

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2009

Laboratory #4:

Vibration measurements:

Strain gauges + MEMS



General information

Office hours

Instructor: Cosme Furlong; cfurlong@wpi.edu
Everyday from 10:00 to 11:00 am
or by appointment

Teaching Assistant: During laboratory sessions



General information

Please refer to posted handouts:

“Laboratory 4: vibration measurements”



Objectives

The objectives of this laboratory are to:

- use two-different types of motion transducers to measure the natural frequencies, damping characteristics, and elastic modulus of a cantilever

For each motion transducer, vibration data will be analyzed to:

1. Determine the vibration amplitude, velocity, and acceleration in various units of measure;
2. Determine natural frequencies;
3. Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
4. Compare measurements with analytical and/or computational models of a cantilever;
5. Determine elastic modulus of a cantilever via vibration measurements



Amplitude (position), velocity, acceleration functions

- Recall that for a position function given by:

$$y(t) = A \sin(\omega t + \phi) \quad [\omega, \text{rad/sec}]$$

- Velocity function is:

$$[f = \frac{\omega}{2\pi}, \text{Hz}]$$

$$\dot{y}(t) = \omega A \cdot \cos(\omega t + \phi)$$

- Acceleration function is:

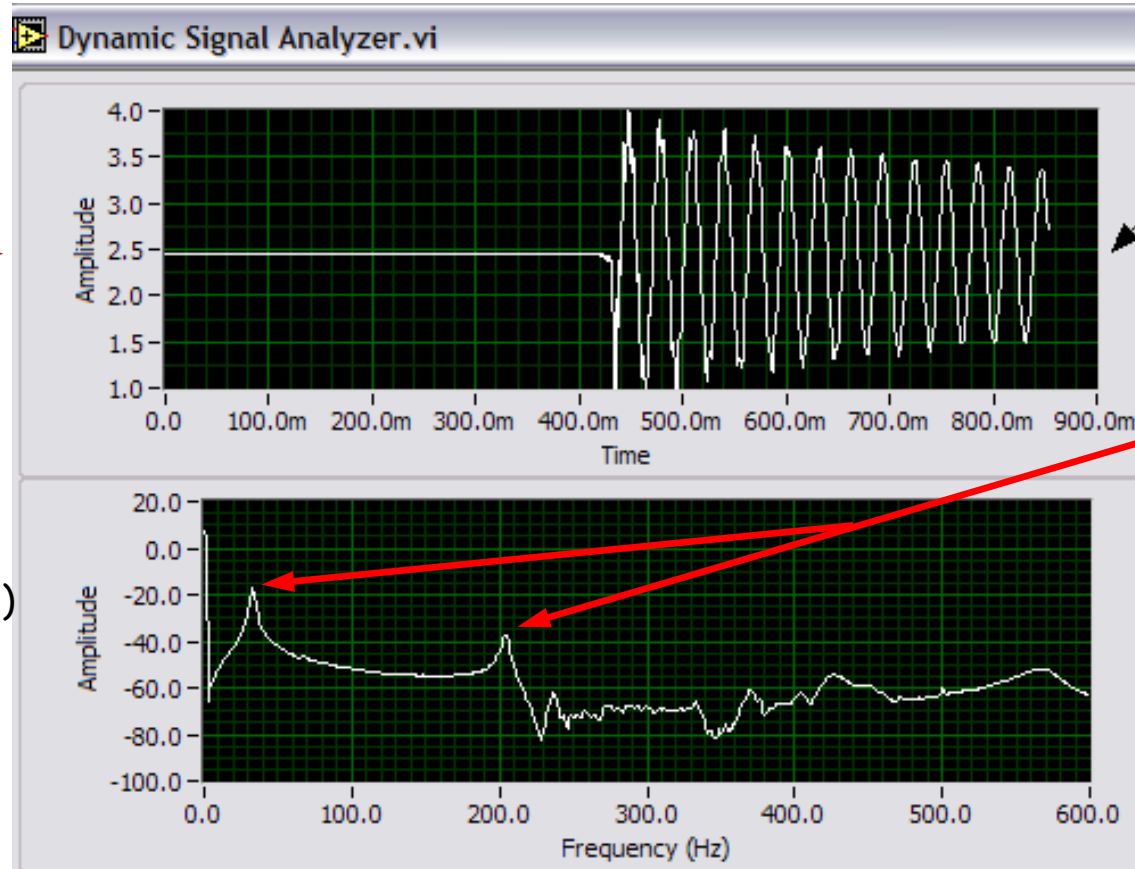
$$\ddot{y}(t) = \underbrace{-\omega^2 A}_{\substack{\uparrow \\ \text{Magnitude of} \\ \text{acceleration}}} \cdot \sin(\omega t + \phi)$$

