WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, C'2010

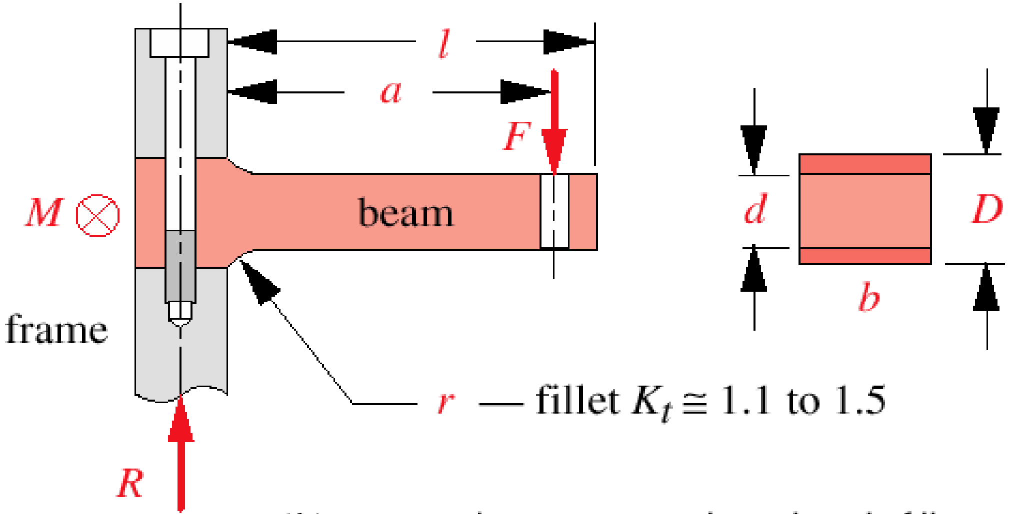
Lecture 16

12 February 2010

Fatigue failure

Designing for HCF

- **Reviewed Example 6-4: cantilever bracket under fully-reversed bending**



(b) Better design — machined with fillets

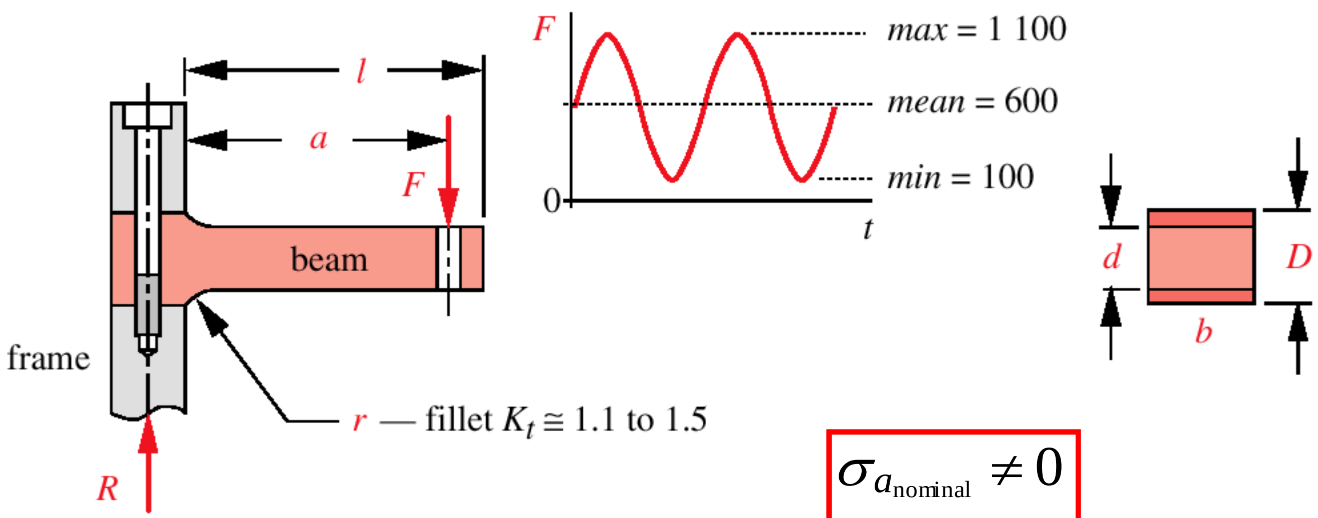
- *Fully reversed load of 1000 lb (amplitude, therefore, is 500 lbs)*
- *Life of about 10^9 cycles*
- *Material: steel/machined*
- *Operating conditions: room temp.*

$$\sigma_{a_{\text{nominal}}} = 26{,}667 \text{ lb/in}^2$$
$$\sigma_{m_{\text{nominal}}} = 0$$

Fully-reversed bending… let's investigate fluctuating loads...

