Fully constrained beam with concentrated load: Example E4, Lecture 07-08

(Using numerical values from Page 167)

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Input parameters:

$L := 10$ $a := 5$ $F := 200$ $E := 30 \cdot 10^6$

$I := 0.2$

Reactions:

$$R1 := \frac{3 \cdot F}{L^2} \cdot (L - a)^2 - \frac{2 \cdot F}{L^3} \cdot (L - a)^3 \qquad R1 = 100$$

$R2 := F - R1$ $R2 = 100$

$$M1 := \frac{-F}{L} \cdot (L - a)^2 + \frac{F}{L^2} \cdot (L - a)^3 \qquad M1 = -250$$

$M2 := M1 + R1 \cdot L - F \cdot (L - a)$ $M2 = -250$

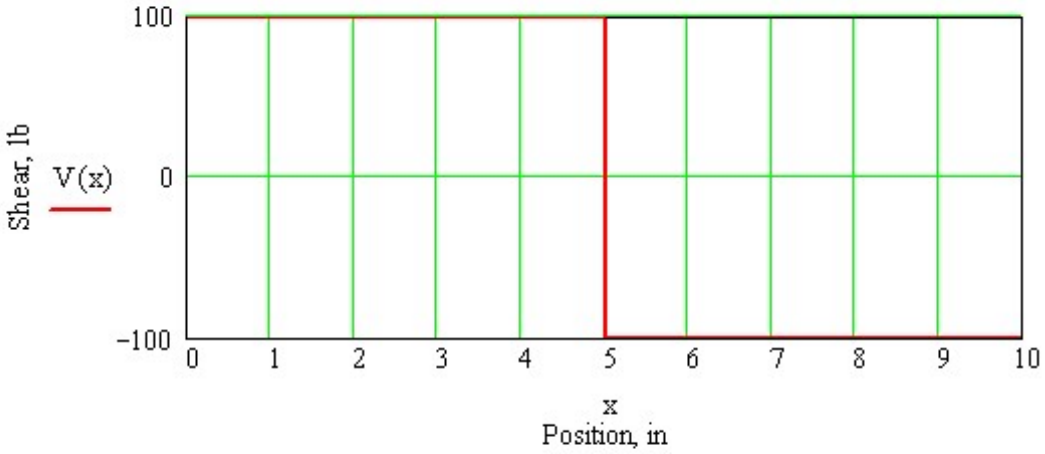
Verifiy equilibrium conditions:

$R1 + R2 - F = 0$ $M1 - M2 + R1 \cdot L - F \cdot (L - a) = 0$

Plot diagrams: $x := 0, 0.001 \cdot L .. L$ $S(x, z) := \text{if}(x \geq z, 1, 0)$

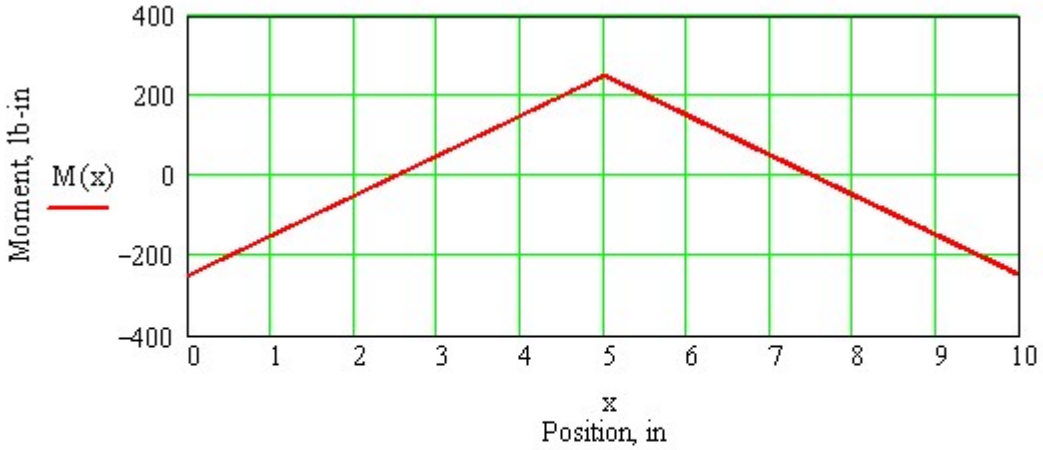
Shear: $V(x) := R1 \cdot S(x, 0) \cdot (x - 0)^0 - F \cdot S(x, a) \cdot (x - a)^0$