Magnitude of
acceleration



Frequency-time domain: dynamic signal analyzer

Sample: single pluck (two frequencies are identified)



Note "peaks," which correspond to natural frequencies of vibration

Time-domain:
 $y(t)$ or $\ddot{y}(t)$

(Uses 2^n discrete points)

Frequency-domain:
 (discrete evaluation of)

$$H(f) = \int_{-\infty}^{\infty} y(t) \exp(-j2\pi ft) dt$$

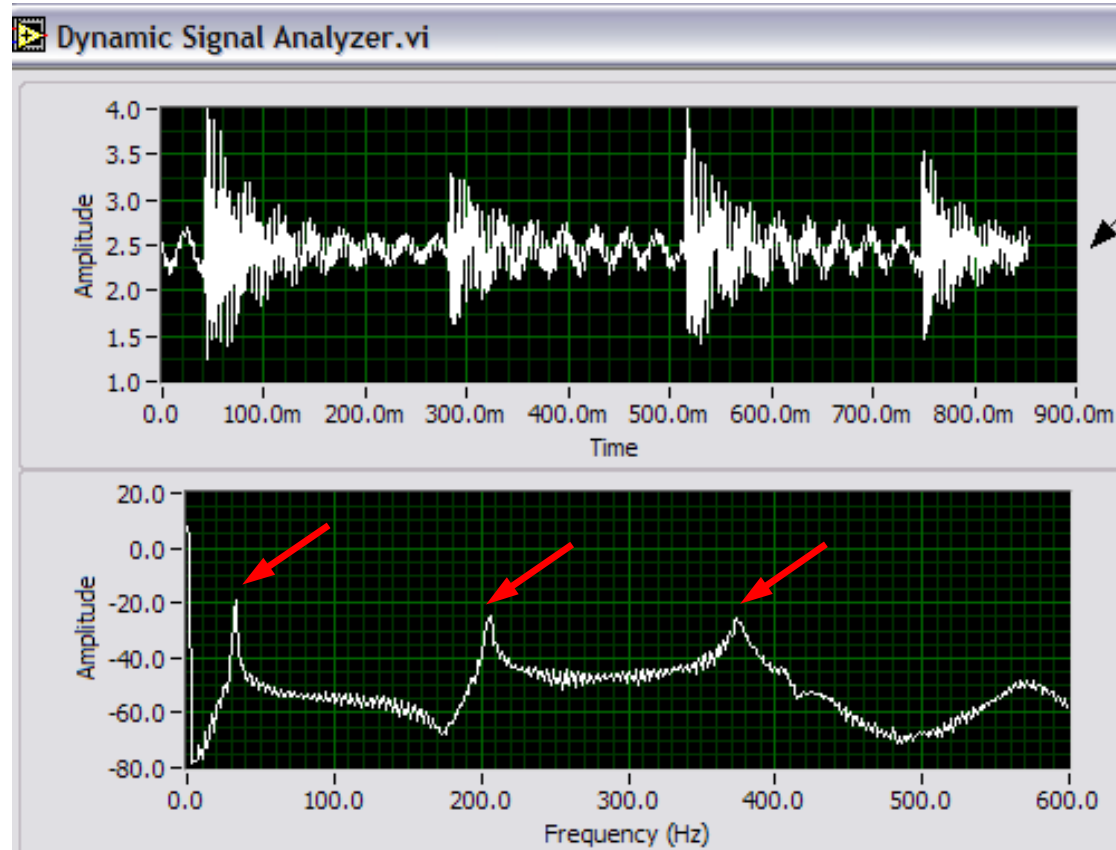
Amplitude or Acceleration (depending on the transducer used)



