WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 05

13 September 2010

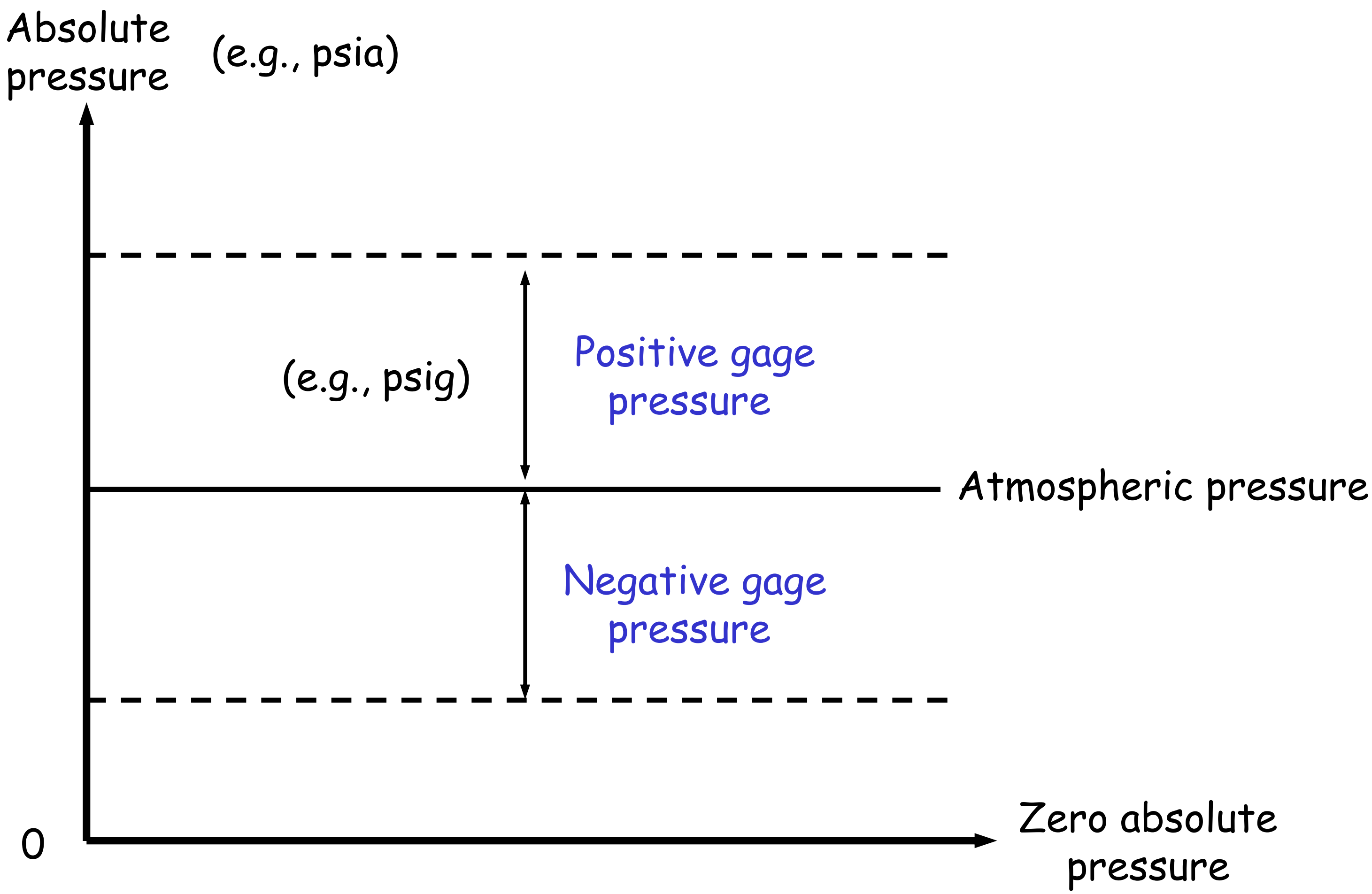
General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

Teaching Assistants: During laboratory sessions

Pressure measurements: definitions



Pressure measurements: definitions

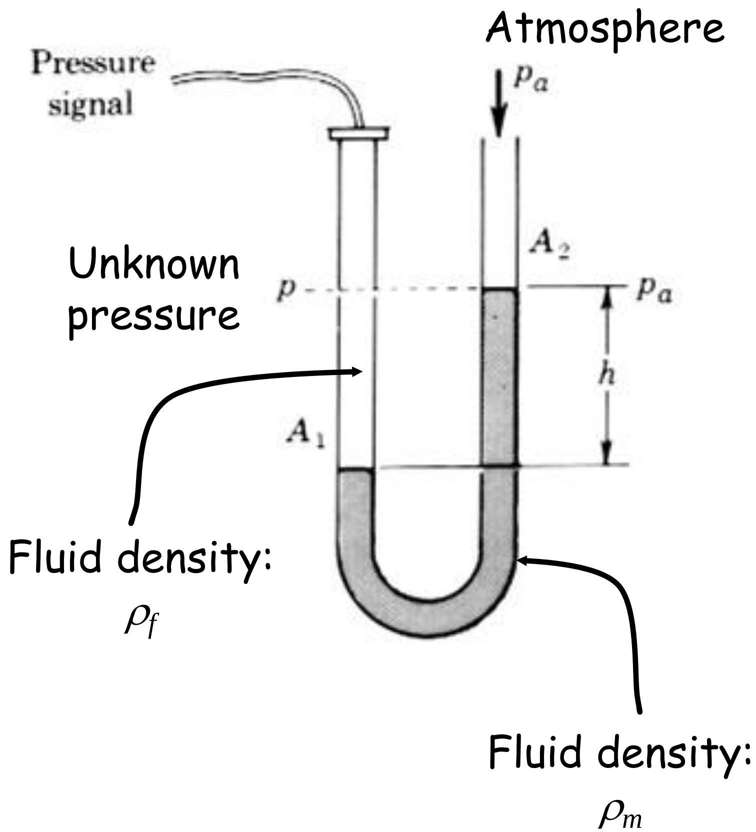
$$1 \text{ atmosphere (atm)} = 14.696 \text{ pounds per square inch absolute}$$
$$= 1.01325 \times 10^5 \text{ newtons per square meter (Pa)}$$
$$= 2116 \text{ pounds-force per square foot } (\text{lbf/ft}^2)$$
$$1 \text{ N/m}^2 \equiv 1 \text{ pascal (Pa)}$$

$$1 \text{ atmosphere (atm)} = 760 \text{ millimeters of mercury (mmHg)}$$
$$1 \text{ bar} = 10^5 \text{ newtons per square meter (100 kPa)}$$
$$1 \text{ microbar} = 1 \text{ dyne per square centimeter}$$
$$= 2.089 \text{ pounds-force per square foot}$$
$$= 0.1 \text{ newtons per square meter (0.1 Pa)}$$

$$1 \text{ millimeter of mercury (mmHg)} = 1333.22 \text{ microbar}$$
$$= 133.322 \text{ newtons per square meter (133.3 Pa)}$$
$$1 \text{ micrometer} = 10^{-6} \text{ meters of mercury } (\mu\text{m, microns})$$
$$= 10^{-3} \text{ millimeters of mercury (mmHg)}$$
$$= 0.133322 \text{ newtons per square meter (0.133 Pa)}$$

$$1 \text{ torr} \equiv 1 \text{ millimeter of mercury (mmHg)}$$
$$1 \text{ inch of mercury} = 70.73 \text{ pounds-force per square foot}$$
$$1 \text{ inch of water} = 6.203 \text{ pounds-force per square foot}$$
$$1 \text{ pound per square inch absolute} = 6894.76 \text{ newtons per square meter (6.894 kPa)}$$
$$= 0.070307 \text{ kilograms-force per square centimeter } (\text{kgf/cm}^2) \text{ [kilopounds per square centimeter } (\text{kp/cm}^2)]$$