Fatigue failure

Designing for HCF. Fluctuating stresses

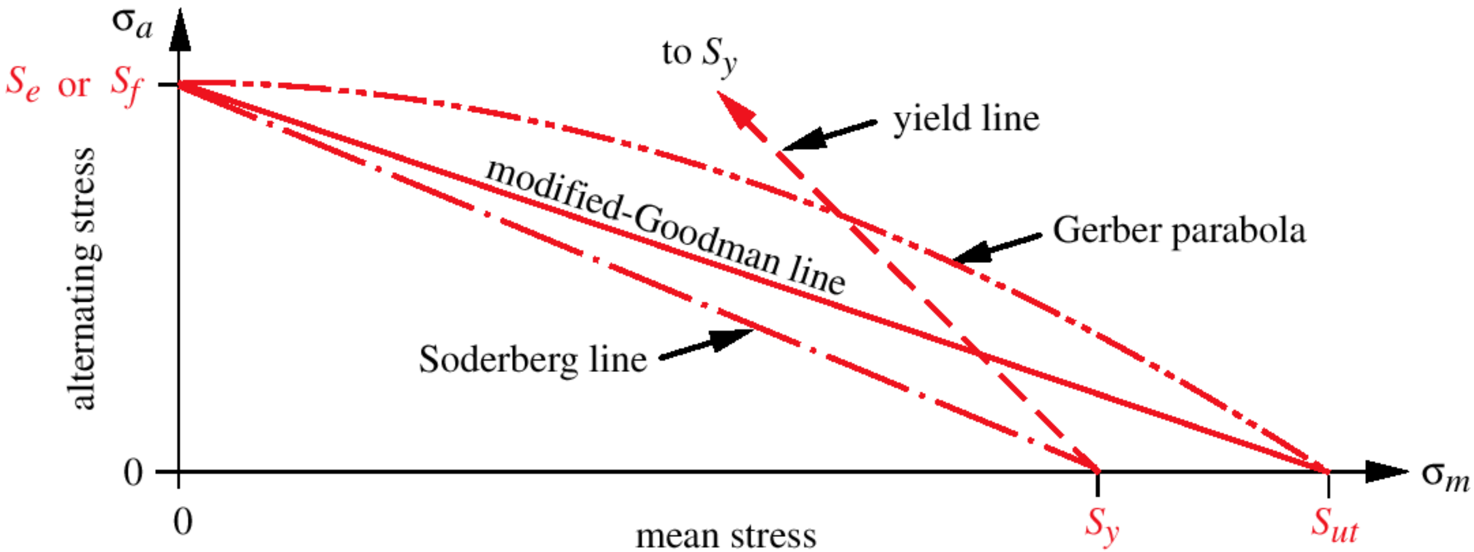


$$\sigma_{a_{\text{nominal}}} \neq 0$$

$$\sigma_{m_{\text{nominal}}} \neq 0$$

Fatigue failure

Designing for HCF: fluctuating uniaxial stresses



Gerber parabola:	$S_a = S_e\left(1 - \frac{\sigma_m^2}{\sigma_{ut}^2}\right)$	*(Fits experimental data: useful to study failed parts)*
Modified-Goodman line:	$S_a = S_e\left(1 - \frac{\sigma_m}{\sigma_{ut}}\right)$	*(Conservative theory)*
Soderberg line:	$S_a = S_e\left(1 - \frac{\sigma_m}{\sigma_y}\right)$	*(Overly conservative theory)*

