta T_e \right)^2 \right]^{\frac{1}{2}}$$

where the symbol  $\delta$  denotes the uncertainty



## Example, cont'd

$$A_c = LW = (1.4 \text{ m})(0.25 \text{ m}) = 0.35 \text{ m}^2$$

$$Q_c = hA_c(T_s - T_e) = \left(15 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}}\right)(0.35 \text{ m}^2)(300 - 20)^\circ\text{C} = 1,470 \text{ W}$$

The fundamental equation is

$$Q_c = Q_c(h, A_c, T_s, T_e)$$

The RSS-type overall uncertainty can be determined from

$$\delta Q_c = \left[ \left( \frac{\partial Q_c}{\partial h} \delta h \right)^2 + \left( \frac{\partial Q_c}{\partial A_c} \delta A_c \right)^2 + \left( \frac{\partial Q_c}{\partial T_s} \delta T_s \right)^2 + \left( \frac{\partial Q_c}{\partial T_e} \delta T_e \right)^2 \right]^{\frac{1}{2}}$$

or

$$\delta Q_c = \left( \partial Q_c \delta h^2 + \partial Q_c \delta A_c^2 + \partial Q_c \delta T_s^2 + \partial Q_c \delta T_e^2 \right)^{\frac{1}{2}}$$



## Example, cont'd

$$\begin{aligned}\partial Q_c \delta h^2 &= \left( \frac{\partial Q_c}{\partial h} \delta h \right)^2 = [A_c (T_s - T_e) \delta h]^2 = & \delta h = \pm 3 \frac{W}{m^2 \cdot ^\circ C} \\ &= \left\{ (0.35 \text{ m}^2) [(300 - 20)^\circ C] \left( 3 \frac{W}{m^2 \cdot ^\circ C} \right) \right\}^2 = 8.64360 \times 10^4 \text{ W}^2\end{aligned}$$

$$\partial Q_c \delta A_c^2 = \left( \frac{\partial Q_c}{\partial A_c} \delta A_c \right)^2 = [h(T_s - T_e) \delta A_c]^2$$

$$A_c = LW \quad \longrightarrow \quad A_c = A_c(L, W)$$

$$\delta A_c = \left[ \left( \frac{\partial A_c}{\partial L} \delta L \right)^2 + \left( \frac{\partial A_c}{\partial W} \delta W \right)^2 \right]^{\frac{1}{2}}$$



## Example, cont'd

$$\delta A_c = \left[ \left( \frac{\partial A_c}{\partial L} \delta L \right)^2 + \left( \frac{\partial A_c}{\partial W} \delta W \right)^2 \right]^{\frac{1}{2}}$$

$$\begin{aligned} \delta A_c &= \left[ (W \delta L)^2 + (L \delta W)^2 \right]^{\frac{1}{2}} = \left\{ [(0.25 \text{ m})(0.03 \text{ m})]^2 + [(1.4 \text{ m})(0.01 \text{ m})]^2 \right\}^{\frac{1}{2}} = \\ &= \left( 5.62500 \times 10^{-5} \text{ m}^2 + 1.96000 \times 10^{-4} \text{ m}^2 \right)^{\frac{1}{2}} = 1.58824 \times 10^{-2} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \partial Q_c \delta A_c^2 &= [h(T_s - T_e) \delta A_c]^2 = \\ &= \left\{ \left( 15 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) [(300 - 20)^\circ\text{C}] (1.58824 \times 10^{-2} \text{ m}^2) \right\}^2 = 4.44970 \times 10^3 \text{ W}^2 \end{aligned}$$



## Example, cont'd

$$\partial Q_c \delta T_s^2 = \left( \frac{\partial Q_c}{\partial T_s} \delta T_s \right)^2 = (hA_c \delta T_s)^2 = \delta T_s = \pm 5^\circ \text{C}$$

$$= \left[ \left( 15 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) (0.35 \text{ m}^2) (5^\circ \text{C}) \right]^2 = 6.89063 \times 10^2 \text{ W}^2$$

$$\partial Q_c \delta T_e^2 = \left( \frac{\partial Q_c}{\partial T_e} \delta T_e \right)^2 = (-hA_c \delta T_e)^2 = \delta T_e = \pm 0.5^\circ \text{C}$$

$$= \left[ - \left( 15 \frac{\text{W}}{\text{m}^2 \cdot ^\circ\text{C}} \right) (0.35 \text{ m}^2) (0.5^\circ \text{C}) \right]^2 = 6.89063 \times 10^0 \text{ W}^2$$

