WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

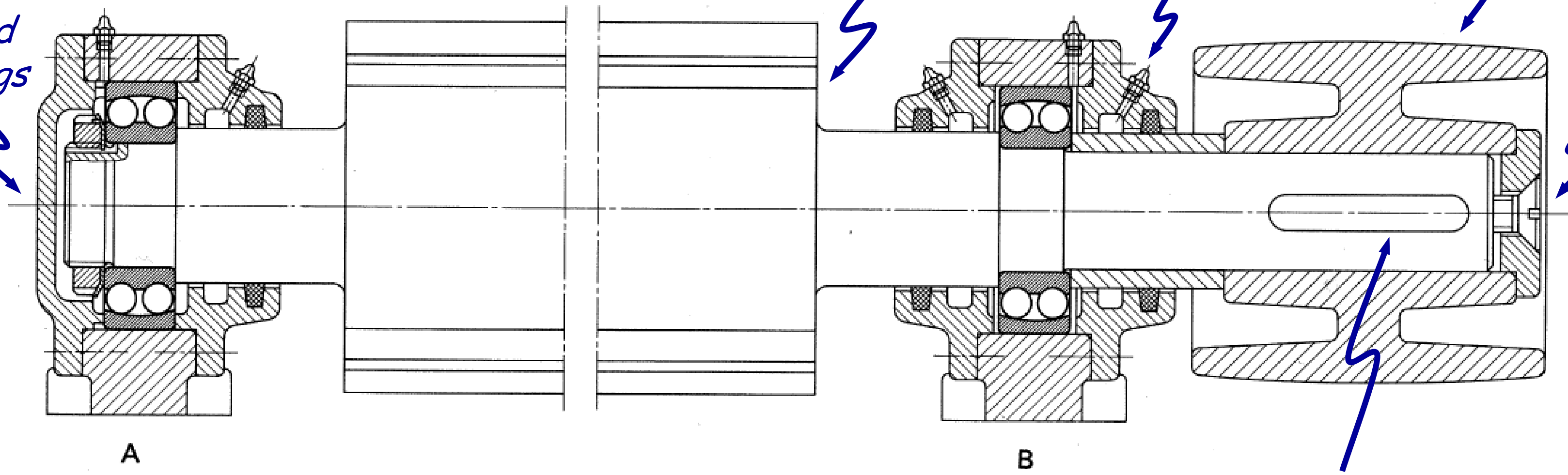
DESIGN OF MACHINE ELEMENTS
ME-3320, A'2011