Frequency-time domain: dynamic signal analyzer

Sample: rapid plucking

(increases frequency resolution, three frequencies are identified)



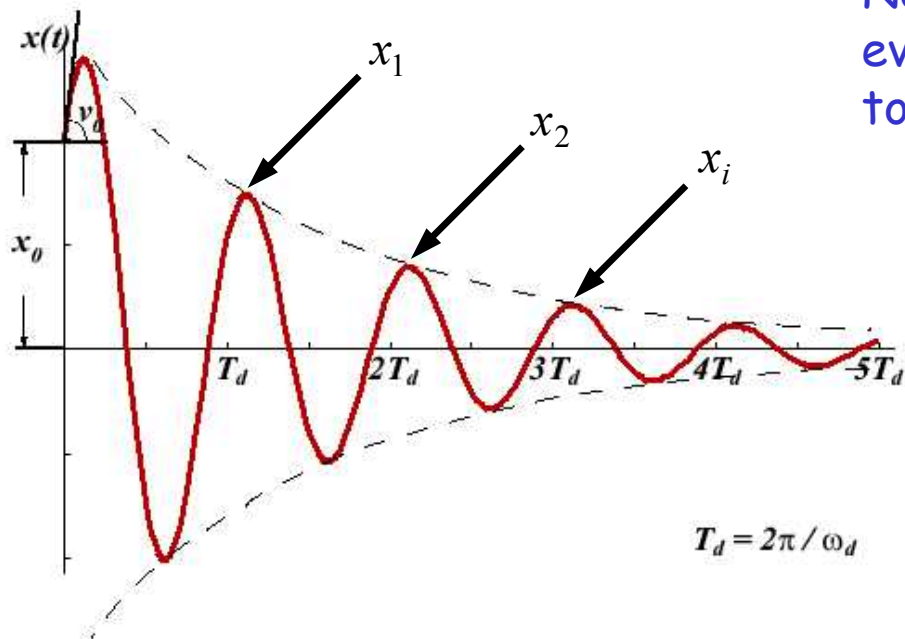
Analysis of a single degree of freedom system

First case: $F(t) = 0$ - Free vibrations

$$\lambda^2 - \omega^2 < 0$$

Under-damped system

Logarithmic decrement:
$$\delta = \ln \frac{x_1}{x_2} = \ln \frac{x_i}{x_{i+1}}$$



Note that you need to evaluate δ multiple times to obtain: $\delta \pm \Delta\delta$

(includes measured uncertainty)



Analysis of a single degree of freedom system

First case: $F(t) = 0$ - Free vibrations

$$\lambda^2 - \omega^2 < 0$$

Under-damped system

Logarithmic decrement:

$$\begin{aligned}\delta &= \ln \frac{x_1}{x_2} = \ln \frac{e^{-\zeta \omega_n t_1} \sin(\sqrt{1-\zeta^2} \omega_n t_1 + \phi)}{e^{-\zeta \omega_n (t_1+T_d)} \sin[\sqrt{1-\zeta^2} \omega_n (t_1 + T_d) + \phi]} \\ &= \ln \frac{x_1}{x_2} = \ln \frac{e^{-\zeta \omega_n t_1}}{e^{-\zeta \omega_n (t_1+T_d)}} = \ln e^{\zeta \omega_n T_d} = \zeta \omega_n T_d\end{aligned}$$

Recall that: $T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}} \Rightarrow$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$

(account for measured uncertainty in δ)

You can determine damping ratio with knowledge of δ

$$\zeta = \frac{b}{b_c} \quad (\text{Damping ratio - percentage of critical damping})$$