Fatigue failure

Modified Goodman-diagram

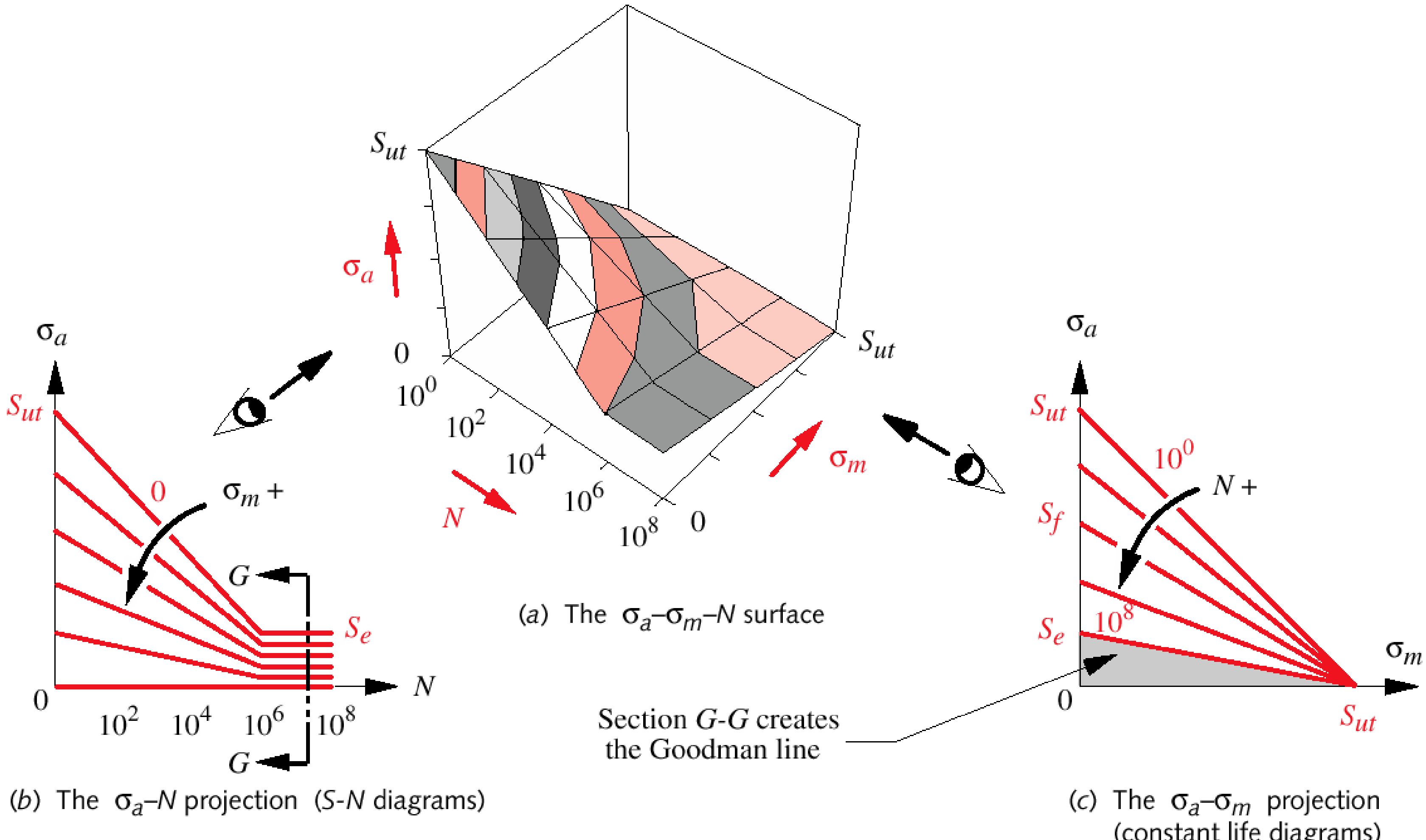
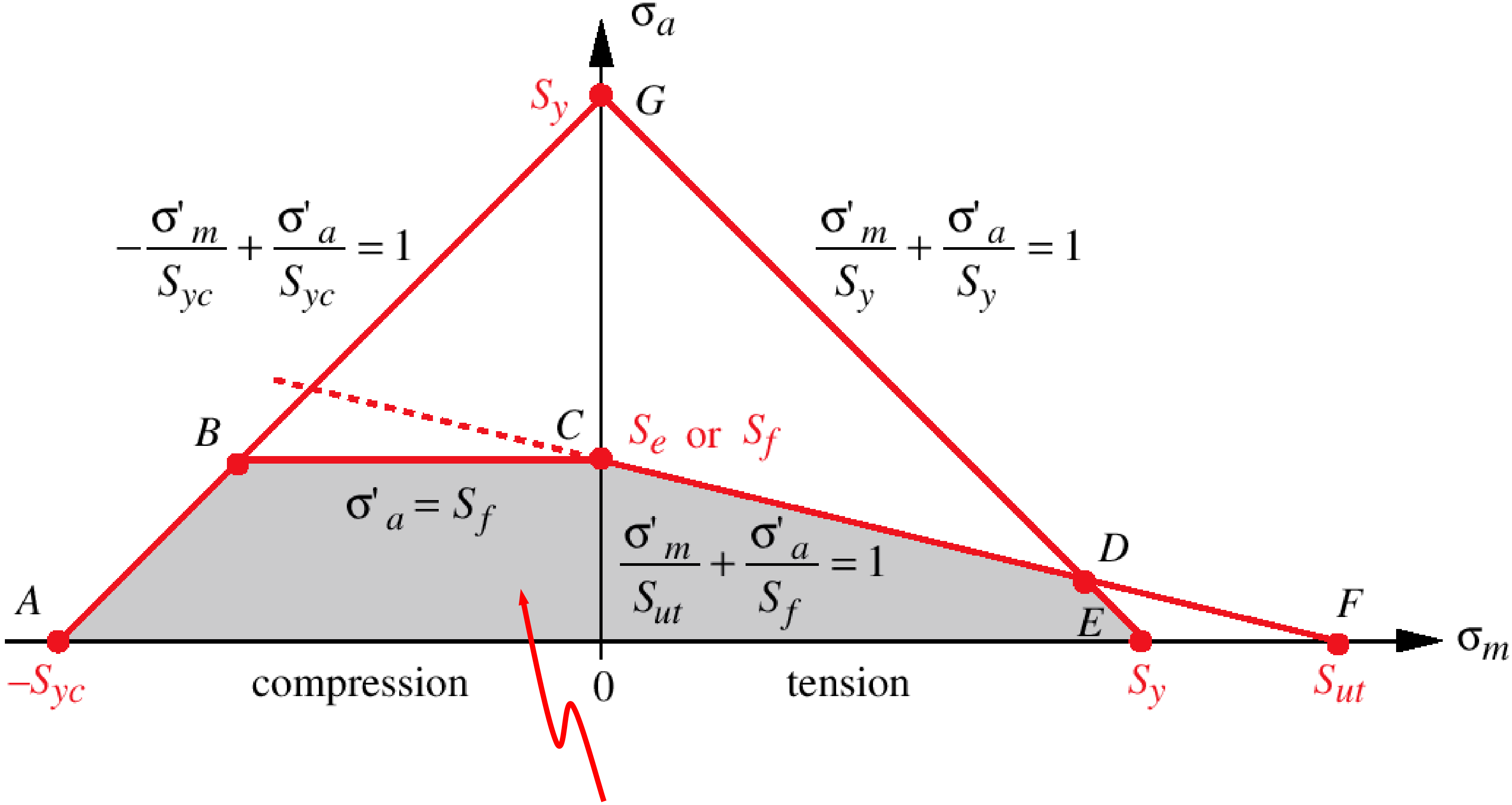


FIGURE 6-43

Fatigue failure

Augmented modified Goodman-diagram

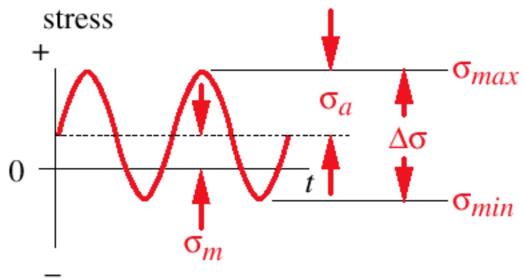


Area in "gray" is the "safe-zone"