$\frac{1}{2}$  LSD



## Example, cont'd

$$\begin{aligned}\delta Q_c &= \left( \partial Q_c \delta h^2 + \partial Q_c \delta A_c^2 + \partial Q_c \delta T_s^2 + \partial Q_c \delta T_s^2 \right)^{\frac{1}{2}} = \\ &= \left[ \left( 8.64360 \times 10^4 + 4.44970 \times 10^3 + 6.89063 \times 10^2 + 6.89063 \times 10^0 \right) W^2 \right]^{\frac{1}{2}} = \\ &= 3.02625 \times 10^2 W = 302.6 W \quad \longrightarrow \quad \text{This is the overall uncertainty}\end{aligned}$$

The percentage overall uncertainty is

$$\% \delta Q_c = \frac{\delta Q_c}{Q_c} \times 100\% = \frac{302.6 W}{1,470 W} \times 100\% = 20.6\%$$

The percentage contributions of the individual uncertainties to the overall uncertainty can be computed as

$$\% \partial Q_c \delta h = \frac{\partial Q_c \delta h^2}{\delta Q_c^2} \times 100\% = \frac{8.64360 \times 10^4 W^2}{\left( 3.02625 \times 10^2 W \right)^2} \times 100\% = 94.3811\%$$



## Example, cont'd

$$\% \partial Q_c \delta h = \frac{\partial Q_c \delta h^2}{\delta Q_c^2} \times 100\% = \frac{8.64360 \times 10^4 \text{ W}^2}{(3.02625 \times 10^2 \text{ W})^2} \times 100\% = 94.3811\%$$

Highest contribution

$$\% \partial Q_c \delta A_c = \frac{\partial Q_c \delta A_c^2}{\delta Q_c^2} \times 100\% = \frac{4.44970 \times 10^3 \text{ W}^2}{(3.02625 \times 10^2 \text{ W})^2} \times 100\% = 4.8587\%$$

Contribution No. 2

$$\% \partial Q_c \delta T_s = \frac{\partial Q_c \delta T_s^2}{\delta Q_c^2} \times 100\% = \frac{6.89063 \times 10^2 \text{ W}^2}{(3.02625 \times 10^2 \text{ W})^2} \times 100\% = 0.7524\%$$

Contribution No. 3

$$\% \partial Q_c \delta T_e = \frac{\partial Q_c \delta T_e^2}{\delta Q_c^2} \times 100\% = \frac{6.89063 \times 10^0 \text{ W}^2}{(3.02625 \times 10^2 \text{ W})^2} \times 100\% = 0.0075\%$$

Lowest contribution

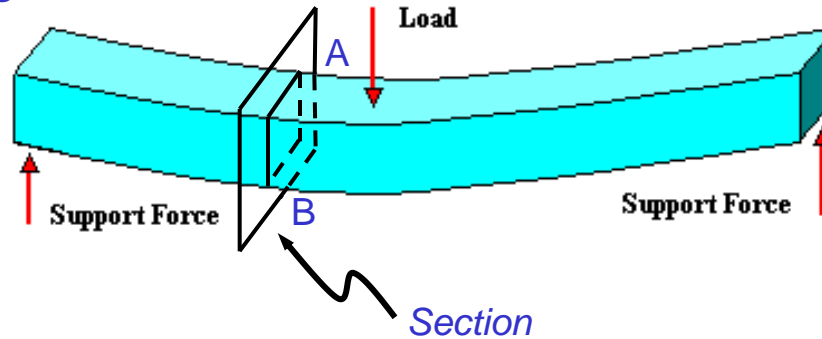
$$\begin{aligned} \text{CHECK} &= \% \partial Q_c \delta h + \% \partial Q_c \delta A_c + \% \partial Q_c \delta T_s + \% \partial Q_c \delta T_e = \\ &= 94.3811\% + 4.8587\% + 0.7524\% + 0.0075\% = 99.9997\% \approx 100.0\% \end{aligned}$$