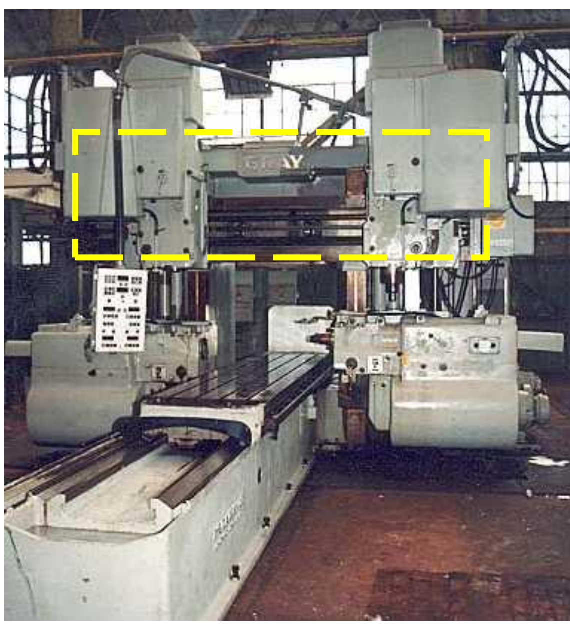
Lecture 19

03 October 2011

Shaft design

Example of rotating machinery: self-aligning ball bearings

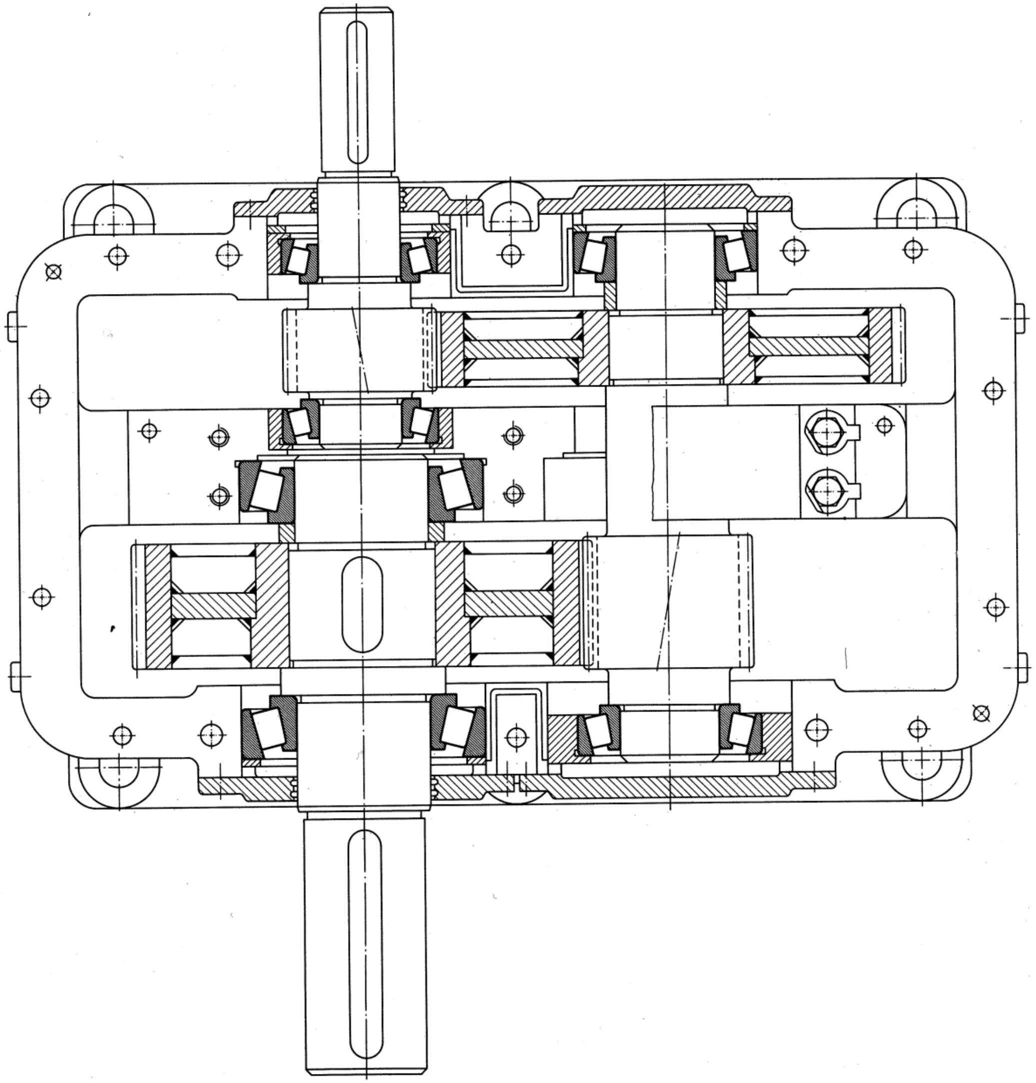




☞ **Cutter shaft of a planer:** shaft diameter (at bearings locations) is 40 mm. Input power is 12 HP at maximum speed of 4,500 rpm

