Worcester Polytechnic Institute
Mechanical Engineering Department

Design of Machine Elements
ME-3320, C'2016

Lecture 07-08
25 January 2016
Deflection in beams
Example E1 -- in class

Recall:

\[
\frac{q}{EI} = \frac{d^4 y}{dx^4}
\]
Load function - deflection

\[
\frac{V}{EI} = \frac{d^3 y}{dx^3}
\]
Shear function - deflection

\[
\frac{M}{EI} = \frac{d^2 y}{dx^2}
\]
Moment function - deflection

\[
\theta = \frac{dy}{dx}
\]
Slope - deflection

\[
y = f(x)
\]
Deflection
Deflection in beams
Example E1 (based on Norton’s example 3-2B)

Determine and plot the shear, moment, slope, and deflection functions for the simply supported beam shown:

(a) Simply supported beam with uniformly distributed loading
Deflection in beams

Example E1 -- in class

Shear Diagram (lb)

Moment Diagram (lb-in)

Slope Diagram (rad)

Deflection Diagram (in)
Plotting singularity functions in MathCad
Example E1 (Based on Norton’s example 3-2B)

To generate the shear and moment functions over the length of the beam, equations (b) and (c) must be evaluated for a range of values of x from 0 to L, after substituting the above values of C, C2, R1, and R2 in them. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

Range of x
\[ x := 0 \text{ in, } 0.01 \ldots L \]

Unit step function
\[ S(x, z) := \text{if}(x \geq z, 1, 0) \]

Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

\[
V(x) := R_1 \cdot S(x, 0.5 \text{ in}) \cdot (x - 0)^0 - \frac{w}{h} \cdot S(x, a) \cdot (x - a)^1 + R_2 \cdot S(x, l) \cdot (x - l)^0
\]

\[
M(x) := R_1 \cdot S(x, 0.5 \text{ in}) \cdot (x - 0)^1 - \frac{w}{2} \cdot S(x, a) \cdot (x - a)^2 + R_2 \cdot S(x, l) \cdot (x - l)^1
\]

Plot the shear and moment diagrams.
Deflection in beams

Example E2 (Based on Norton’s example 3-3B)

Determine and plot the shear, moment, slope, and deflection functions for the cantilever beam shown:

(a) Loading Diagram

(b) Cantilever beam with concentrated loading
Deflection in beams
Example E2 -- in class

Recall:

\[
\frac{q}{EI} = \frac{d^4 y}{dx^4}
\]
Load function - deflection

\[
\frac{V}{EI} = \frac{d^3 y}{dx^3}
\]
Shear function - deflection

\[
\frac{M}{EI} = \frac{d^2 y}{dx^2}
\]
Moment function - deflection

\[
\theta = \frac{dy}{dx}
\]
Slope - deflection

\[
y = f(x)
\]
Deflection
Deflection in beams
Example E2 -- in class

Shear Diagram (lb)

Slope Diagram (rad)

Moment Diagram (lb-in)

Deflection Diagram (in)
Plotting singularity functions in MathCad
Example E2 (Based on Norton's example 3-3B)

To generate the shear and moment functions over the length of the beam, equations (b) and (c) must be evaluated for a range of values of x from 0 to l, after substituting the above values of C₁, C₂, R₁, and M₁ in them. For a Mathcad solution, define a step function $S$. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

$$x := 0 \text{ in, 0.01 \ldots l}$$
$$S(x, z) := \text{if}(x \geq z, 1, 0)$$

Write the shear and moment equations in Mathcad form, using the function $S$ as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R₁ \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 - F \cdot S(x, a) \cdot (x - a)^0$$

$$M(x) := -M₁ \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 + R₁ \cdot S(x, 0 \text{ in}) \cdot (x - 0)^1 - F \cdot S(x, a) \cdot (x - a)^1$$

(b) Shear Diagram
(c) Moment Diagram
Deflection in beams: solve in class, if time permits

Determine and plot the shear, moment, slope, and deflection functions for the beams shown:

Example E3
(Based on Norton's Example 4-6)

Example E4
(Statically indeterminate)

(c) Overhung beam with concentrated force and uniformly distributed loading

(can be solved with the method of superposition)
Designing to minimize stress concentrations

Initial design

Improved design

(a) Force flow around a sharp corner

(b) Force flow around a radiused corner

Modifications to reduce stress concentrations at a sharp corner

(a) Stress concentration at a sharp corner

(b) Stress concentration reduced with radius

(c) Stress concentration reduced with groove

(d) Stress concentration reduced with washer
Stress concentration: demo during class lecture

Experimentally obtained fringe patterns using photoelasticity: patterns reveal distribution of internal stresses.

Note locations subjected to stress concentrations.
Stress concentration

**Bending**

Nominal bending stress:

\[ \sigma_{nom} = \frac{Md}{I} \]

\[ \sigma_{max} = K_t \frac{Md}{I} \]

\( K_t \) is the geometric stress concentration factor -- normal stress

**Shear**

Nominal shear stress:

\[ \tau_{nom} \]

\[ \tau_{max} = K_{ts} \tau_{nom} \]

\( K_{ts} \) is the geometric stress concentration factor -- shear stress
Stress concentration factors

Stress concentration at the edge of an elliptical hole in a plate (axial load)

\[ K_t = 1 + 2 \left( \frac{a}{c} \right) \]
Stress concentration factors

Stress concentration in a stepped flat bar subjected to bending

\[ \sigma_{nom} = \frac{Mc}{I} = 6 \frac{M}{hd^2} \]

\[ \sigma_{max} = K_t \sigma_{nom} \]

and:

\[ K_t = A \left( \frac{r}{d} \right)^b \]

where:

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Stress distribution in cross-sections

Example: geometric stress concentration factors

Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points A and B. Assume a stress concentration factor of 2.5 in both bending and torsion.
Reading

- Chapter 9: design case studies
- Chapters 4 of textbook: Sections 4.12 to 4.19
- Review notes and text: ES-2501, ES-2502

Homework assignment

- Author's: 4-9
- Solve: 4-23f, 4-24f, 4-25f, 4-26f