Mechanical pressure measurement devices: U-manometer



Pressure balance indicates:

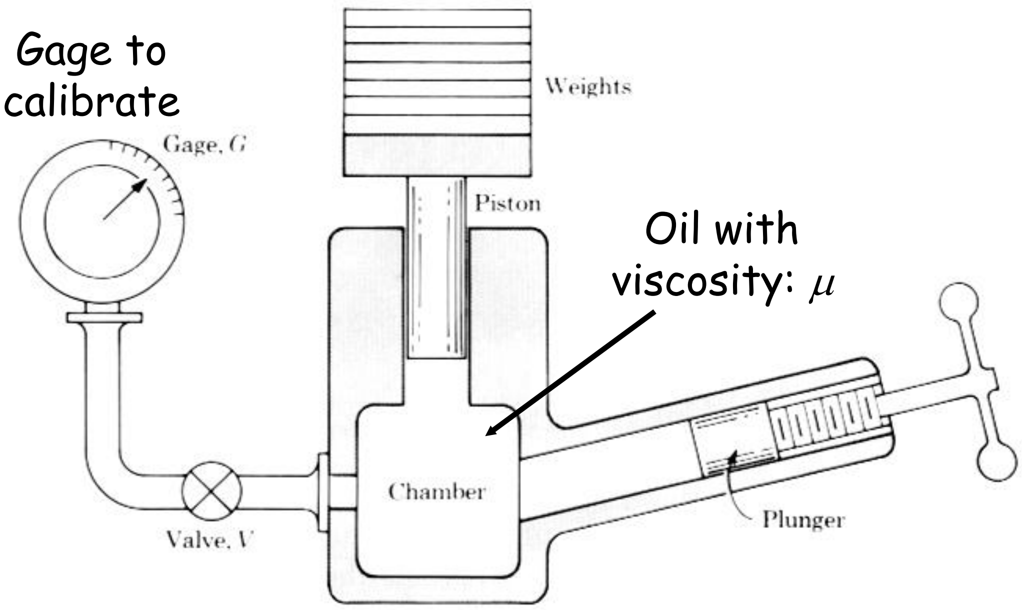
$$p_a + \frac{g}{g_c} h\rho_m = p + \frac{g}{g_c} h\rho_f$$

$$p - p_a = \frac{g}{g_c} h(\rho_m - \rho_f)$$

Recall:

1. $g_c = 32.174 \text{ lbm} \cdot \text{ft} / \text{lbf} \cdot \text{s}^2$
2. $g_c = 1 \text{ slug} \cdot \text{ft} / \text{lbf} \cdot \text{s}^2$
3. $g_c = 1 \text{ g} \cdot \text{cm} / \text{dyn} \cdot \text{s}^2$
4. $g_c = 1 \text{ kg} \cdot \text{m} / \text{N} \cdot \text{s}^2$
5. $g_c = 9.80665 \text{ kgm} \cdot \text{m} / \text{kgf} \cdot \text{s}^2$

Static calibration of gages: dead-weight tester



Accuracy limited by: (1) friction between cylinder and piston; and (2) uncertainty in the area of the piston.

Percentage error due to cylinder-piston clearance:

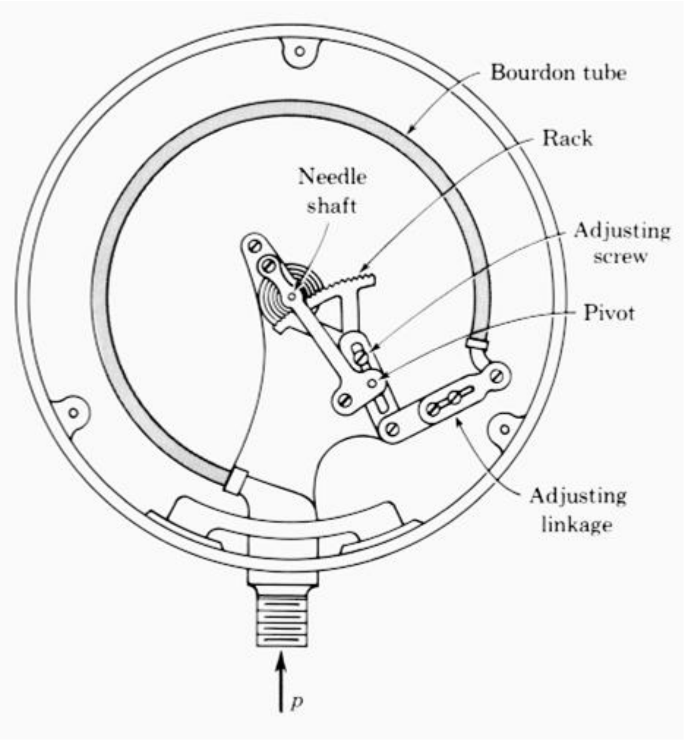
$$\text{Percent error} \sim \frac{(\rho\,\Delta p)^{1/2} b^3}{\mu D L}$$

where
ρ = density of the oil
Δp = pressure differential on the cylinder
b = clearance spacing
μ = viscosity
D = piston diameter
L = piston length

Bourdon-tube pressure gage

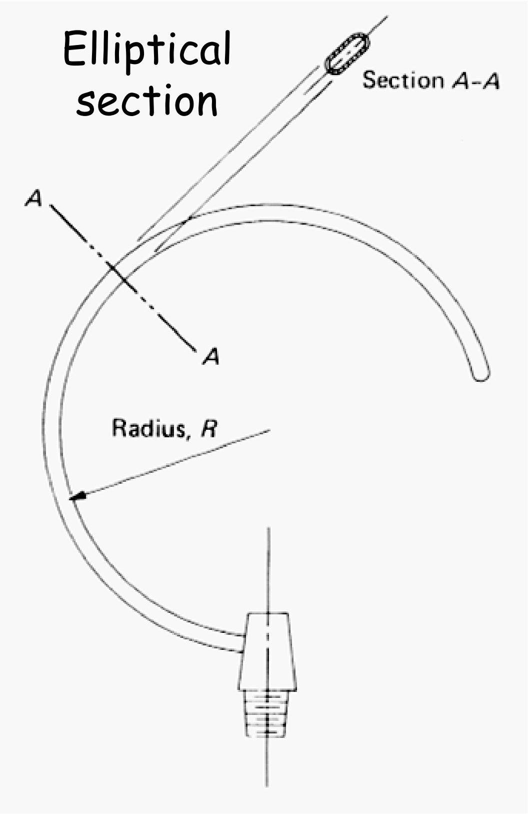
For consistent and inexpensive measurements of static pressure