Shaft design

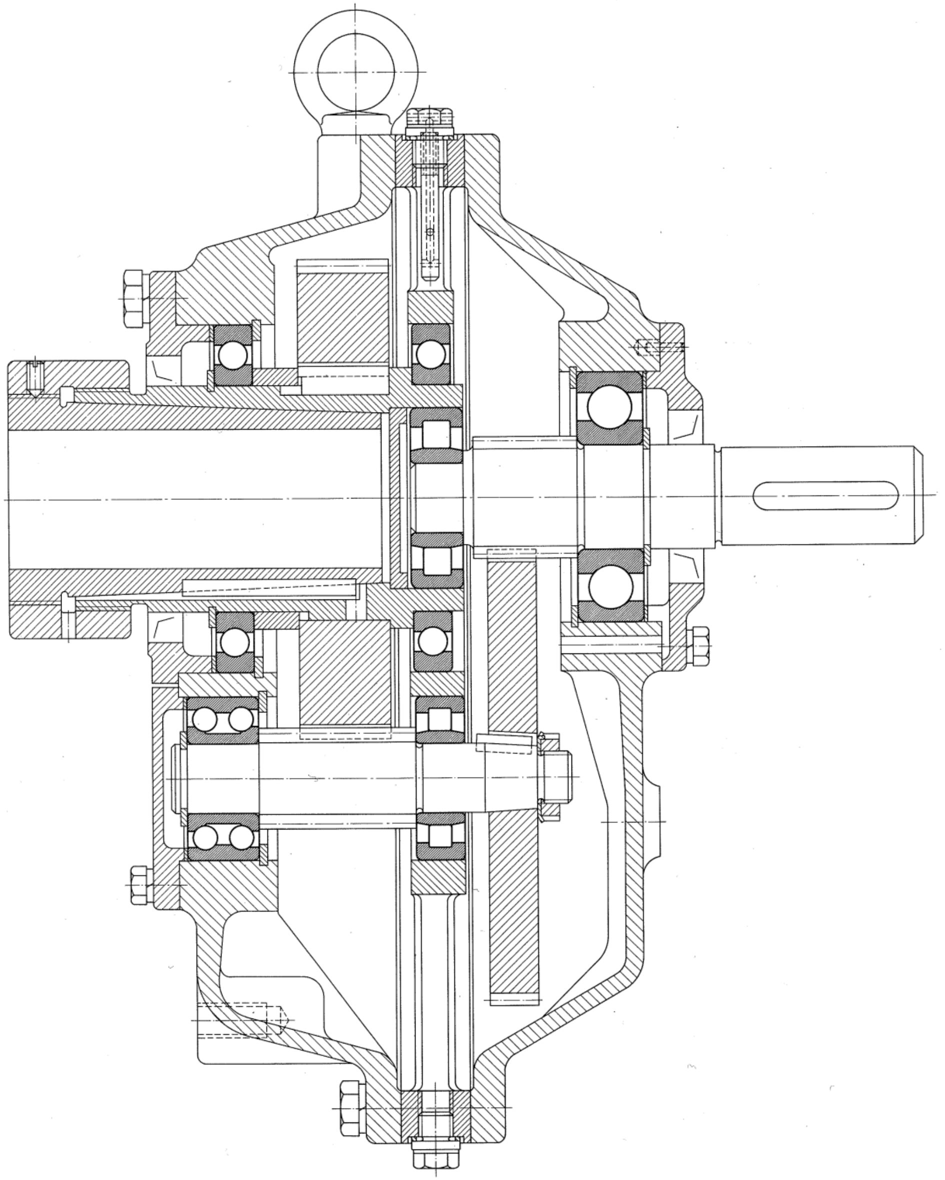
Example of rotating machinery: tapered roller bearings



- **Double reduction spur gear:** maximum shaft diameter (at largest bearing location) is 180 mm. Input power is 1,500 HP at maximum speed of 1,500 rpm. Gear ratio is 8:1

Shaft design

Example of rotating machinery



- **Slip-on gearbox:** maximum shaft diameter (at largest bearing location) is 65 mm. Input power is 4 HP at maximum speed of 1,560 rpm. Gear ratio is 12.5:1

Shaft design

Loads and stresses

☞ **Refer to previous lectures**

- ⇨ Bending stresses: mean, amplitude, maximum (appropriately corrected for stress concentration factors, K_f, K_{fm})
- ⇨ Shear stresses: mean, amplitude, maximum (appropriately corrected for stress concentration factors, K_{fs}, and K_{fsm})
- ⇨ Direct shear, bearing, and tearout