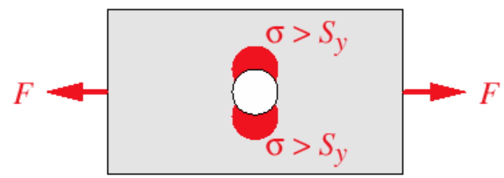
Fatigue failure

Stress-concentration factors in fluctuating stresses

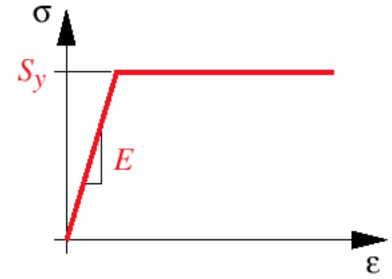
Note that component may "yield" locally



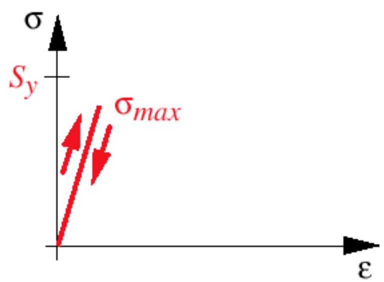
(a) Fluctuating stress



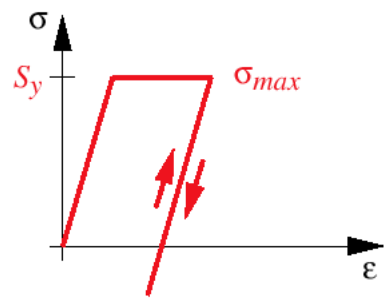
(b) Possible plastic zones



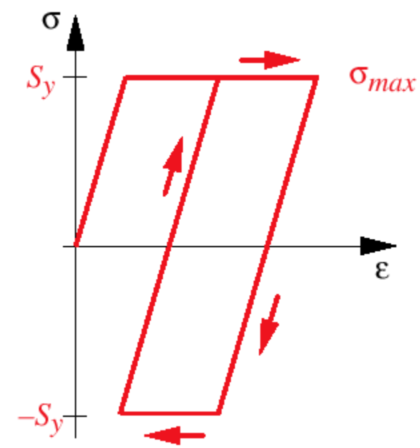
(c) Elastic-perfectly plastic material



(d) No yielding



(e) Yielding on first cycle

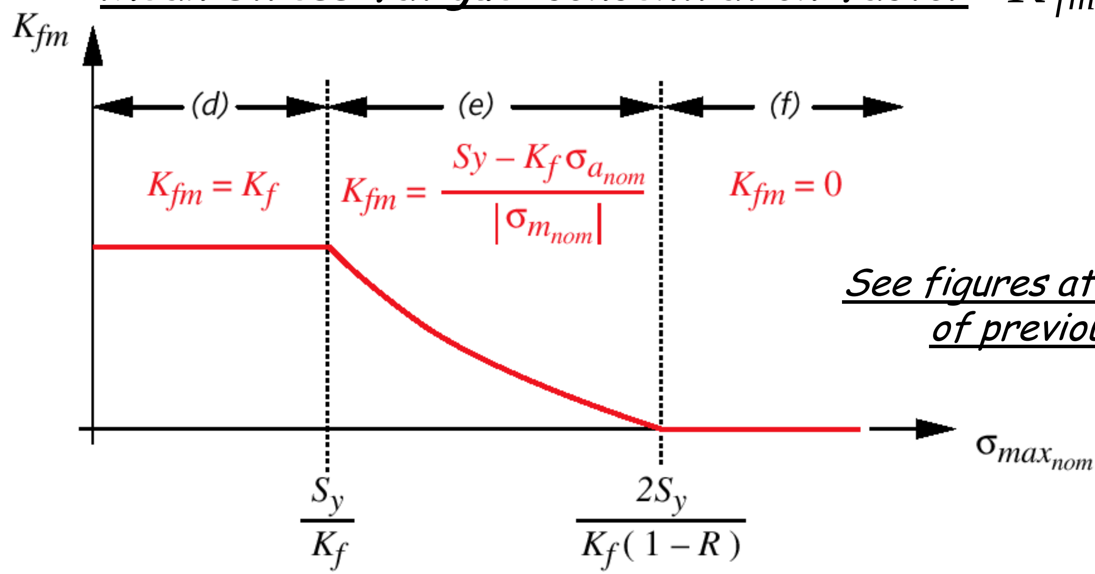


(f) Reversed yielding

Fatigue failure

Stress-concentration factors in fluctuating stresses

Mean stress fatigue-concentration factor: K_{fm}



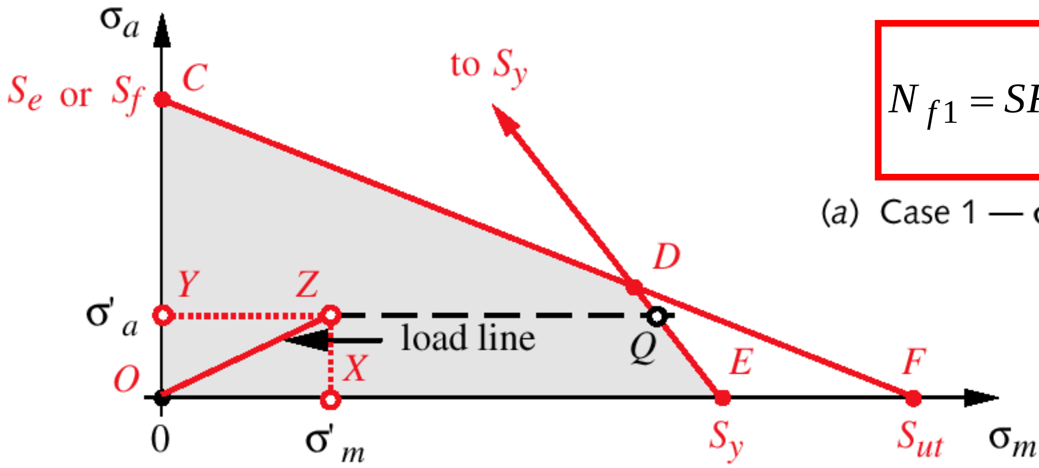
See figures at the bottom of previous page