Linear, uniform, deformations of the tube are desired



Unknown pressure

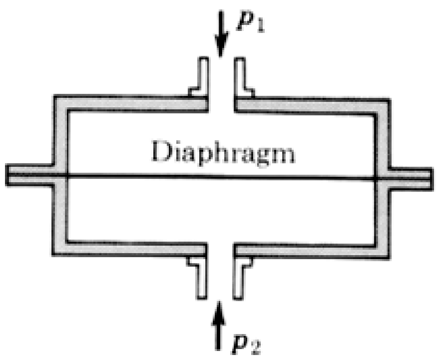
Elliptical section

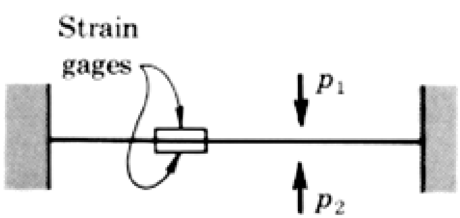


Diaphragm gages

Differential pressure

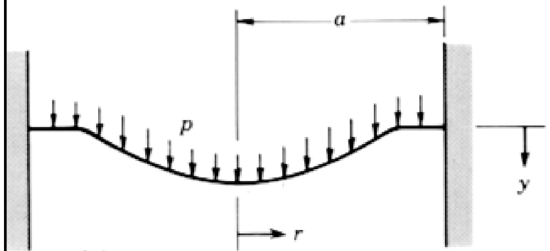
Schematic of a gage





Schematic of a gage with "strain gages"

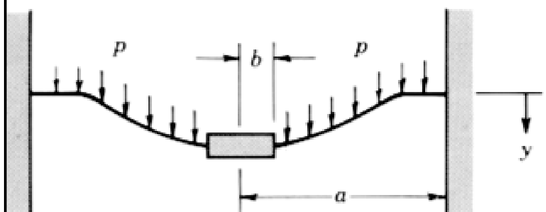
Deflection characteristics of three diaphragm arrangements:



(a)

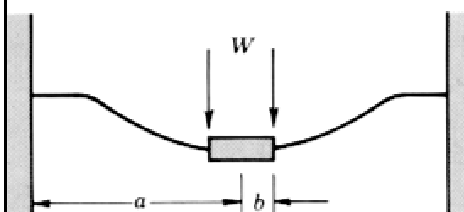
$$y_{max} = \frac{3p}{16Et^3} a^4 (1-\mu^2)$$

$$y(r) = \frac{3p(1-\mu^2)}{16Et^3} (a^2 - r^2)^2$$



(b)

$$y_{max} = \frac{3p(1-\mu^2)}{16Et^3} \left[a^4 + 3b^4 - 4a^2b^2 - 4a^2b^2 \ln(a/b) + \frac{16a^2b^2}{a^2-b^2} \left(\ln \frac{a}{b} \right)^2 \right]$$

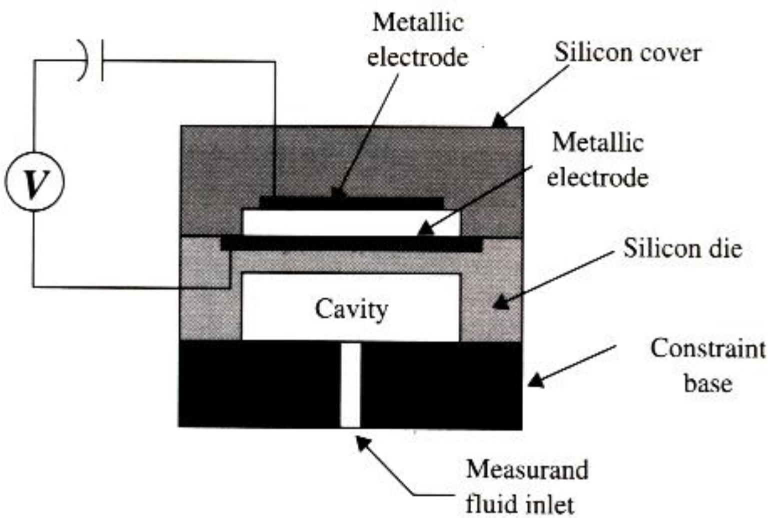


(c)

$$y_{max} = \frac{3W(1-\mu^2)}{4\pi Et^3} \left[a^2 - b^2 - \frac{4a^2b^2}{a^2-b^2} \left(\ln \frac{a}{b} \right)^2 \right]$$

Diaphragm gages: capacitive sensors

Cross section of a die



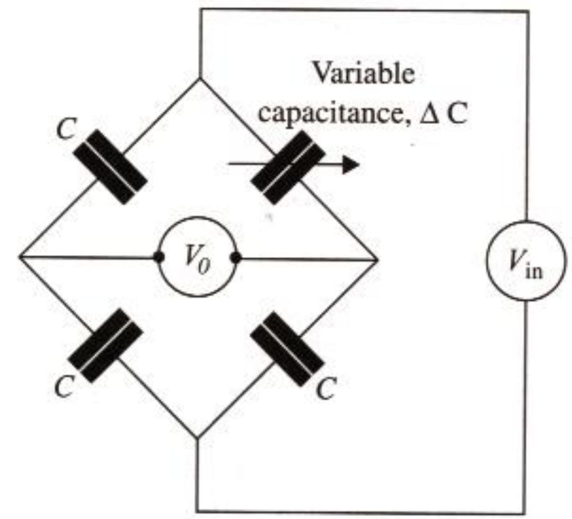
Capacitance:

$$C = \varepsilon_r \varepsilon_o \frac{A}{d}$$

Wheatstone bridge:

$$V_o = V_{in} \frac{\Delta C}{2(2C + \Delta C)}$$

Electronic circuit

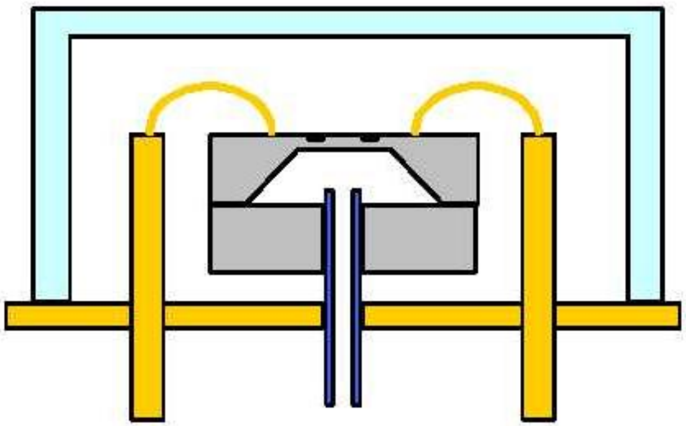


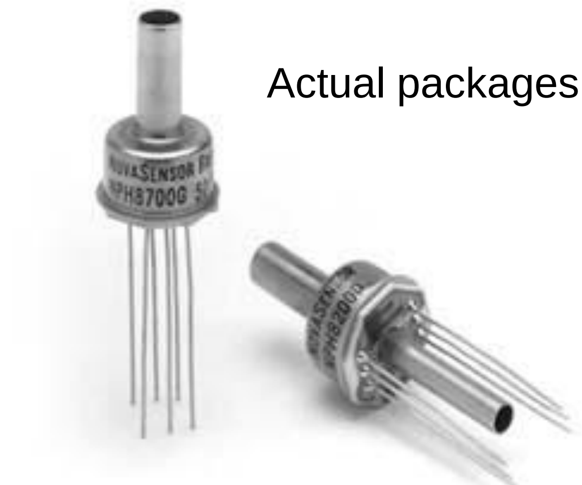
Diaphragm gages: resistive sensors

Make sure to know the meaning of this term

Typical piezoresistive pressure sensor die and package

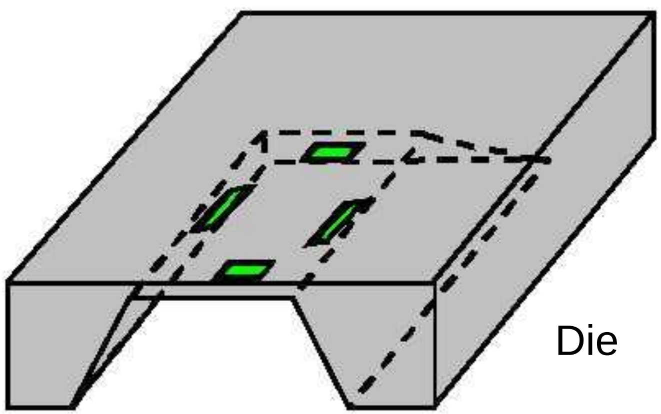
Schematic





Actual packages

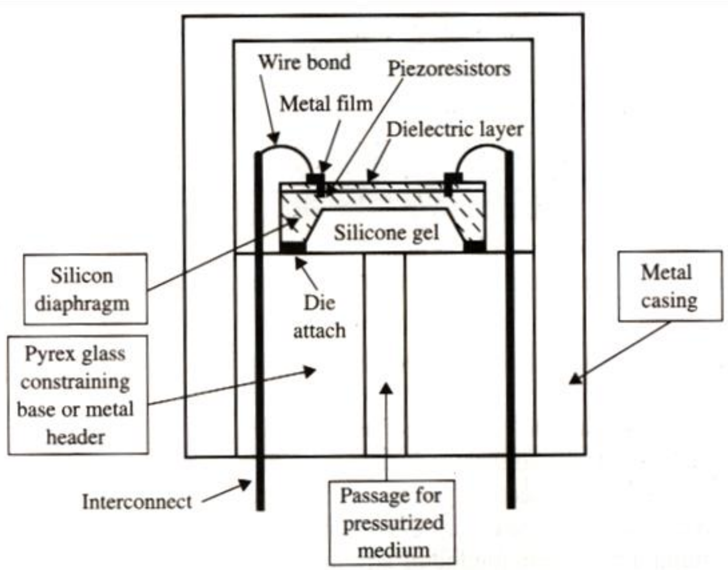
http://www.novasensor.com



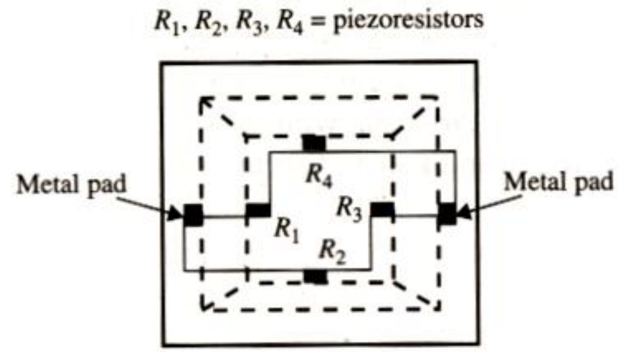
Die

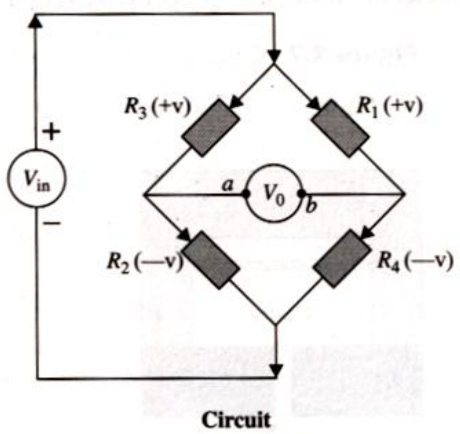
Diaphragm gages: resistive sensors

Cross section of an electronic package



Top view of silicon die





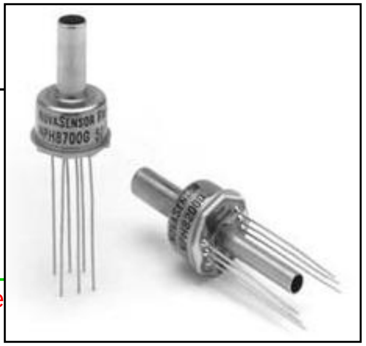
Wheatstone bridge:

$$V_o = V_{in}\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$$

NovaSensor's NPH solid state sensor

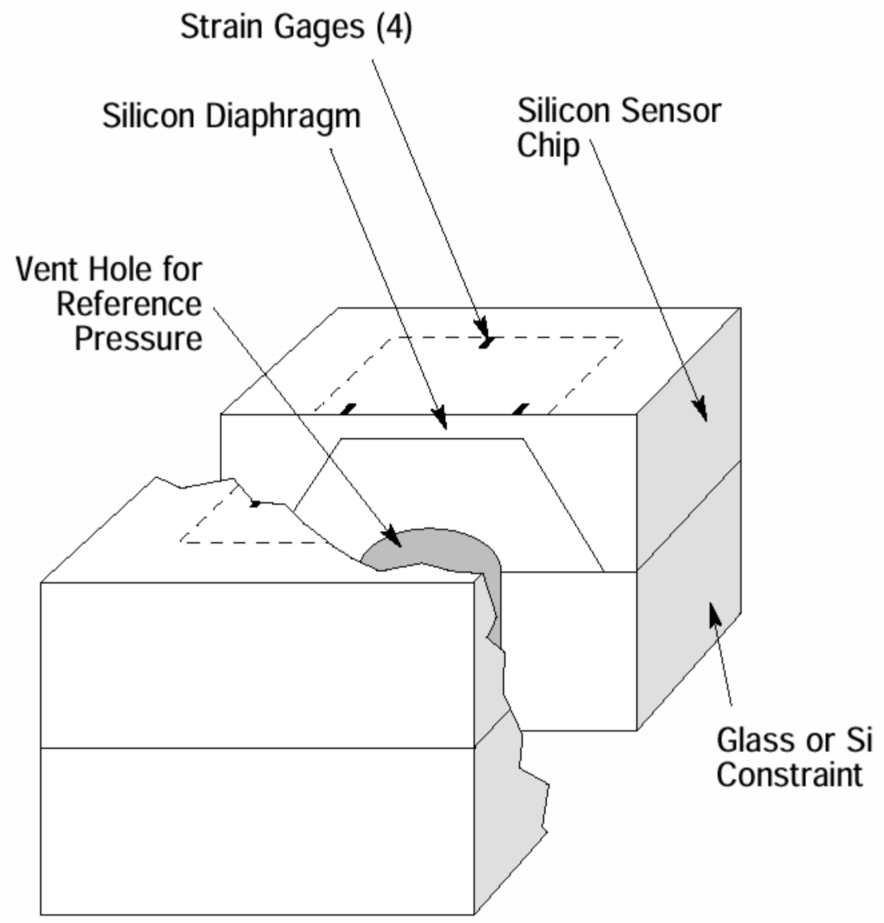
Sensor characteristics

PARAMETER	VALUE	UNITS	NOTES
GENERAL			
Pressure Range	2.5	kPa	≈ 0–10" H2O
	7	kPa	≈ 0–1 psi
	30	kPa	≈ 0–5 psi
Maximum Pressure	5x		rated pressure(8)
ELECTRICAL @ 25°C (77°F) unless otherwise stated			
Input Excitation	1.5	mA	2mA max.
Insulation Resistance	10^7	Ω	@ 50 V_{DC}
Input Impedance (10", 1PSI)	3,200	Ω	± 25%
Input Impedance (30kPa)	4,000	Ω	± 20%
Output Impedance	5,000	Ω	± 20%
Bridge Impedance	5,000	Ω	± 20%
ENVIRONMENTAL			
Temperature Range			
Operating(9)	–40 to +125	°C	–40° to +257°F
Compensation Range	0 to +70	°C	+32° to +158°F
Vibration	10	g_{RMS}	20 to 2000Hz
Shock	100	g	11 milliseconds
Life (Dynamic Pressure Cycle)	1×10^6	cycles	
MECHANICAL			
Weight	<5	grams	<0.2 oz.
Media Compatibility	Noncorrosive gases and dry air		
Wetted Materials — Top Port:	Nickel, gold plated Kovar, silicone gel, gold wire, RTV, silicon & glass.		
Wetted Materials — Bottom Port:	Gold plated Kovar, silicon, glass and RTV.(10)		



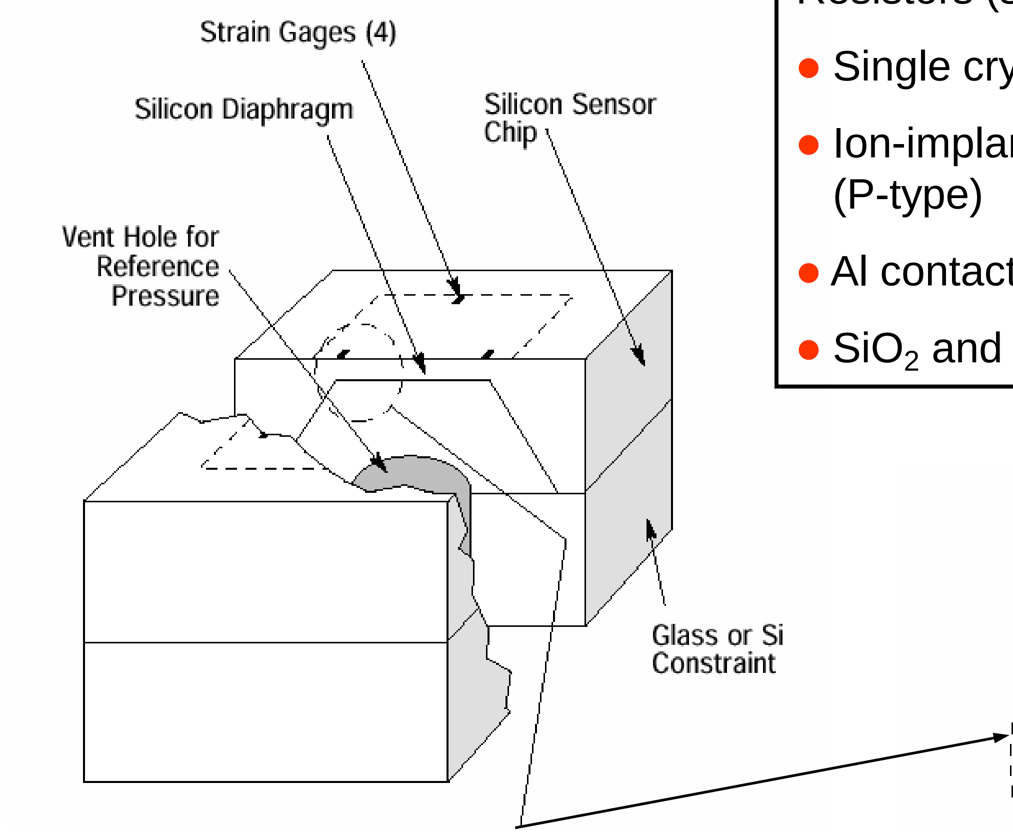
NovaSensor's NPH solid state sensor

Sensor characteristics



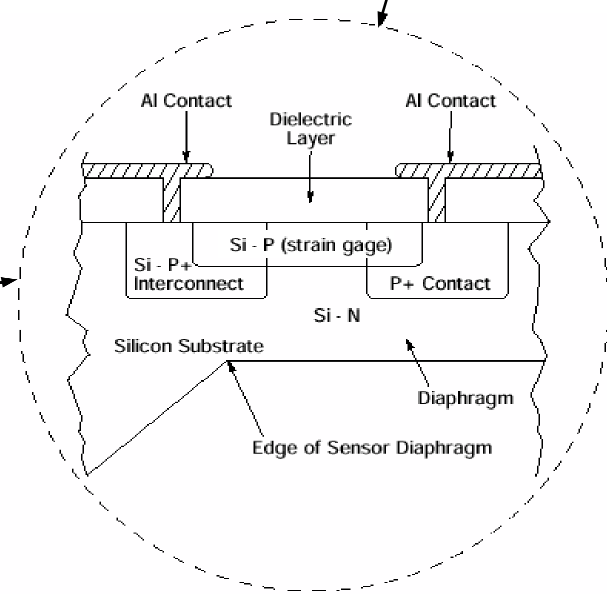
NovaSensor's NPH solid state sensor

Sensor characteristics



Resistors (strain gages):

- Single crystal silicon substrate (N-type)
- Ion-implantation (or diffused) Boron into silicon (P-type)
- Al contacts
- SiO_2 and Si_3N_4 as insulating layers



NovaSensor's NPH solid state sensor

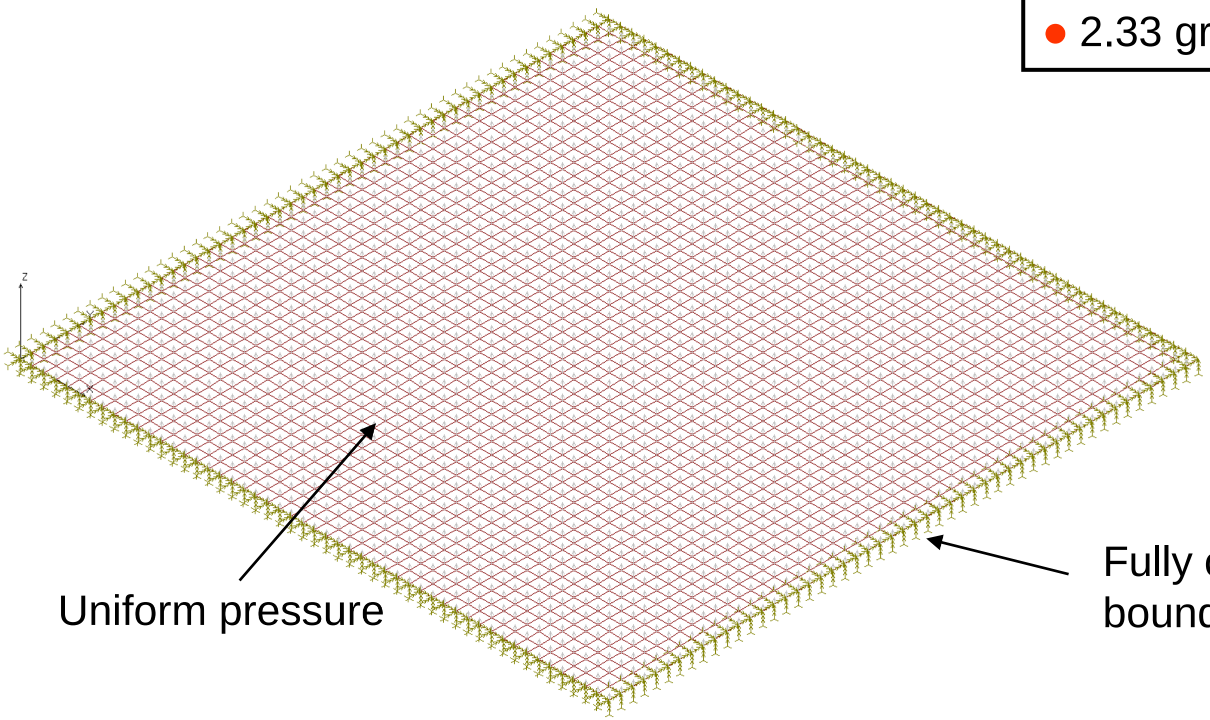
Computational (Finite Element) simulations