☞ **Shafts are designed to account for both, stresses and deflections**

Shaft design

Materials

- **Ground precision shafts of different materials can be purchased. Make sure to take into account hardness and machinability of material -- geometrical features such as grooves, holes, threads, and keyways may need to be machined**

- **Low- to medium-carbon steels:** cold- or hot-rolled
  - Cold rolled steels used for shaft diameters, ϕ, lower than 3 inch
  - Hot-rolled must be machined in order to remove carburized outer layers
  - Rolled steels may contain residual stresses

- **Stainless steels**
- **Bonze**
- **Other: depending on application, stress levels, deflection, etc.**
- **… consult standards (ANSI, ASME, etc…)**

Shaft design

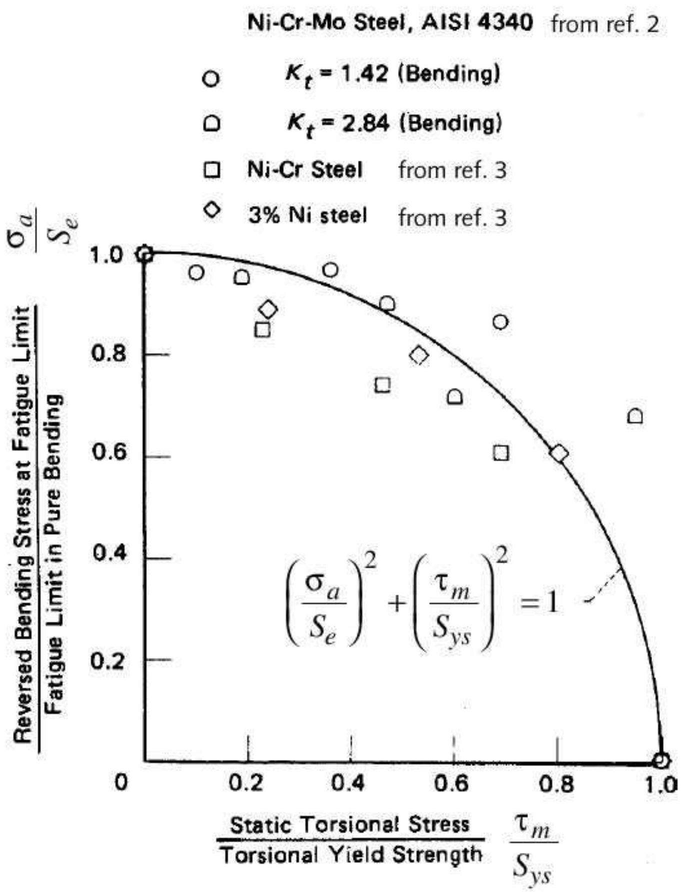
Some design considerations

- **Refer to previous lectures**
  - Shafts can be design to minimize both, stresses and deflections: use the shortest shaft possible
  - Cantilever configurations typically have larger deflections
  - Sometimes, hollow shafts are utilized (better stiffness/mass ratio), however, may be more expensive
  - Minimize stress-concentration features -- particularly in regions subjected to high bending stresses
  - Use low-carbon steels when designing shafts subjected to minimum deflection considerations
  - When using gears, shaft deflections at gear locations cannot exceed 0.005 in (127 μm) and slopes cannot exceed 0.03 degrees
  - If plain bearings are used, slopes at gear locations cannot exceed 0.04 degrees
  - If plain bearings are used, deflections at gear locations should be less than the oil-film thickness in the bearing
  - First natural frequency should be at least 3-times larger than the highest frequency expected in service
  - … Consult standards for additional considerations… (ASME, AGMA… etc…)