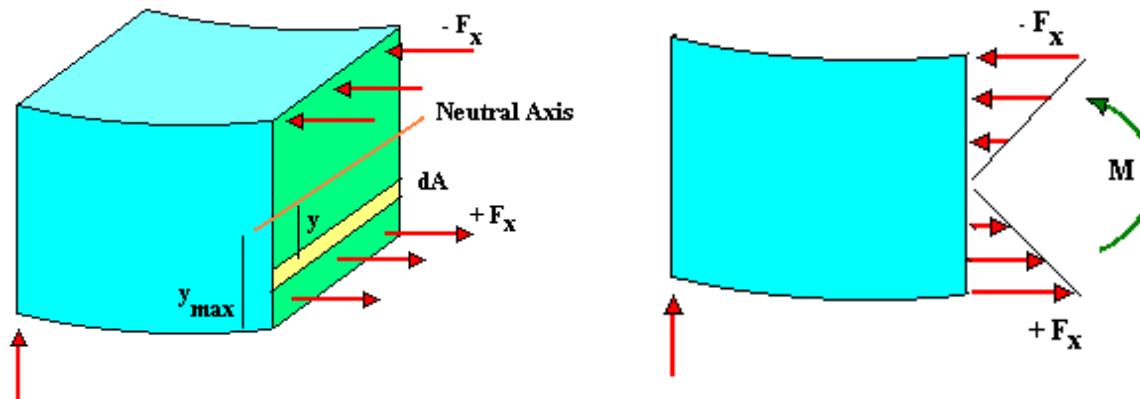
# Stress distribution in cross-sections

## Bending stress

### Bending load



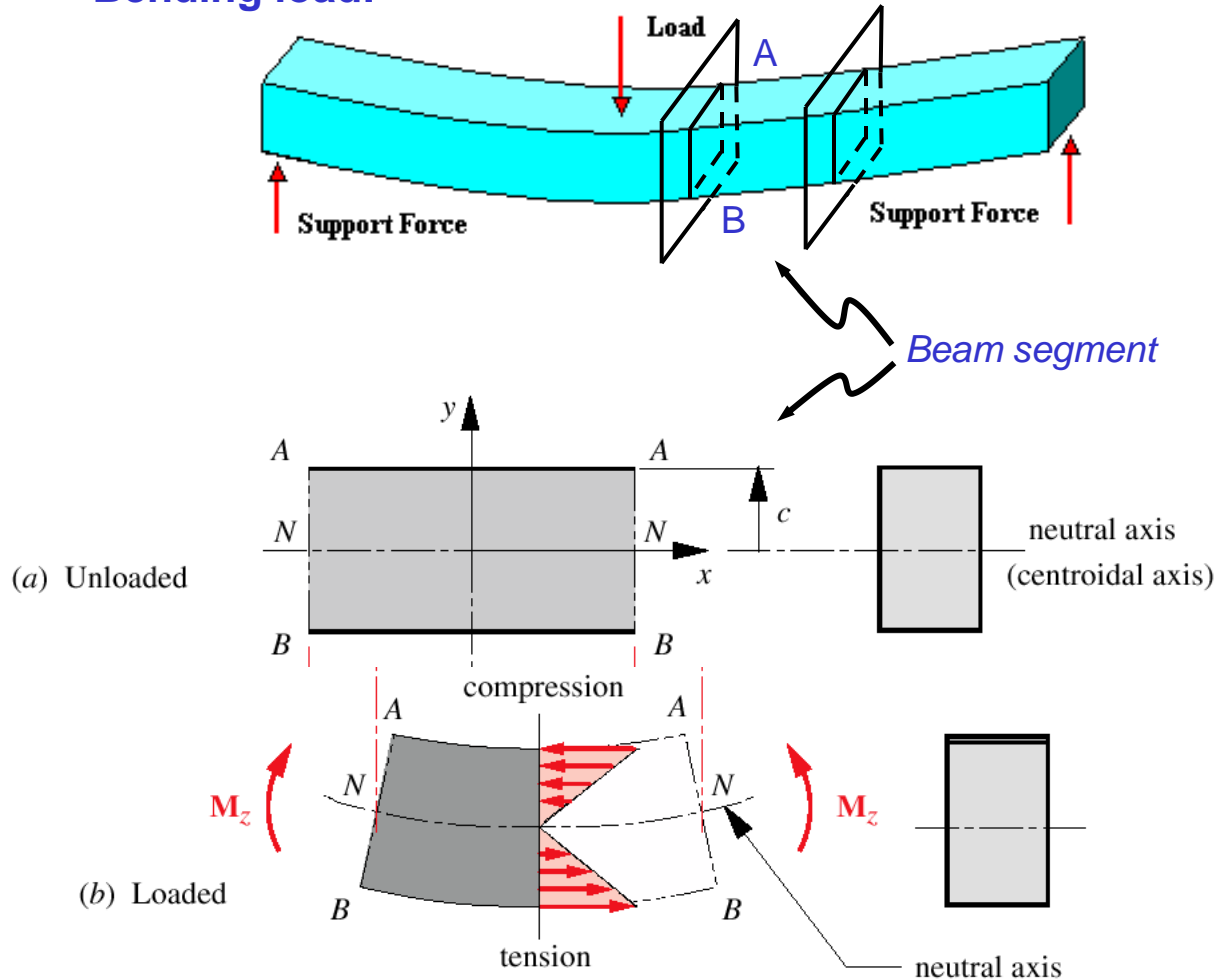
### Internal distribution of bending forces