Shell formulation: constant thickness of 3 μm

1000 μm × 1000 μm diaphragm

Material properties utilized:

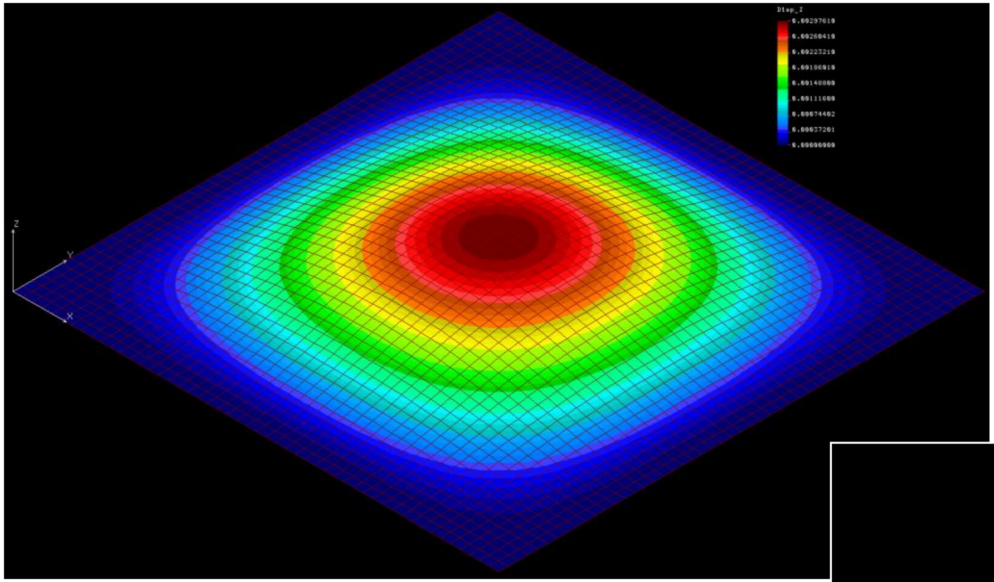
- 160 GPa
- 2.33 gr/cm^3



NovaSensor's NPH solid state sensor

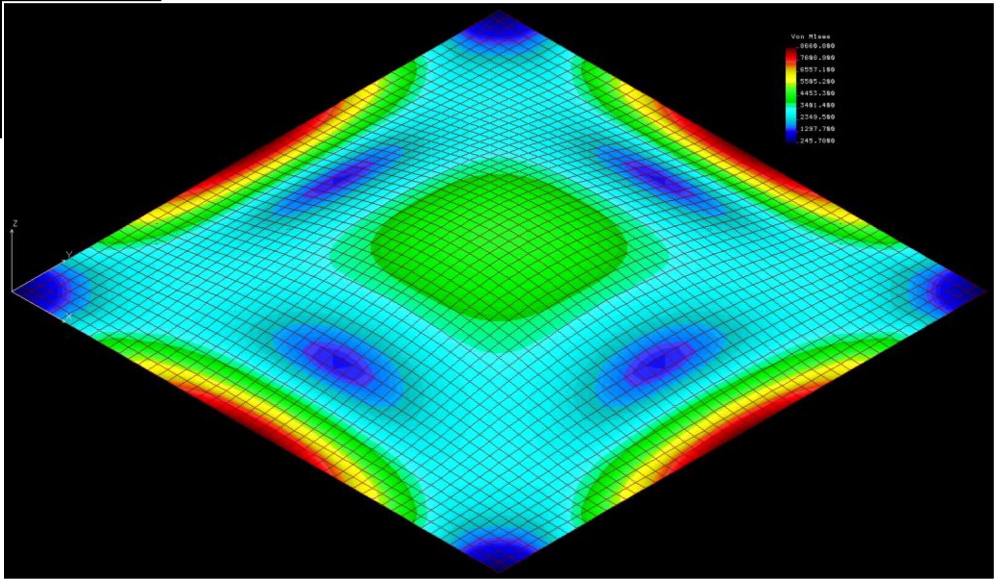
Computational (Finite Element) simulations

Out-of-plane displacements



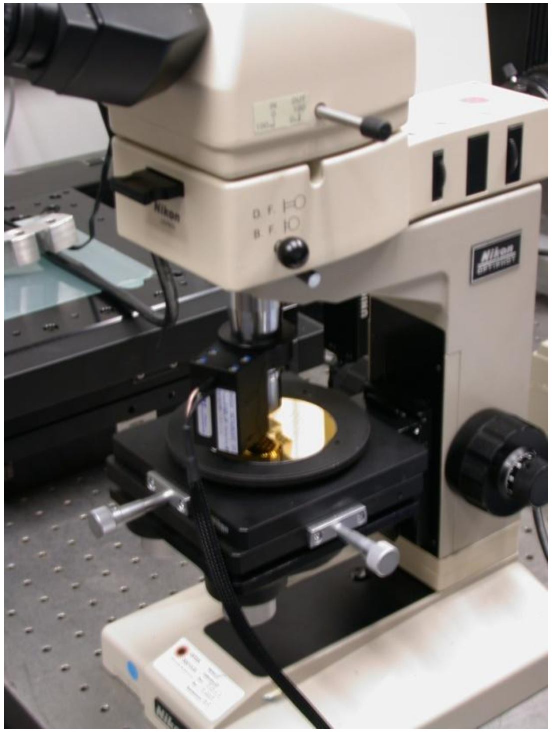
Note locations where maximum stresses, as related to maximum strains, appear – those locations are used for placing of strain gauges

Equivalent stresses (von Mises)



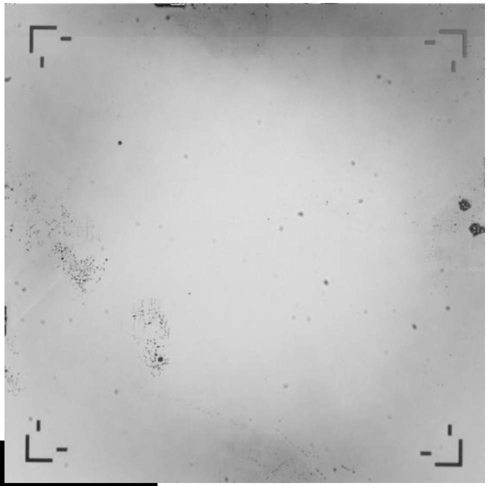
NovaSensor's NPH solid state sensor
Interferometric measurements of displacements