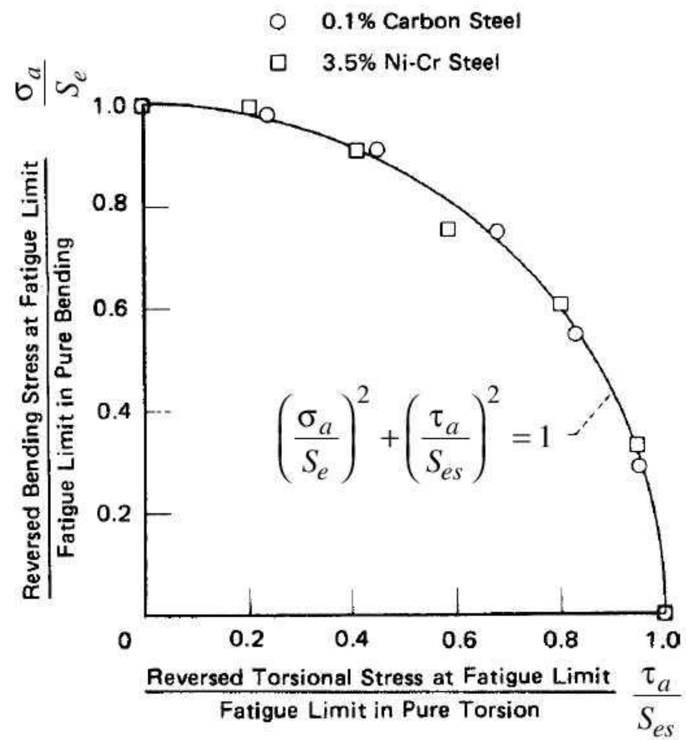
Shaft design

Failure envelope: <u>fatigue failure is taken into account</u>

Experimental data and fatigue failure envelope. See ANSI/ASME standard B106.1M-1985 on "design of transmission shafting."



(*a*) Combined stress fatigue-test data for reversed bending combined with static torsion (from ref. 4)



(*b*) Combined stress fatigue-test data for reversed bending combined with reversed torsion (from ref. 5)

Shaft design

ASME method: fully-reversed bending and constant torsion

- Based on failure envelope (shown before):

$$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\tau_m}{S_{ys}}\right)^2 = 1$$

- Safety factor: N_f

- von Mises stress in shear (strain-energy theory): $S_{ys} = \dfrac{S_y}{\sqrt{3}}$

- Amplitude stress in bending and mean torsional stresses: σ_a, τ_m (corrected for fatigue stress-concentration factors)

- Shaft diameter is calculated as:

$$\boxed{d = \left\{ \frac{32 N_f}{\pi} \left[\left(K_f \frac{M_a}{S_f} \right)^2 + \frac{3}{4} \left(K_{fsm} \frac{T_m}{S_y} \right)^2 \right]^{1/2} \right\}^{1/3}}$$

Shaft design

Fluctuating bending and torsion

- **Based on von Mises stresses (amplitude and mean):** σ'_a, σ'_m

- **Failure envelope given as:**

$$\frac{1}{N_f} = \frac{\sigma'_a}{S_f} + \frac{\sigma'_m}{S_{ut}}$$

- **von Mises stress in shear (strain-energy theory):** $S_{ys} = \dfrac{S_y}{\sqrt{3}}$

- **Amplitude and mean stress components in bending and shear: (corrected for fatigue stress-concentration factors)** $\sigma_a, \sigma_m, \tau_a, \tau_m$