(g) K_{fm} as a function of the maximum nominal stress $\sigma_{max_{nom}}$

$$
\begin{cases}
\text{if } K_f\left|\sigma_{\max_{\text{nominal}}}\right| < S_y \longrightarrow K_{fm} = K_f \\
\text{if } K_f\left|\sigma_{\max_{\text{nominal}}}\right| > S_y \longrightarrow K_{fm} = \dfrac{S_y - K_f\ \sigma_{a_{\text{nominal}}}}{\sigma_{m_{\text{nominal}}}} \\
\text{if } K_f\left|\sigma_{\max_{\text{nominal}}} - \sigma_{\min_{\text{nominal}}}\right| > 2S_y \longrightarrow K_{fm} = 0
\end{cases}
$$

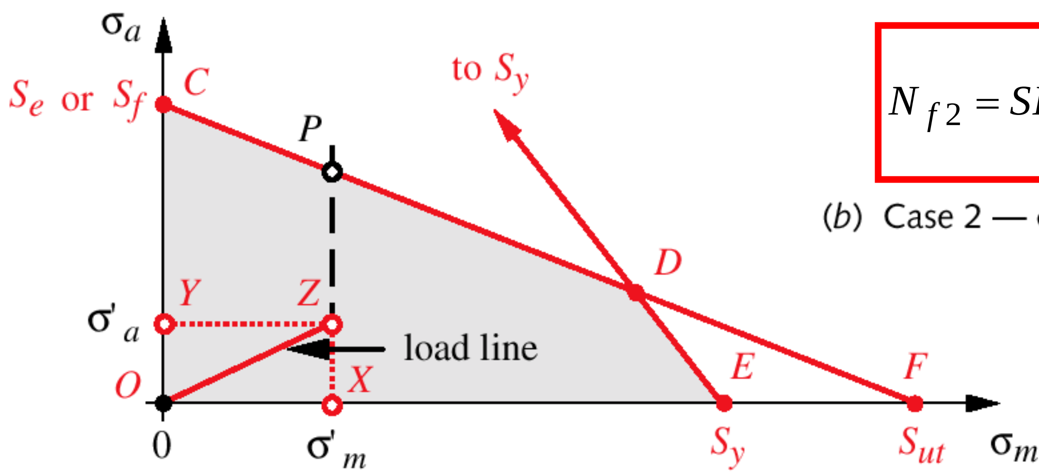
Fatigue failure – Modified Goodman's diagram

Safety factors in fluctuating stresses: Cases 1 and 2



$$N_{f1} = SF = \frac{\overline{YQ}}{\overline{YZ}} = \frac{S_y}{\sigma'_m}\left(1 - \frac{\sigma'_a}{S_y}\right)$$

(*a*) Case 1 — σ_a constant and σ_m varies

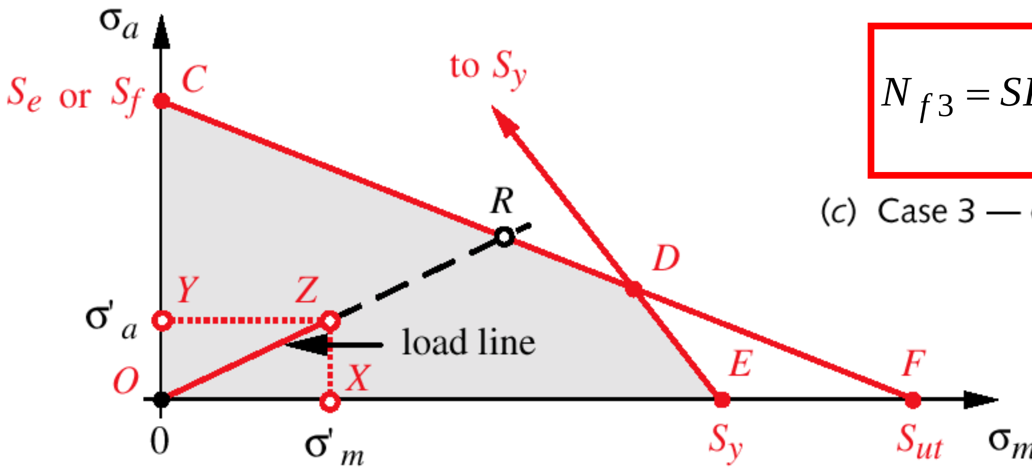


$$N_{f2} = SF = \frac{\overline{XP}}{\overline{XZ}} = \frac{S_f}{\sigma'_a}\left(1 - \frac{\sigma'_m}{S_{ut}}\right)$$

(*b*) Case 2 — σ_a varies and σ_m constant

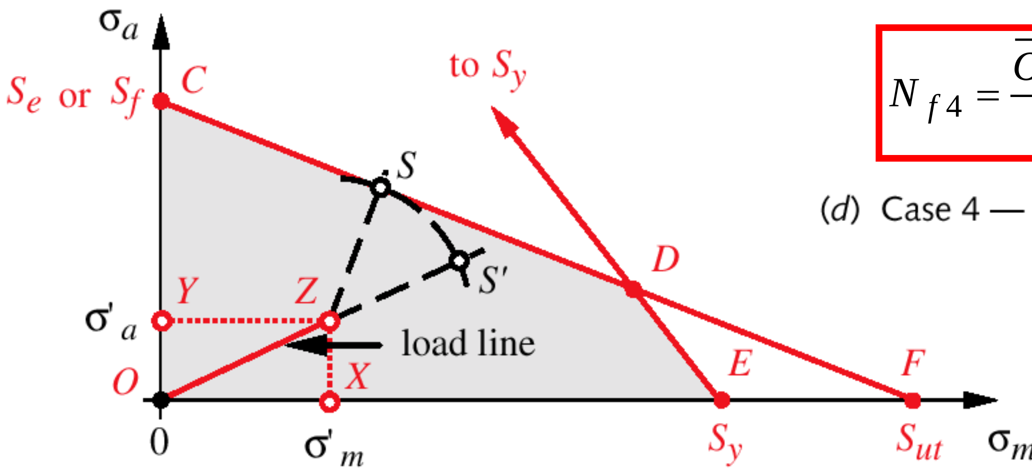
Fatigue failure – Modified Goodman's diagram