# Stress distribution in cross-sections

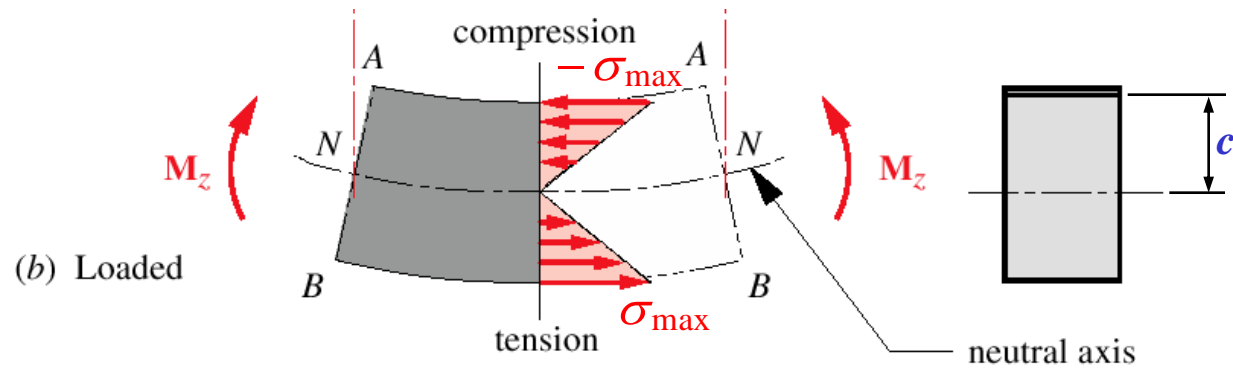
## Bending stress

Bending load:



# Stress distribution in cross-sections

## Bending stress



**Bending stress:**

$$\sigma_x(x, y) = -\frac{My}{I}$$

Recall that  $M = M(x)$

**Maximum bending stress:**

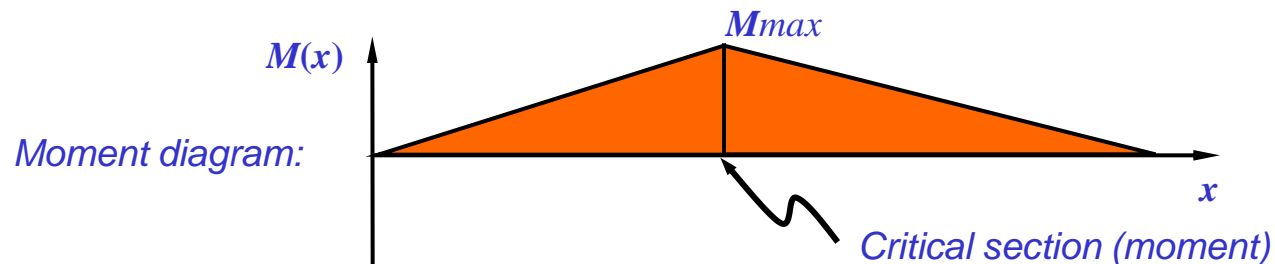
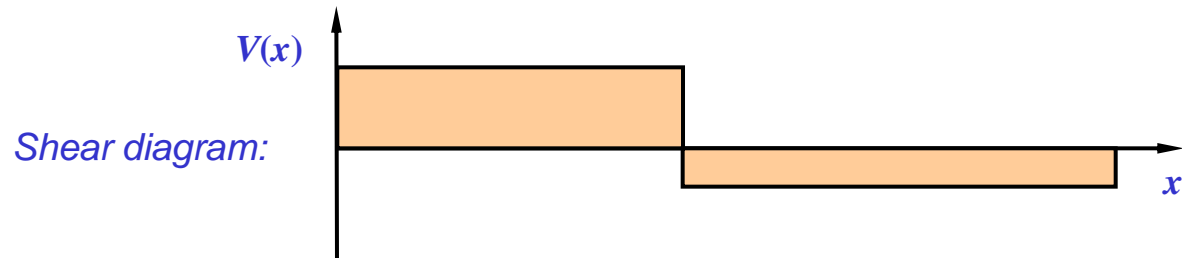
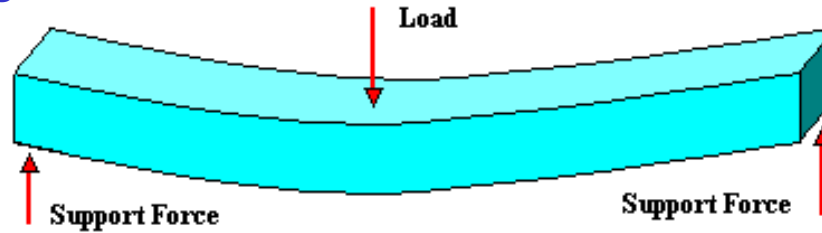
$$\sigma_{max}(x) = \frac{Mc}{I}$$



# Stress distribution in cross-sections

## Bending stress

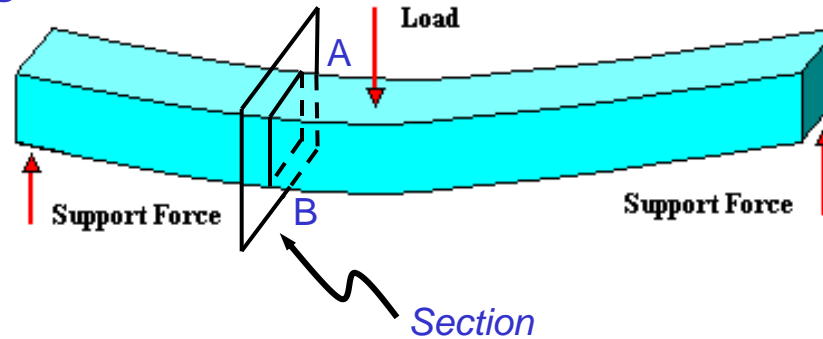
Bending load:



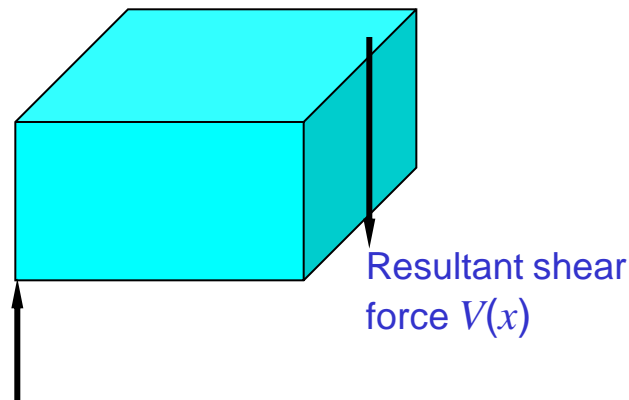
# Stress distribution in cross-sections

## Transverse shear stress

Bending load

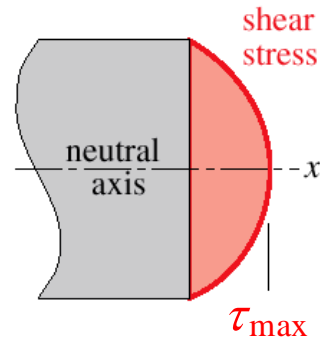


Internal distribution of transversal shear forces



# Stress distribution in cross-sections

## Transverse shear stress

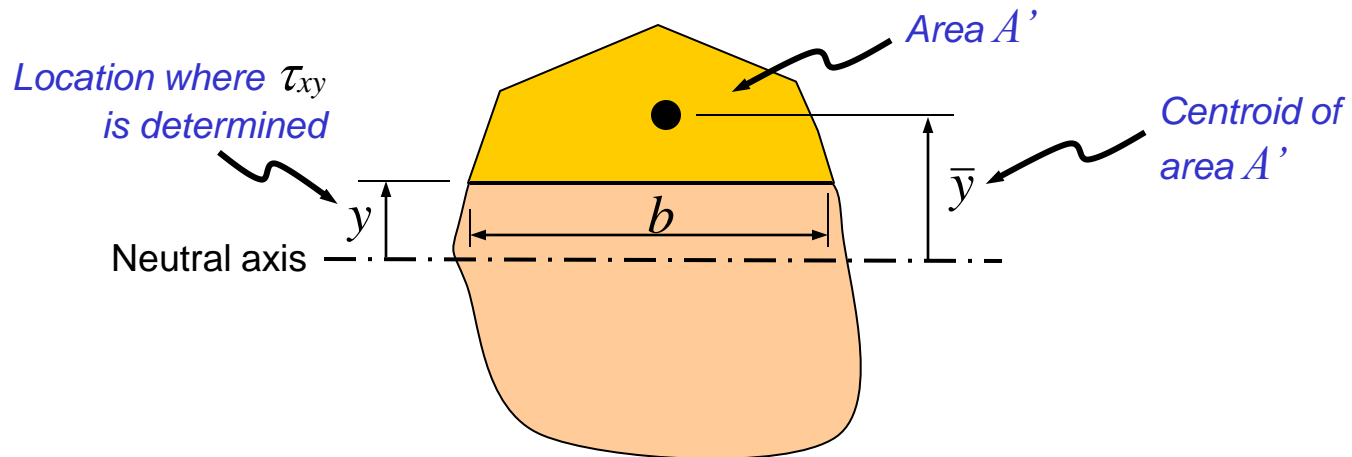


Transverse shear stress:

$$\tau_{xy} = \frac{V Q}{I b} \quad \text{with} \quad Q = \bar{y} A'$$

Recall that  $V = V(x)$

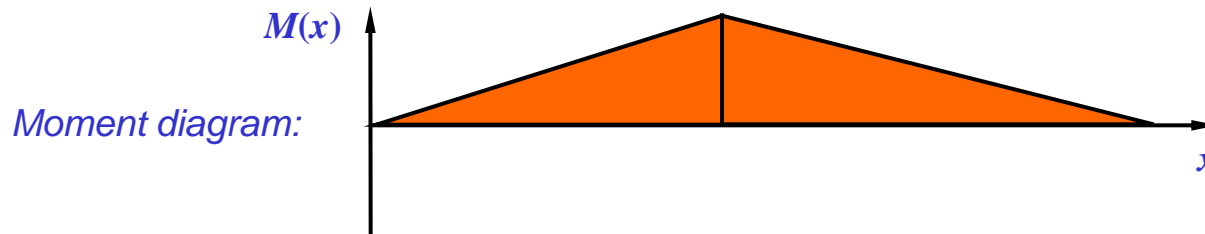
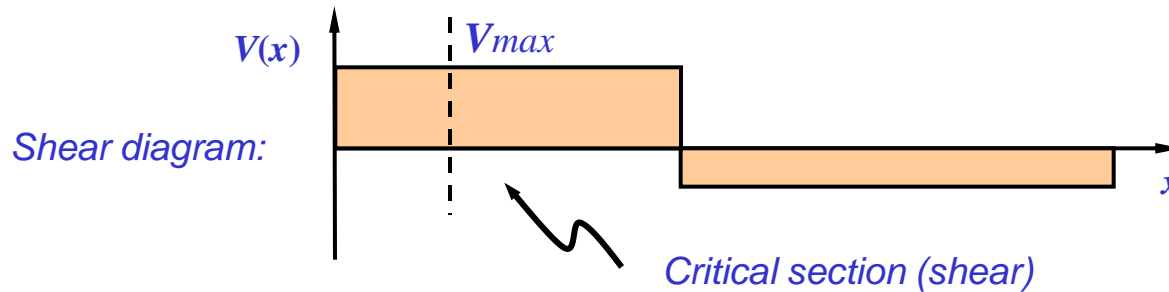
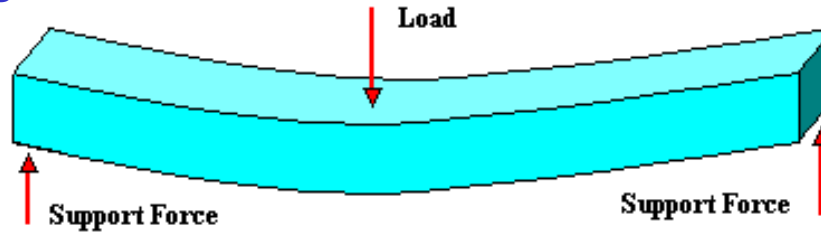
Generic cross-section:



# Stress distribution in cross-sections

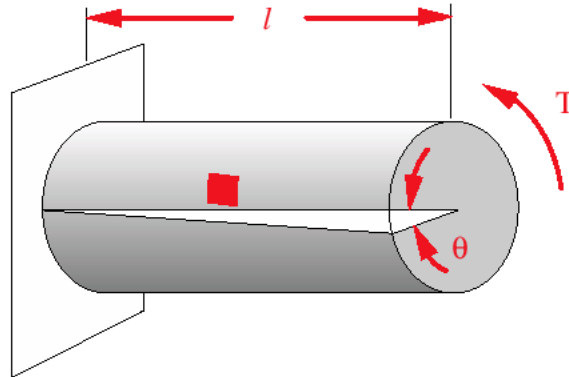
## Transverse shear stress

Bending load:



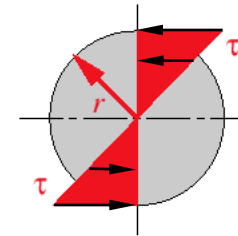
# Stress distribution in cross-sections

## Torsional stress



(a) Deflection  $\theta$

Pure shear stress



(b) Shear-Stress Distribution  $\tau$

**Shear stress distribution:**

$$\tau = \frac{T \rho}{J}, \quad \text{recall that } J = I_x + I_y$$

**Angular deflection:**

$$\theta = \frac{T l}{K G}, \quad \text{recall that the shear modulus is } G = \frac{E}{2(1+\nu)}$$

( $K$  is a geometric factor -- see *Table 4-2*)



# Reading assignment

- Beckwith: Ch. 7, 12
- Bishop: Ch. 10



# Homework

- **Beckwith:** 12.1, 12.2, 12.8
- **Bishop:** Section 10.2.1