- **Shaft diameter is calculated as:**

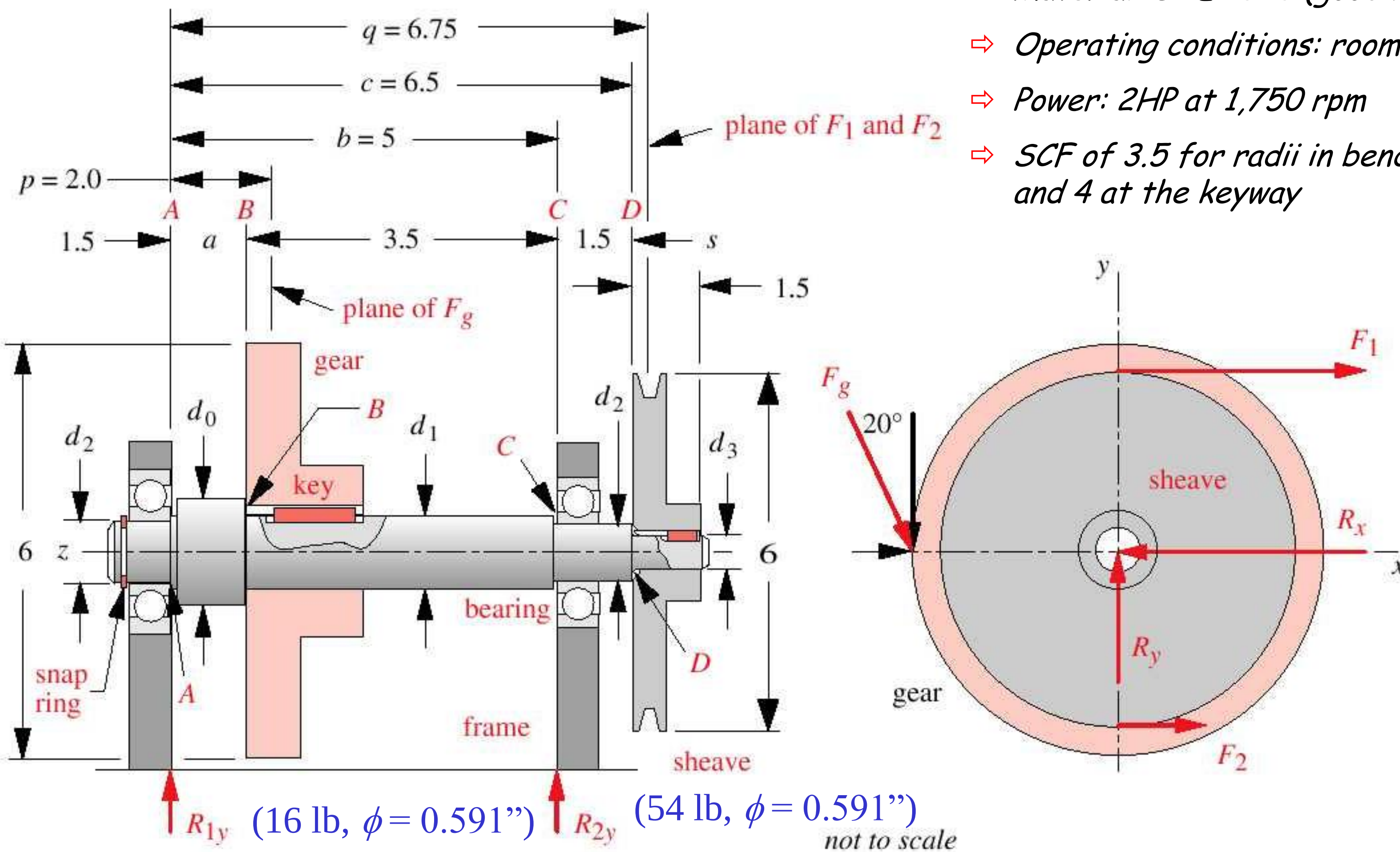
$$\boxed{d = \left\{ \frac{32 N_f}{\pi} \left[\frac{\sqrt{(K_f M_a)^2 + \frac{3}{4}(K_{fs} T_a)^2}}{S_f} + \frac{\sqrt{(K_{fm} M_m)^2 + \frac{3}{4}(K_{fsm} T_m)^2}}{S_{ut}} \right] \right\}^{1/3}}$$

Shaft design

Fully-reversed bending and constant torsion

☞ **Review and Master: Example 10-1**

- ⇨ *Safety factor: 2.5*
- ⇨ *Infinite life*
- ⇨ *Material: SAE 1020 (good notch sensitivity)*
- ⇨ *Operating conditions: room temperature*
- ⇨ *Power: 2HP at 1,750 rpm*
- ⇨ *SCF of 3.5 for radii in bending, 2 in torsion, and 4 at the keyway*