Safety factors in fluctuating stresses: Cases 3 and 4



$$N_{f3} = SF = \frac{\overline{OR}}{\overline{OZ}} = \frac{S_f\, S_{ut}}{\sigma'_a S_{ut} + \sigma'_m S_f}$$

(*c*) Case 3 — σ_a / σ_m ratio constant



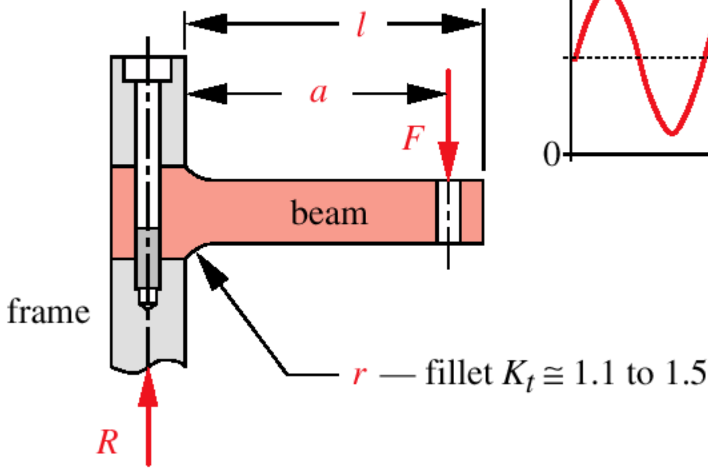
$$N_{f4} = \frac{\overline{OZ} + \overline{ZS}}{\overline{OZ}}$$

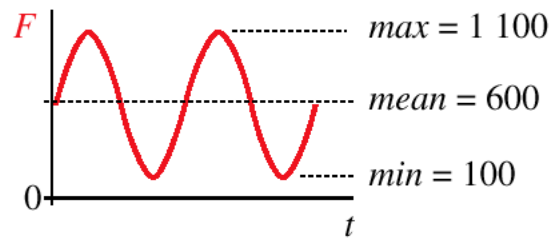
(*d*) Case 4 — σ_a and σ_m vary independently

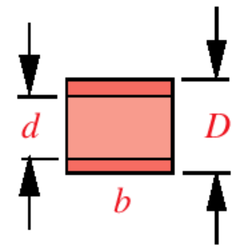
Fatigue failure

Designing for HCF. Review examples: fluctuating load

- **Examples 6-5:** fatigue under fluctuating bending. *Design a cantilever bracket to support a fluctuating bending load of 100 to 1100 lb amplitude for 10^9 cycles with no failure. Its dynamic deflection cannot exceed 0.02 in.*







$b = 2.0\,in$

$d = 1.0\,in$

$D = 1.125\,in$

$r = 0.5\,in$

$a = 5.0\,in$

$l = 6.0\,in$

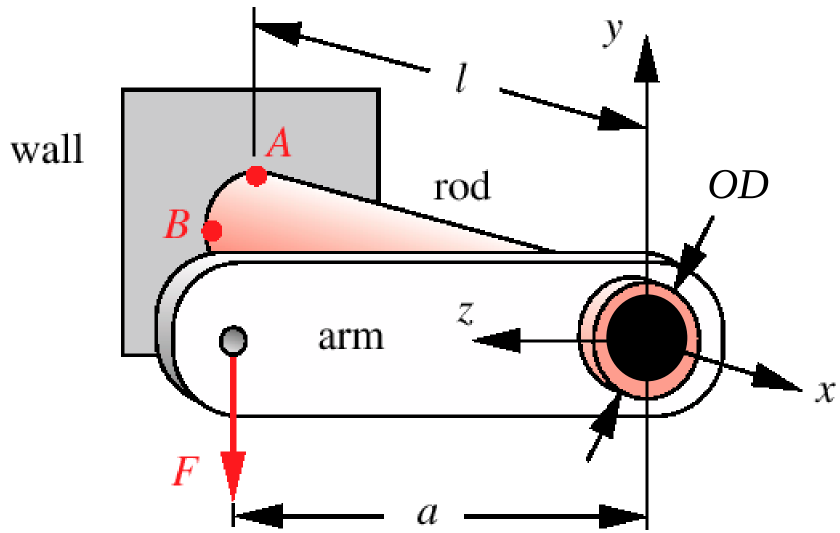
- *Fluctuating load*
- *Life of about 10^9 cycles*
- *Material: SAE-1040 normalized/machined*
- *Operating conditions: air at max. temperature of 120 °F*

- **Comment:** *use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)*

Fatigue failure

Review examples

☞ **Example 6-6: multiaxial fluctuating stresses**



Notch radius (wall) is 0.25",
K_t=1.70, K_{ts}=1.35

- ⇨ *Applied load: sinusoidal [–200,340] lb*
- ⇨ *<u>Finite life</u> of about 6x10^7 cycles*
- ⇨ *Material: Al 2024-T4*
- ⇨ *Operating conditions: room temp.*

Initial dimensions:

$ID = 1.5\,in$

$OD = 2\,in$

$a = 8.0\,in$

$l = 6.0\,in$

☞ **Comment:** *use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)*

Fatigue failure

Review examples

☞ **Example 6-6:** **Multiaxial Fluctuating Stresses**

Problem — Determine the safety factors for the bracket tube shown in Figure 5-7.

Units — $ksi := 10^3 \cdot psi$

Given — The material is 2024-T4 aluminum

Yield strength	$S_y := 47 \cdot ksi$
Tensile strength	$S_{ut} := 68 \cdot ksi$
Tube length	$l := 6 \cdot in$
Arm length	$a := 8 \cdot in$
Tube OD	$od := 2.0 \cdot in$
Tube ID	$id := 1.5 \cdot in$
Load	$F_{min} := -200 \cdot lbf$ $\quad F_{max} := 340 \cdot lbf$

Assumptions — The load is dynamic and the assembly is at room temperature. Consider shear due to transverse loading as well as other stresses.

