WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, C’2016

Lecture 10-11
04 February 2016
Static failure theories

Accepted failure theories that apply to **ductile** materials:

- Total strain energy theory
- Distortion energy theory
- Pure shear-stress theory
- Maximum shear-stress theory
- Maximum normal stress theory (limited application)

Accepted failure theories that apply to **brittle** materials:

- Maximum normal stress theory (even material)
- Maximum normal stress theory (uneven material)
- Coulomb-Mohr theory
- Modified Mohr theory
Static failure theories
Ductile materials

**Safety factors:**

Distortion energy theory:

\[ SF = N = \frac{S_y}{\sigma} \]

Yield strength of the material

von Mises effective stress

(Obtained from)

Distortion energy theory (pure shear):

\[ SF = N = \frac{S_{ys}}{\tau_{max}} \]

Max. shear-stress

Max. shear-stress theory:

\[ SF = N = \frac{S_{ys}}{\tau_{max}} \]

Maximum shear-stress

\[ S_{ys} = 0.577S_y \]

\[ S_{ys} = 0.5S_y \]
Static failure theories

Ductile materials

\[ \sigma_2 \text{ is assumed to be the zero stress} \]

\[ S_{ys} = 0.5 \ S_y \]

**Figure 5-5**
The 2-D Shear-Stress Theory Hexagon Inscribed Within the Distortion-Energy Ellipse
Static failure theories: experimental verifications
Ductile & brittle materials

Figure 5-8
Experimental Data from Tensile Tests Superposed on Three Failure Theories (Reproduced from Fig. 7.11, p. 252, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993)
Static failure theories

Brittle materials

Mohr’s circle: pure tension

FIGURE 2-5
A Tensile Test Specimen of Brittle Cast Iron After Fracture

FIGURE 2-4
Stress-Strain Curve of a Brittle Material
Static failure theories
Brittle materials

Mohr’s circle: pure shear

Pure shear condition
Static failure theories
Brittle materials: even and uneven materials

(a) An even material — $S_{uc} = -S_{ut}$

(b) An uneven material — $|S_{uc}| > |S_{ut}|$

Figure 5-10
Mohr's Circles for Both Compression and Tensile Tests Showing the Failure Envelopes for (a) Even and (b) Uneven Materials
Static failure theories

Brittle materials: *Coulomb-Mohr, modified-Mohr, and normal stress theories*

**Figure 5-11**

Coulomb-Mohr, Modified-Mohr, and Maximum Normal-Stress Theories for Uneven Brittle Materials
Static failure theories: brittle materials

Coulomb-Mohr, modified-Mohr, and normal stress theories

Experimental observations

FIGURE 5-12

Static failure theories: brittle materials

*Modified-Mohr theory: quadrants of interest*

**FIGURE 5-13**

Modified-Mohr Failure Theory for Brittle Material
Static failure theories: brittle materials

Modified-Mohr theory

Safety factor: zone I:

Modified-Mohr theory:

\[ SF = N = \frac{S_{ut}}{\sigma_1} \]

Ultimate strength of the material in tension

Max. principal normal stress

Modified-Mohr theory: applicable inside this area

Graph showing stress states 
- \((S_{ut}, S_{ut})\)
- \((S_{ut}, -S_{ut})\)
**Static failure theories: brittle materials**

**Modified-Mohr theory**

**Safety factor: zone II**

\[
SF = N = \frac{S_{ut}\left|S_{uc}\right|}{\left|S_{uc}\right|\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}
\]

\[\sigma_3\]

\[(S_{ut}, S_{ut})\]

\[(S_{ut}, -S_{ut})\]

\[(0, -S_{uc})\]

**Modified-Mohr theory: applicable inside this area**
Static failure theories: brittle materials

Modified-Mohr theory

Safety factor: zone II

Modified-Mohr theory:

\[ SF = N = \frac{S_{ut} |S_{uc}|}{|S_{uc}| \sigma_1 - S_{ut}(\sigma_1 + \sigma_3)} \]

EC: derive expression for the SF in Zone II

Understand: state of stresses at points A, B, and C.

What do points A', B', and C' represent?

FIGURE 5-13
Modified-Mohr Failure Theory for Brittle Material
Static failure theories: brittle materials

Effective stress: Dowling indexes

(Similar concept as the equivalent von Mises stress in ductile materials)

\[
C_1 = \frac{1}{2} \left[ |\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_1 + \sigma_2) \right]
\]

\[
C_2 = \frac{1}{2} \left[ |\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_2 + \sigma_3) \right]
\]

\[
C_3 = \frac{1}{2} \left[ |\sigma_1 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_1 + \sigma_3) \right]
\]
Static failure theories: brittle materials

*Modified-Mohr theory: effective stress*

**Safety factor:**

\[ SF = N = \frac{S_{ut}}{\tilde{\sigma}} \]

Modified-Mohr theory. Effective stress:

Effective stress. Obtained as:

\[ \tilde{\sigma} = \text{MAX} (\sigma_1, \sigma_2, \sigma_3, C_1, C_2, C_3) \]

and

\[ \tilde{\sigma} = 0 \quad \text{if } \text{MAX} < 0, \text{ use a different approach} \]
Determine the safety factors for the bracket rod shown considering: (a) ductile; and (b) brittle materials.

Ductile case:
Al 2024-T4 (consult Appendix C)
\[ S_y = 47 \text{ kpsi} \]

Brittle case:
Class 50 gray cast iron (consult Appendix C)
\[ S_{ut} = 52.5 \text{ kpsi}, \quad S_{uc} = 164 \text{ kpsi} \]
Reading assignment

- Chapters 5 of textbook: Sections 5.2 to 5.5
- Review notes and text: ES2501, ES2502

Homework assignment

- Author's: 5-2
- Solve: 5-10, 5-12, 5-30, 5-35m