

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS ME-3320, A'2011

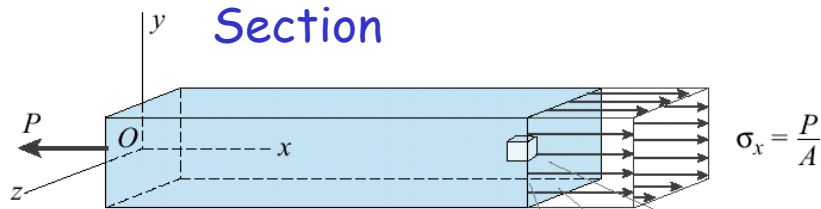
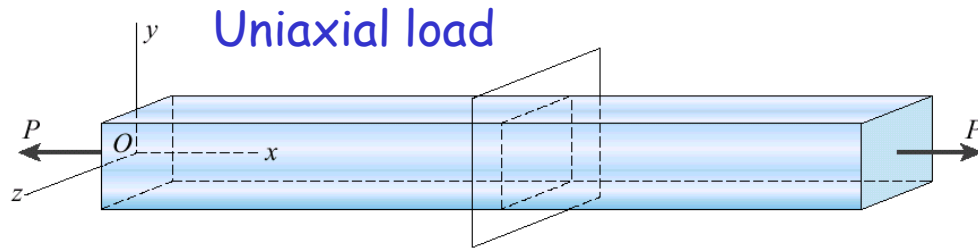
Lecture 06

02 September 2010



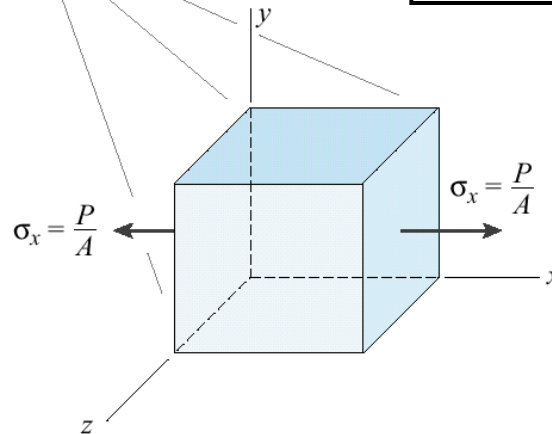
Stress distribution in cross-sections

Uniaxial load



Induced change in length:

$$\Delta s = \frac{P l}{A E}$$

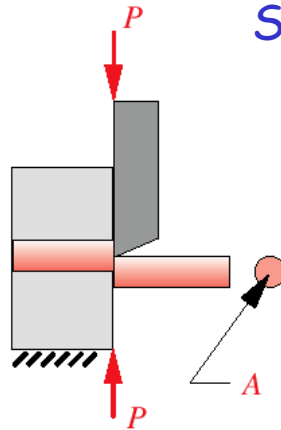


Stress cube
for uniaxial
stress
loading



Stress distribution in cross-sections

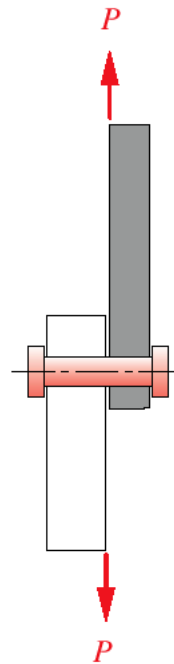
Direct shear



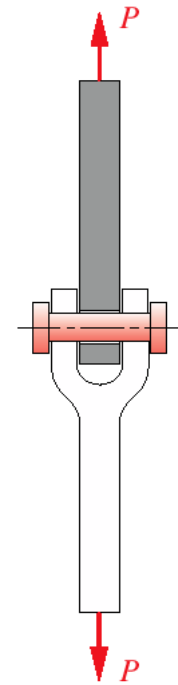
(a) Direct shear

Single shear stress:

$$\tau_{xy} = \frac{P}{A_{shear}}$$



(a) Pivot in single shear



(b) Pivot in double shear

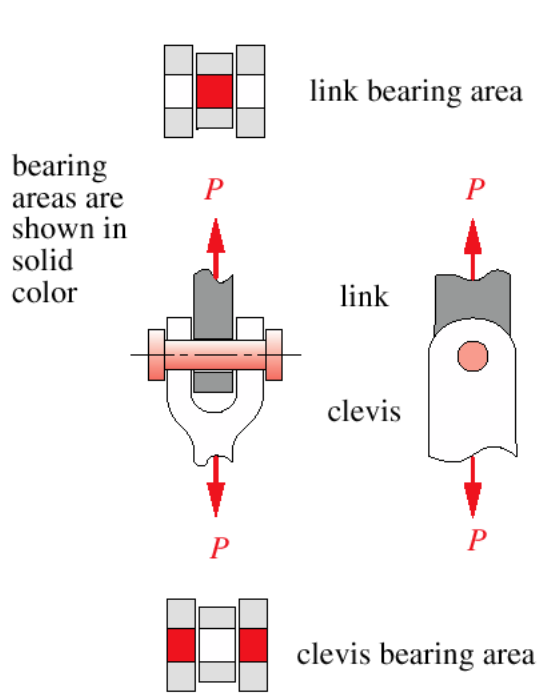
Double shear stress:

$$\tau_{xy} = \frac{P}{2A_{shear}}$$



Stress distribution in cross-sections

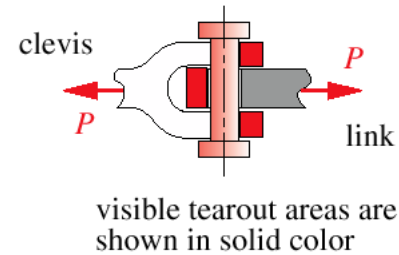
Bearing and tearout



(a) Bearing-stress areas

$$\sigma = -\frac{P}{A_{bearing}}, \quad A_{bearing} = l d$$

(Areas colored in solid red)



(b) Tearout failure

$$\sigma = \frac{P}{A_{tearout}}, \quad A_{tearout}$$

(Areas colored in solid red)