A finite life design will be sought with a life of $N := 6 \cdot 10^7$ cycles.

The notch radius at the wall is $r := 0.25 \cdot in$ and stress-concentration factors are for bending $K_t := 1.7$, and for shear, $K_{ts} := 1.35$.

Solution — See Figure 5-7 and Mathcad file EX06-06. Also see Example 4-9 for a more complete explanation of the stress analysis for this problem.

1 Aluminum does not have an endurance limit. Its endurance strength at 5E8 cycles can be estimated from equation 6.5c. Since the S_{ut} is larger than 48 ksi, the uncorrected $S'_{f@5E8}$ is

$$S'_{f5E8} := 19 \cdot ksi$$

fatigue → 2 The correction factors are calculated from equations 6.7 and used to find a corrected ~~endurance~~ strength at the standard 5E8 cycles.

$C_{load} := 1.0$ for bending

Note: hollow tube evaluation →

$A_{95} := 0.0105 \cdot od^2$ $A_{95} = 0.042 in^2$

$d_{eq} := \sqrt{\dfrac{A_{95}}{0.0766}}$ $d_{eq} = 0.740 in$

← Make sure to know how to evaluate A_{95}

$C_{size} := 0.869 \cdot \left(\dfrac{d_{eq}}{in}\right)^{-0.097}$ $C_{size} = 0.895$

Table 6-3 constants $A := 2.7$ $b := -0.265$ ← Note "negative" exponent

S_{ut} is used in kpsi →

$C_{surf} := A \cdot \left(\dfrac{S_{ut}}{ksi}\right)^{b}$ $C_{surf} = 0.883$ (a)

$C_{temp} := 1$

$C_{reliab} := 0.753$ for 99.9%

$S_{f5E8} := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_{f5E8}$ (b)

Note that this is only 16.6% of the S_{ut} (and 24 % of the S_y) →

$S_{f5E8} = 11.30 ksi$

Note that the bending value of C_{load} is used despite the fact that there is both bending and torsion present. The torsional shear stress will be converted to an equivalent tensile stress with the von Mises calculation. C_{surf} is calculated from equation 6.7*e* using data from Table 6-3. This corrected fatigue strength is still at the tested number of cycles, $N = 5E8$.

3 This problem calls for a life of 6E7 cycles, so a strength value at that life must be estimated from the S-N line of Figure 6-33b using the corrected fatigue strength at that life. Equation 6.10a for this line can be solved for the desired strength after we compute the values of its coefficients a and b from equation 6.10c.

$$S_m := 0.90 \cdot S_{ut} \qquad S_m = 61.2\,ksi$$

From Table 6-5 for 5E8 $\qquad z := 5.699$

This is calculated differently for materials with an S_e →

$$b := -\frac{1}{z} \cdot \log\left(\frac{S_m}{S_{f5E8}}\right) \qquad b = -0.1288 \qquad (c)$$

$$a := \frac{S_m}{10^{3 \cdot b}} \qquad a = 148.9\,ksi$$

$$S_n := a \cdot N^b \qquad S_n = 14.84\,ksi$$

Note that S_m is calculated as 90% of S_{ut} because loading is bending rather than axial (see Eq. 6.9). The value of z is taken from Table 6-5 for N = 5E8 cycles. This is a corrected fatigue strength for the shorter life required in this case and so is larger than the corrected test value, which was calculated at a longer life.

4 The notch sensitivity of the material must be found to calculate the fatigue stress-concentration factors. Table 6-8 shows the Neuber factors for hardened aluminum. Interpolation gives a value of $a := 0.147^2 \cdot in$ at the material's S_{ut}. Equation 6.13 gives the resulting notch sensitivity for the assumed notch radius.

$$q := \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \qquad q = 0.773 \qquad (d)$$

5 The fatigue stress-concentration factors are found from equation 6.11b using the given geometric stress-concentration factors for bending and torsion, respectively.

K_t and K_{ts} are given (we are lucky!)

$$K_f := 1 + q \cdot (K_t - 1) \qquad K_f = 1.541 \qquad (e)$$

$$K_{fs} := 1 + q \cdot (K_{ts} - 1) \qquad K_{fs} = 1.270 \qquad (f)$$

6 The bracket tube is loaded in both bending (as a cantilever beam) and in torsion. The shapes of the shear, moment and torque distributions are shown in Figure 4-30. All are maximum at the wall. The alternating and mean components of the applied force, moment, and torque at the walls are

Forces: *evaluated for amplitude and mean components*

Loads $F_a := \frac{F_{max} - F_{min}}{2}$ $F_a = 270\,lbf$

$F_m := \frac{F_{max} + F_{min}}{2}$ $F_m = 70\,lbf$ (g)

Moments: *evaluated for amplitude, mean, and maximum components*

Moments $M_a := F_a \cdot l$ $M_a = 1620\,lbf \cdot in$

$M_m := F_m \cdot l$ $M_m = 420\,lbf \cdot in$ (h)

$M_{max} := M_a + M_m$ $M_{max} = 2040\,lbf \cdot in$

Torques $a := 8.0 \cdot in$

$T_a := F_a \cdot a$ $T_a = 2160\,lbf \cdot in$ (i)

$T_m := F_m \cdot a$ $T_m = 560\,lbf \cdot in$

Torques: *evaluated for amplitude and mean components*

7 The fatigue stress-concentration factor for the mean stresses depends on the relationship between the maximum local stress in the notch and the yield strength as defined in equation 6.17, a portion of which is shown here.