Interferometric microscope

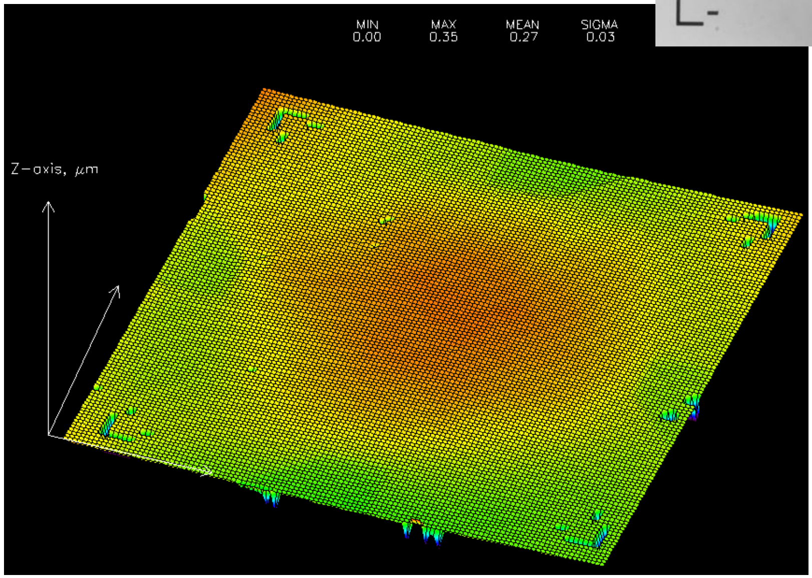


Diaphragm as fabricated: 0.0 psi

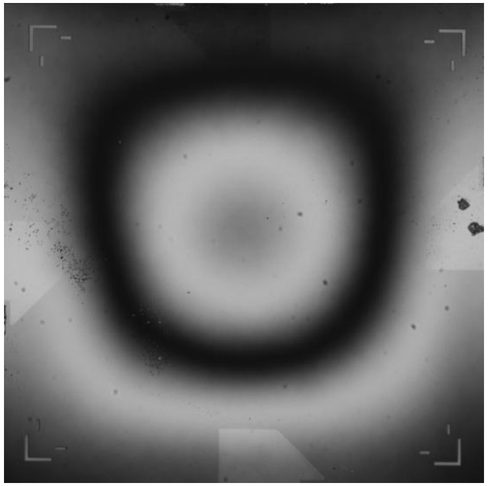
Interferogram of top surface of the diaphragm



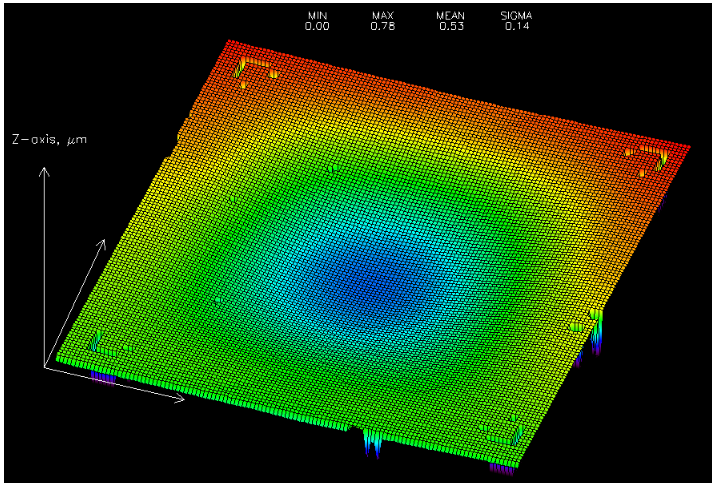
Shape information: 3D surface

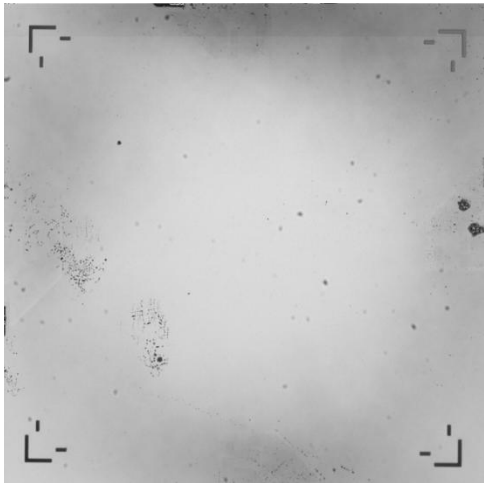


NovaSensor's NPH solid state sensor
Interferometric measurements of displacements