Stress distribution in cross-sections

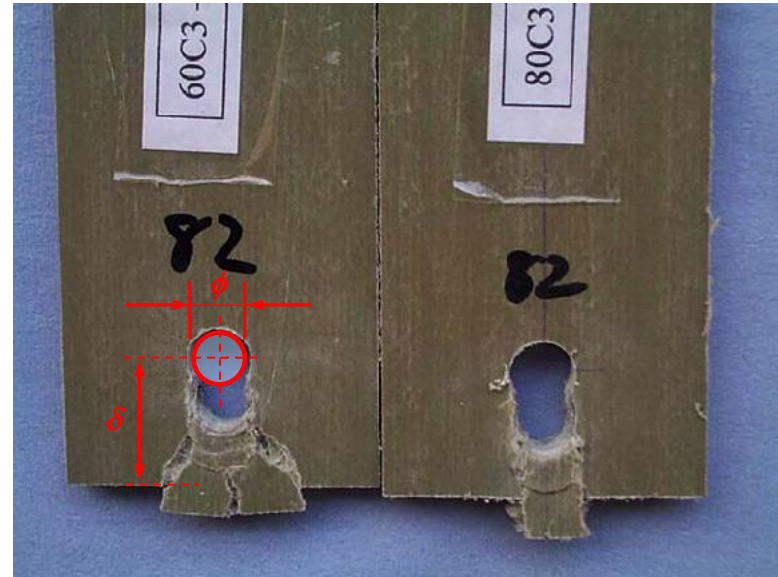
Bearing and tearout: modes of failure

Bearing load



$$\sigma = \frac{P}{A_{bearing}}, \quad A_{bearing} = \phi \cdot T$$

Tearout load



$$\sigma = \frac{P}{A_{tearout}}, \quad A_{tearout} = 2 \cdot \delta \cdot T$$

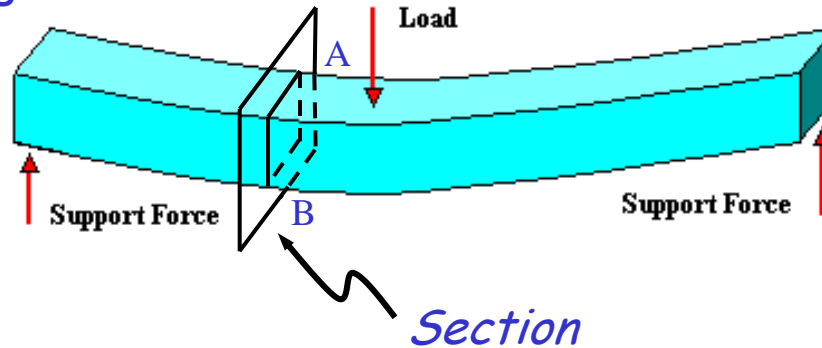
T = Thickness of the plate



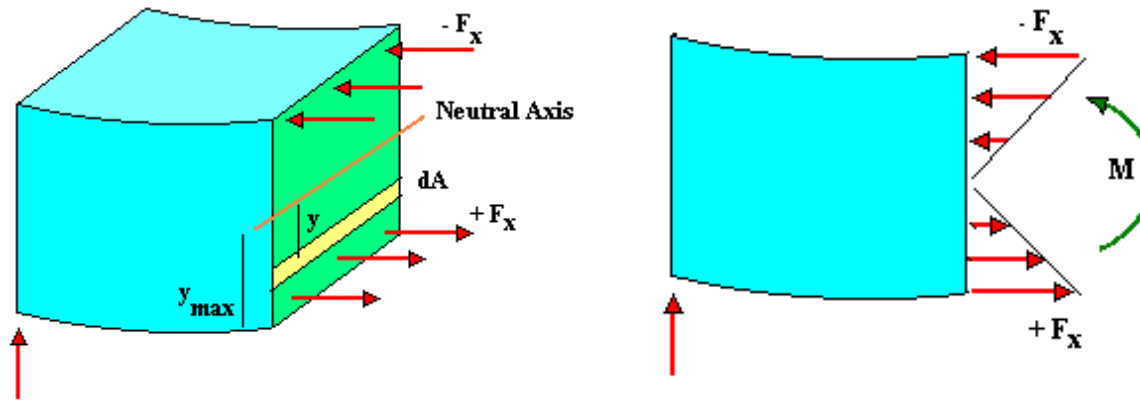
Stress distribution in cross-sections