Background: cantilevers

Natural frequencies

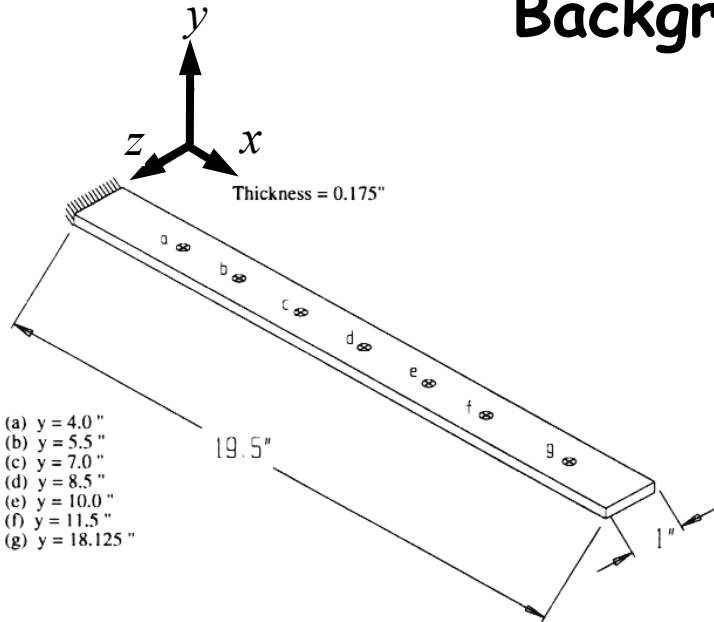


Fig. 3 Low carbon steel beam showing locations of interest

$$\text{First: } \omega_n = 3.5160 \sqrt{\frac{EI}{\bar{m}L^4}}, \quad \text{rad/s} \quad (3)$$

where,

E = Young's modulus = 30×10^6 psi,

I = Moment of inertia of the beam section = 4.46614×10^{-4} in⁴,

\bar{m} = mass per unit of length = 1.26805×10^{-4} lbf·s²/in², and

L = length of the beam = 19.5 in,

$$\text{Second: } \omega_n = 22.0345 \sqrt{\frac{EI}{\bar{m}L^4}}, \quad (4)$$

$$\text{Third: } \omega_n = 61.6972 \sqrt{\frac{EI}{\bar{m}L^4}}, \quad (5)$$

$$\text{Fourth: } \omega_n = 120.0902 \sqrt{\frac{EI}{\bar{m}L^4}}, \quad (6)$$

$$\text{Fifth: } \omega_n = 199.8600 \sqrt{\frac{EI}{\bar{m}L^4}}, \quad (7)$$

$$EI \frac{\partial^4 y}{\partial x^4} + \bar{m} \frac{\partial^2 y}{\partial t^2} = 0. \quad (8)$$

Note: first frequency
can also be expressed as

$$\omega_n = 1.015 \sqrt{\frac{E}{\rho}} \left(\frac{t}{L^2} \right), \quad \text{rad/s}$$

Table 4. Natural Frequencies obtained using Eq. 3 to 7

Natural Frequency	Magnitude, Hz
1	15.1270
2	94.8010
3	265.446
4	516.675
5	859.876



Recovery of elastic modulus from vibration measurements

Assume the use of the fundamental frequency (higher frequencies can also be used):

$$\omega_n = 1.015 \sqrt{\frac{E}{\rho}} \left(\frac{t}{L^2} \right), \text{ rad/s}$$

Solve for the elastic modulus, E , at the measured ω_n :

$$E = E(\omega_n, \rho, t, L)$$

Note that uncertainty analysis is necessary:

$$\delta E = \delta E(\omega_n, \rho, t, L)$$



Reading assignment

- **Beckwith:** Ch. 6, 17, Appendix B
- **Bishop:**



Homework assignment: Handout-M

M1.- Refer to Appendix B of Beckwith. Do Fourier transformation of the square wave shown in Fig. 4.9(a).

Plot the transformed function using:

1. Three-terms of the series;
2. Five-terms of the series; and
3. Seven-terms of the series

Discuss your observations.