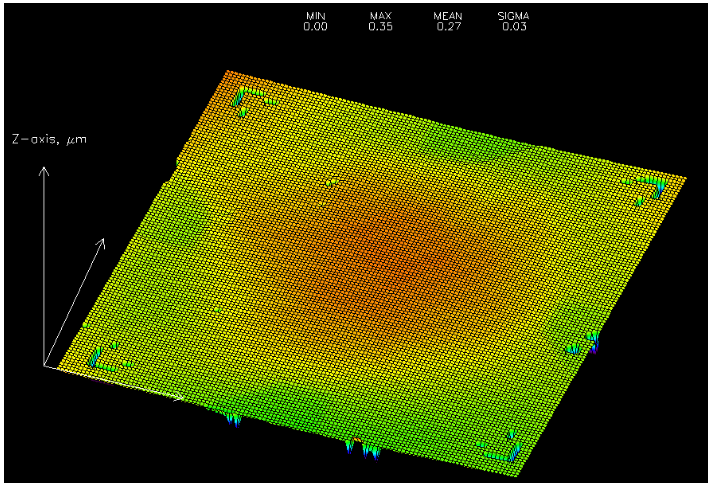


-1.0 psi



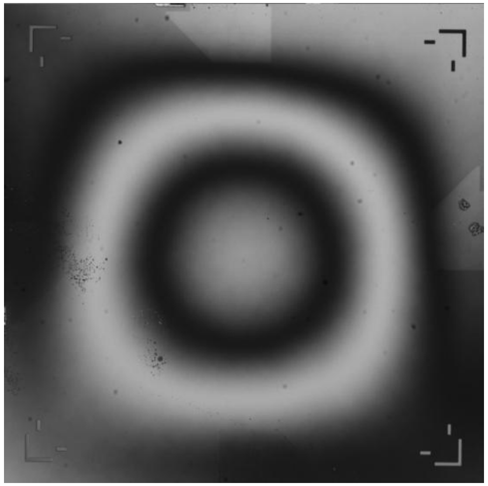


0.0 psi

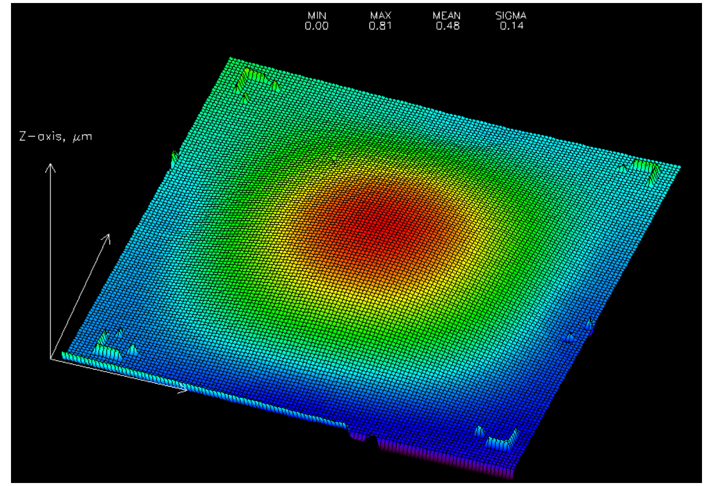


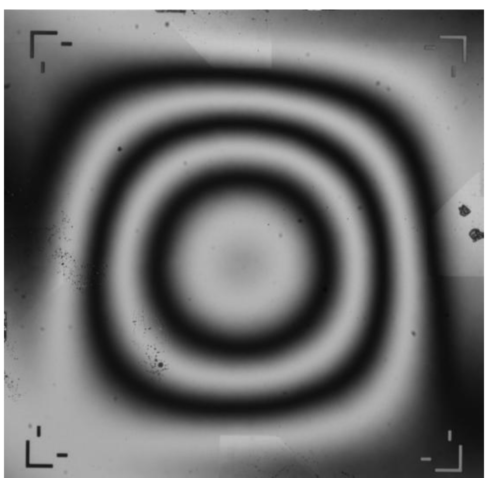
NovaSensor's NPH solid state sensor

Interferometric measurements of displacements

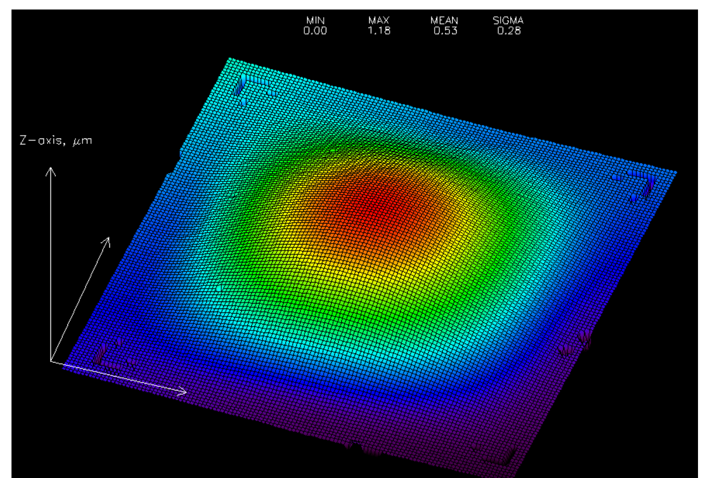


+1.0 psi





+2.0 psi

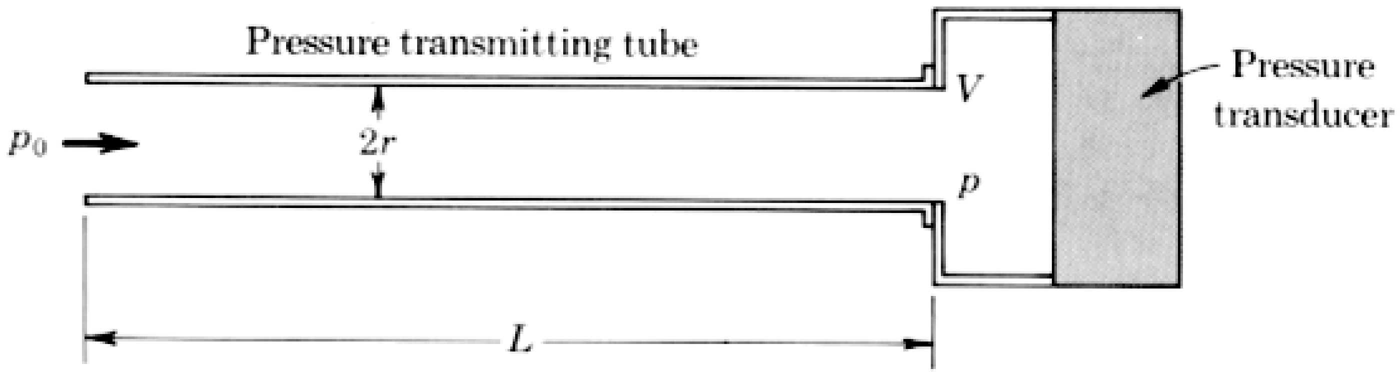


Dynamic response considerations

Transient response considerations

Transient response depends on: (1) response of the transducer (sensor); and (2) response of the pressure-transmitting fluid.

Schematic of a pressure-transmitting system



Dynamic response considerations

Transient response considerations

Pressure amplitude ratio: $$\left|\frac{p}{p_0}\right| = \frac{1}{\{[1-(\omega/\omega_n)^2]^2 + 4h^2(\omega/\omega_n)^2\}^{1/2}} \tag{6-4}$$

In this equation, p is the amplitude of the pressure signal impressed on the transducer. The natural frequency ω_n is given by

$$\omega_n = \sqrt{\frac{3\pi r^2 c^2}{4LV}} \tag{6-5}$$

and the damping ratio h is

$$h = \frac{2\mu}{\rho c r^3}\sqrt{\frac{3LV}{\pi}} \tag{6-6}$$

In the above formulas, c represents the velocity of sound in the fluid, μ is the dynamic viscosity of the fluid, and ρ is the fluid density. The phase angle for the pressure signal is

$$\phi = \tan^{-1}\frac{-2h(\omega/\omega_n)}{1-(\omega/\omega_n)^2} \tag{6-7}$$

The velocity of sound for air may be calculated from

$$c = 49.1\,T^{1/2}\ \text{ft/s} \qquad \text{with } T \text{ in °R}$$
$$c = 20.04\,T^{1/2}\ \text{m/s} \qquad \text{with } T \text{ in K}$$

When the tube diameter is very small, as in a capillary, it is possible to produce a very large damping ratio, so that Eq. (6-4) will reduce to the following for frequencies below the natural frequencies:

$$\left|\frac{p}{p_0}\right| = \frac{1}{[1+4h^2(\omega/\omega_n)^2]^{1/2}} \tag{6-8}$$

If the transmitting fluid is a gas, the entire system can act as a Helmholtz resonator with a resonant frequency of

$$\omega_n = \left[\frac{\pi r^2 c^2}{V(L+\frac{1}{2}\sqrt{\pi^2 r^2})}\right]^{1/2} \tag{6-9}$$

Reading assignment

- **Beckwith**: Ch. 3, 14
- **Bishop:** Ch. 6

References:

- J.P.Holman, *Experimental methods for engineers*, McGraw-Hill, 1989
- T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 5th ed., Addison-Wesley, 1995
- C. Furlong, *MEMS: introduction and applications*, Course notes on MEMS, ISTFA, Worcester, MA, 2004
- GE NovaSensors, http://www.gesensing.com/

Homework assignment

- **Bishop:** P6.6

Handout-B

B1.- Derive **complete** uncertainty equation (i.e., RSS uncertainty) for the unknown pressure p in a U-manometer. Discuss your observations.

B2.- Derive **complete** uncertainty equation (i.e., RSS uncertainty) for the "percent error" in the dead-weight tester described in this notes. Discuss your observations.