Outer fiber $c := 0.5 \cdot od$ $c = 1.000\,in$

Moment of inertia $I := \frac{\pi}{64} \cdot \left(od^4 - id^4\right)$ $I = 0.5369\,in^4$

$J := 2 \cdot I$ $J = 1.0738\,in^4$

If $K_f \cdot \left|\sigma_{max}\right| < S_y$ then $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

$$K_f \cdot \left|\frac{M_{max} \cdot c}{I}\right| = 5.86\,ksi$$

which is less than $S_y = 47\,ksi$ so, $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$ (j)

Compensate for local "yield," if any

In this case, there is no reduction in stress-concentration factors for the mean stress because there is no yielding at the notch to relieve the stress concentration.

8 The largest tensile bending stress will be in the top or bottom outer fiber at points A or A'. The largest torsional shear stress will be all around the outer circumference of the tube. (See Example 4-9 for more details.) First take a differential element at point A or A' where both of these sresses combine. (See Figure 4-32.) Find the alternating and mean components of the normal bending stress and of the torsional shear stress on point A using equations 4.11b and 4.24b, respectively.

Evaluate applied, amplitude and mean, stresses – **point A** is subjected to bending and shear

$$\sigma_a := K_f \cdot \frac{M_a \cdot c}{I} \qquad \sigma_a = 4.65\,ksi$$

$$\tau_a := K_{fs} \cdot \frac{T_a \cdot c}{J} \qquad \tau_a = 2.56\,ksi \qquad (k)$$

$$\sigma_m := K_{fm} \cdot \frac{M_m \cdot c}{I} \qquad \sigma_m = 1.21\,ksi$$

$$\tau_m := K_{fsm} \cdot \frac{T_m \cdot c}{J} \qquad \tau_m = 0.66\,ksi \qquad (l)$$

9 Find the alternating and mean von Mises effective stresses at point A from equation 6.22b.

Evaluate equivalent Mises, amplitude and mean, stresses

$$\sigma_{xa} := \sigma_a \qquad \sigma_{ya} := 0 \cdot psi \qquad \tau_{xya} := \tau_a$$

$$\sigma'_a := \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xya}^2} \qquad \sigma'_a = 6.42\,ksi$$

$$\sigma_{xm} := \sigma_m \qquad \sigma_{ym} := 0 \cdot psi \qquad \tau_{xym} := \tau_m \qquad (m)$$

$$\sigma'_m := \sqrt{\sigma_{xm}^2 + \sigma_{ym}^2 - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^2} \qquad \sigma'_m = 1.66\,ksi$$

10 Because the moment and torque are both caused by the same applied force, they are synchronous and in-phase and any change in them will be in a constant ratio.

This is a Case 3 situation and the safety factor is found using equation 6.18e. ← Note the use of S_n in this equation (finite life)

Evaluate for all cases, if unsure about which case →

$$N_f := \frac{S_n \cdot S_{ut}}{\sigma'_a \cdot S_{ut} + \sigma'_m \cdot S_n} \qquad N_f = 2.2$$

At point A

11 Since the tube is a short beam, we need to check the shear due to transverse loading at point B on the neutral axis where the torsional shear is also maximal. The maximum transverse shear stress at the neutral axis of a hollow, thin-walled, round tube was given as equation 4.15d.

Cross-section area $A := \frac{\pi}{4}\cdot\left(od^2 - id^2\right)$ $A = 1.374\, in^2$

Account for transversal shear – **point B** is subjected to only shear stresses

$$\tau_{abend} := K_{fs}\cdot\frac{2\cdot F_a}{A} \qquad \tau_{abend} = 499\, psi$$

$$\tau_{mbend} := K_{fsm}\cdot\frac{2\cdot F_m}{A} \qquad \tau_{mbend} = 129\, psi \qquad (o)$$

Point B is in pure shear. The total shear stress at point B is the sum of the transverse shear stress and the torsional shear stress which act on the same planes of the element.

$$\tau_{atotal} := \tau_{abend} + \tau_a \qquad \tau_{atotal} = 3055\, psi$$

$$\tau_{mtotal} := \tau_{mbend} + \tau_m \qquad \tau_{mtotal} = 792\, psi \qquad (p)$$

12 Find the alternating and mean von Mises effective stresses at point B from equation 6.22b.

Evaluate equivalent Mises, amplitude and mean, stresses

$$\sigma_{xa} := 0\cdot psi \qquad \sigma_{ya} := 0\cdot psi \qquad \tau_{xya} := \tau_{atotal}$$

$$\sigma'_a := \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa}\cdot\sigma_{ya} + 3\cdot\tau_{xya}^2} \qquad \sigma'_a = 5.29\, ksi$$

$$\sigma_{xm} := 0\cdot psi \qquad \sigma_{ym} := 0\cdot psi \qquad \tau_{xym} := \tau_{mtotal} \qquad (q)$$

$$\sigma'_m := \sqrt{\sigma_{xm}^2 + \sigma_{ym}^2 - \sigma_{xm}\cdot\sigma_{ym} + 3\cdot\tau_{xym}^2} \qquad \sigma'_m = 1.37\, ksi$$

13 The safety factor for point B is found using equation 6.18e.

Note the use of S_n in this equation (finite life)

Evaluate for all cases, if unsure about which case

$$N_f := \frac{S_n\cdot S_{ut}}{\sigma'_a\cdot S_{ut} + \sigma'_m\cdot S_n} \qquad N_f = 2.7$$

At point B

Both points A and B are safe against fatigue failure.

Reading

- Chapters 6 of textbook: Sections 6.9 to 6.13
- Review notes and text: ES2501, ES2502

Homework assignment

- Author's: 6-4, 6-5, 6-6
- Solve: 6-23(a,b,c), 6-33m, 6-34m, 6-42