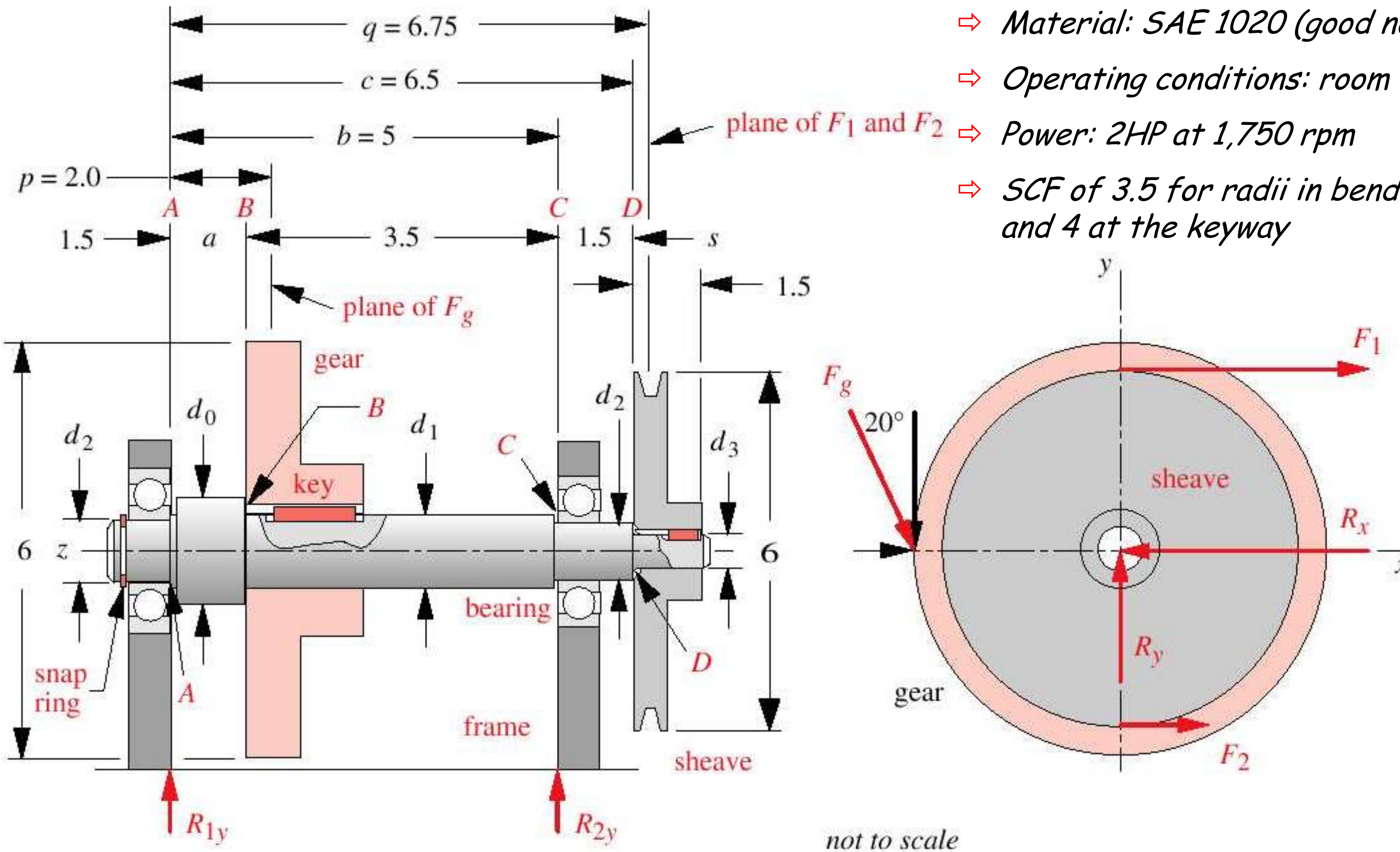


Shaft design

Fluctuating bending and torsion

☞ **Review and Master: Example 10-2**

- *Mean and alternating torque are both 74 lb-in*
- *Safety factor: 2.5*
- *Infinite life*
- *Material: SAE 1020 (good notch sensitivity)*
- *Operating conditions: room temperature*
- *Power: 2HP at 1,750 rpm*
- *SCF of 3.5 for radii in bending, 2 in torsion, and 4 at the keyway*



Reading

- Chapters 10 of textbook: Sections 10.0 to 10.8
- Review notes and text: ES2501, ES2502, ES2503

Homework assignment

- Author's: 10-1, 10-2
- Solve: 10-1e, 10-4e, 10-9e