Values @ ends: $V(0) = 100$ $V(L) = -100$



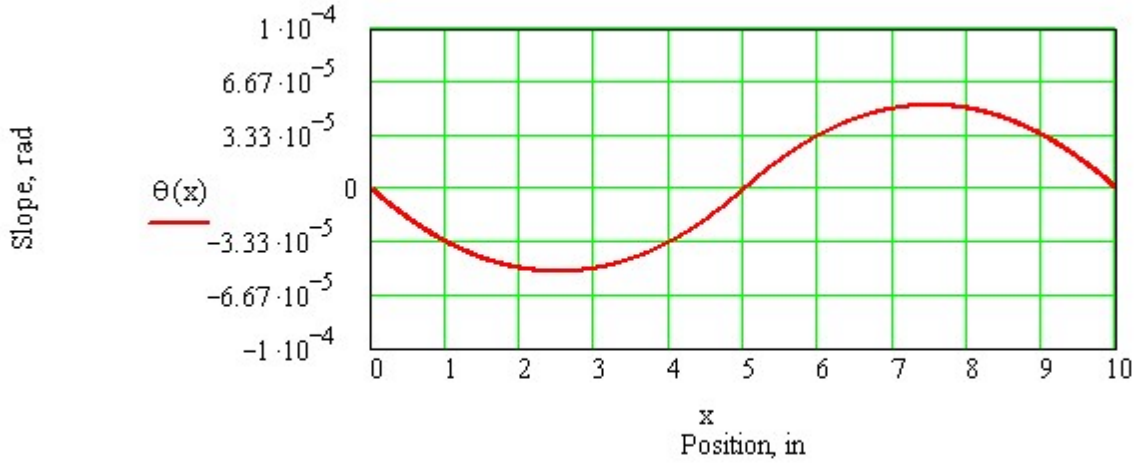
Moment: $M(x) := R1 \cdot S(x, 0) \cdot x + M1 \cdot S(x, 0) - F \cdot S(x, a) \cdot (x - a) + R2 \cdot S(x, L) \cdot (x - L)$

Values @ ends: $M(0) = -250$ $M(L) = -250$



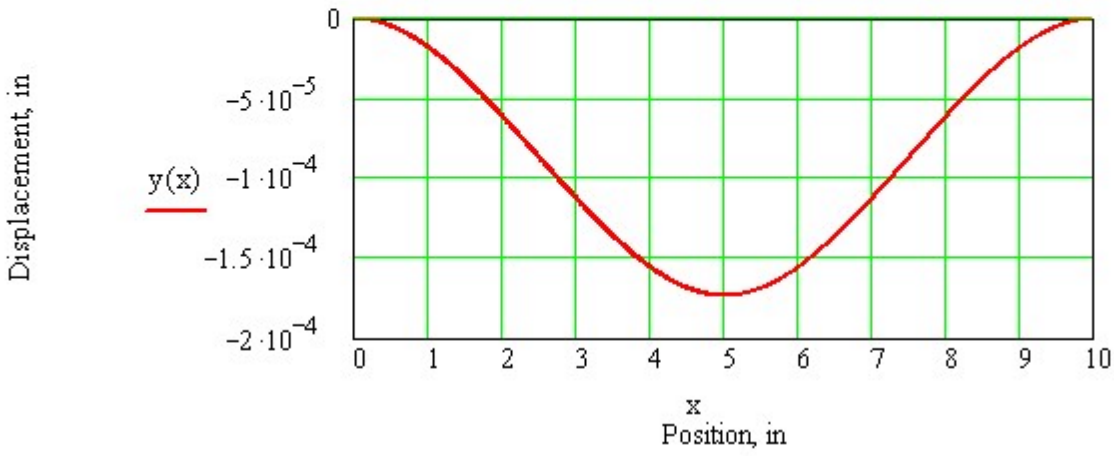
Slope: $$\theta(x) := \frac{1}{E \cdot I}\left[\frac{R1}{2}\cdot S(x,0)\cdot(x-0)^2 + M1\cdot S(x,0)\cdot(x-0) - \frac{F}{2}\cdot S(x,a)\cdot(x-a)^2\right]$$

Values @ ends: $\theta(0) = 0$ $\theta(L) = 0$ $\theta(a) = 0$



Displacement: $$y(x) := \frac{1}{E \cdot I}\left[\frac{R1}{6}\cdot S(x,0)\cdot(x-0)^3 + \frac{M1}{2}\cdot S(x,0)\cdot(x-0)^2 - \frac{F}{6}\cdot S(x,a)\cdot(x-a)^3\right]$$

Values @ ends: $y(0) = 0$ $y(L) = 0$ $y(a) = -1.736\times10^{-4}$



Indexing values of the function to find extreme values for displacement:

Number of points: $N := 1000$

Index: $i := 0, 1 .. N$

$xmin := 0$ $xmax := L$

$$dx := \frac{xmax - xmin}{N}$$

Indexed x-values: $Ax_i := xmin + i \cdot dx$

Indexed y-values: $Ay_i := y(Ax_i)$

Min/max values of y: $miny := min(Ay)$ $miny = -1.736 \times 10^{-4}$

$maxy := max(Ay)$ $maxy = 0$

Iterate to find x-locations of the min & max values of y:

$xmax :=$ | $xmax \leftarrow Ax_0$ | for $i \in 0, 1 .. N$ | $xmax \leftarrow$ | Ax_i if $Ay_i \geq maxy$ | $xmax$ otherwise | $xmax$

$xmin :=$ | $xmin \leftarrow Ax_0$ | for $i \in 0, 1 .. N$ | $xmin \leftarrow$ | Ax_i if $Ay_i \leq miny$ | $xmin$ otherwise | $xmin$

max value at: $xmax = 10$

Verify: $y(xmax) = 0$

Min value at: $xmin = 5$

Verify: $y(xmin) = -1.736 \times 10^{-4}$