Bending stress

Bending load



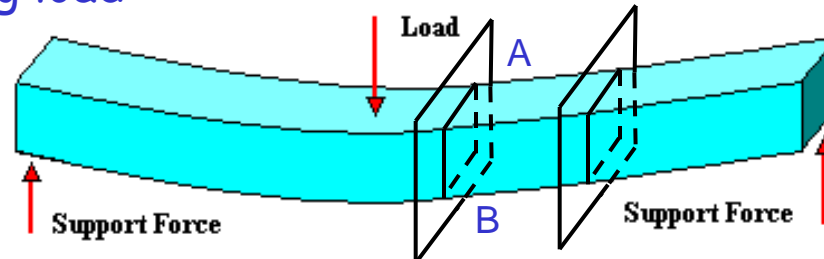
Internal distribution of bending forces



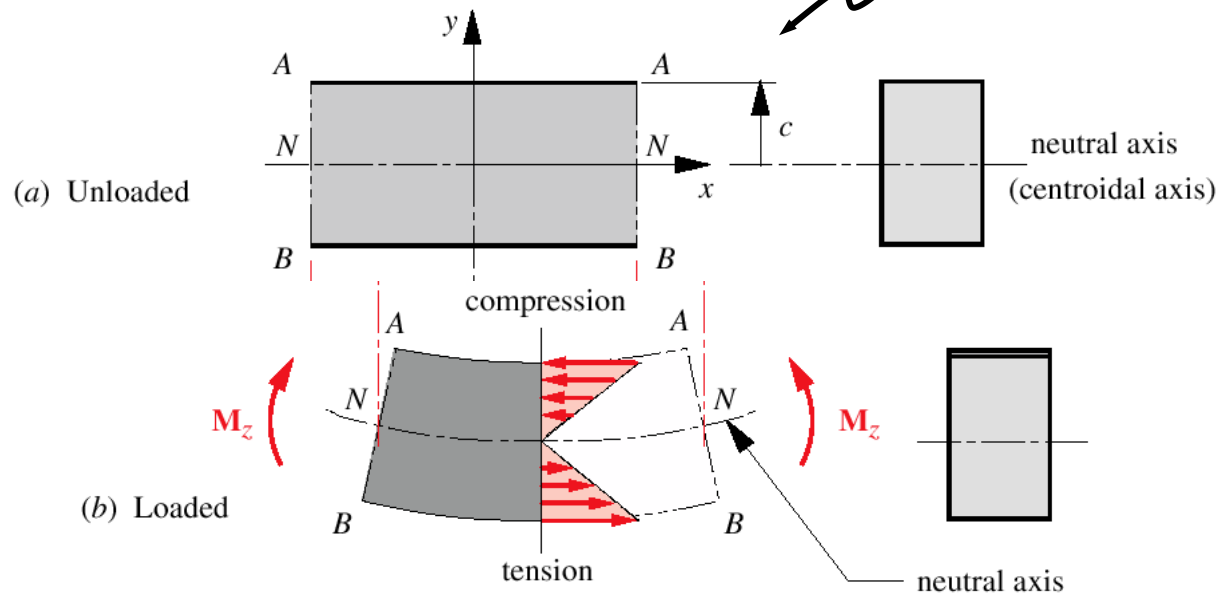
Stress distribution in cross-sections

Bending stress

Bending load:

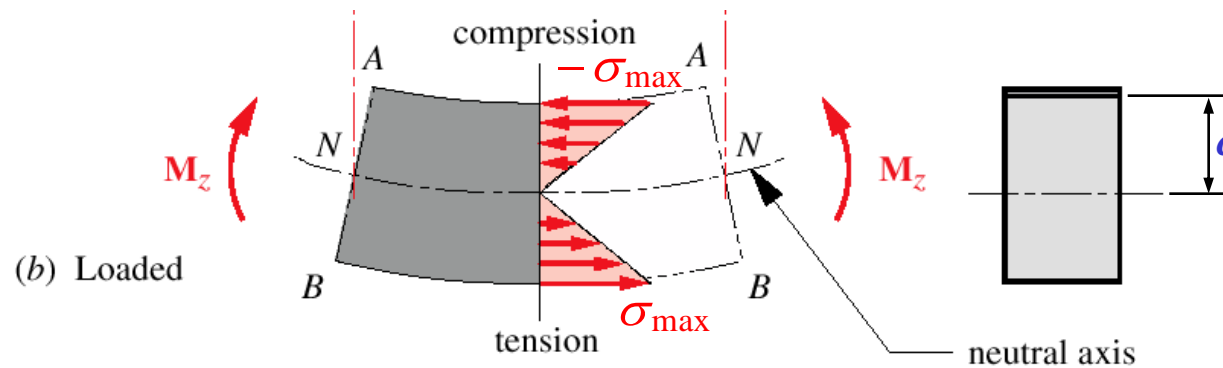


Beam segment



Stress distribution in cross-sections

Bending stresses



Bending stresses:

$$\sigma_x(x, y) = -\frac{My}{I}$$

Recall that $M = M(x)$

Maximum bending stress:

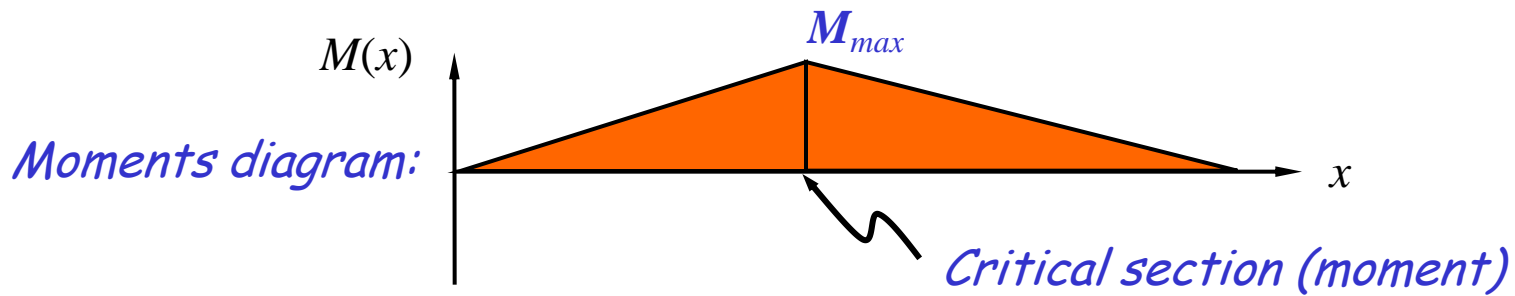
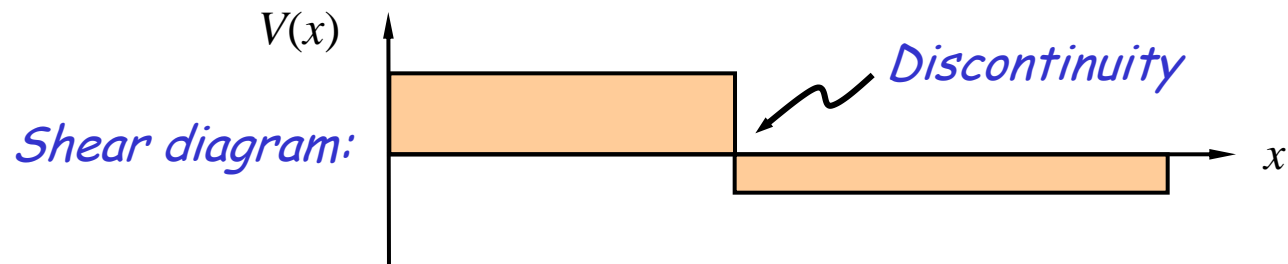
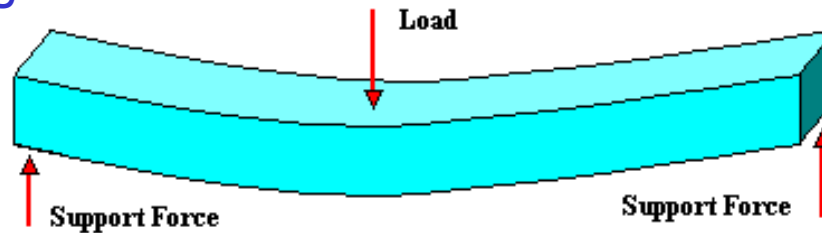
$$\sigma_{\max}(x) = \frac{Mc}{I}$$



Stress distribution in cross-sections

Bending stresses

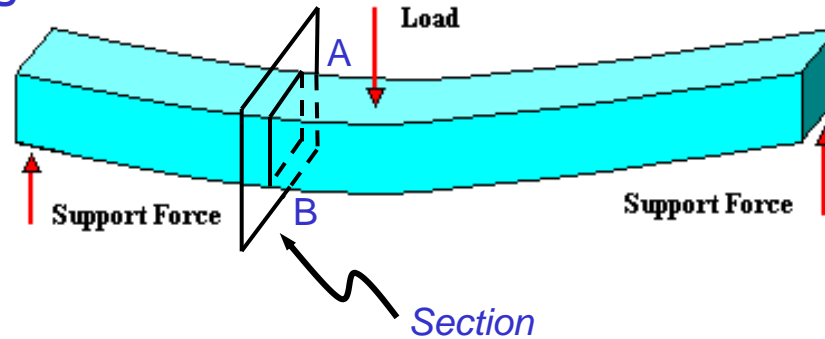
Bending load:



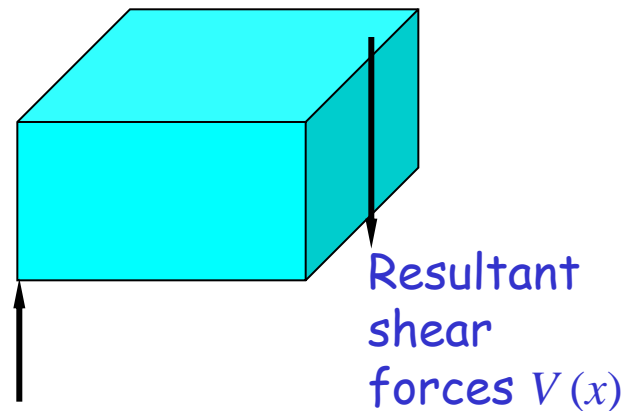
Stress distribution in cross-sections

Transverse shear stress

Bending load:

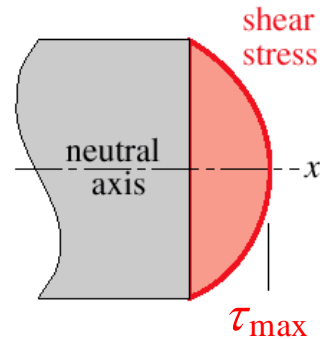


Internal distribution of transversal shear forces



Stress distribution in cross-sections

Transverse shear stress

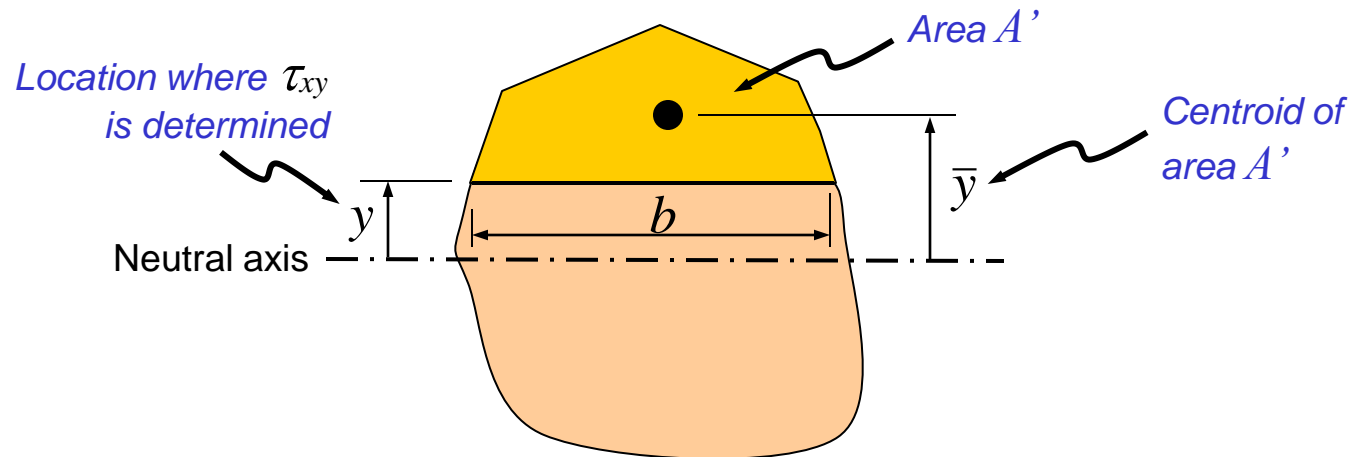


Transverse shear stress:

$$\tau_{xy} = \frac{V Q}{I b} \quad \text{with} \quad Q = \bar{y} A'$$

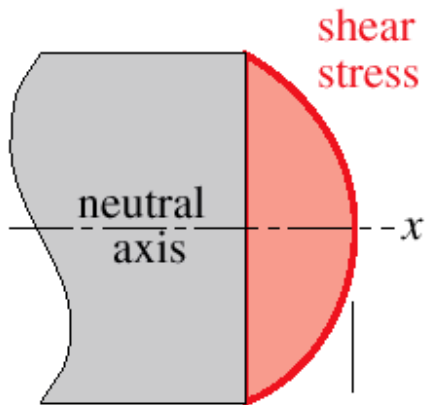
Recall that $V = V(x)$

Generic cross-section:



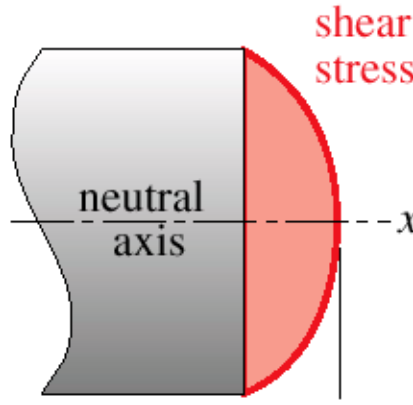
Stress distribution in cross-sections

Transverse shear stress: distribution in common sections



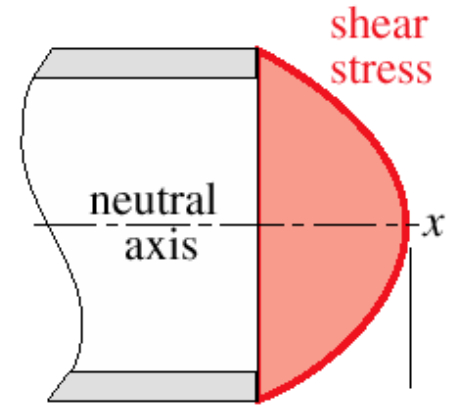
$$\tau_{max} = \frac{3 V}{2 A}$$

(a) rectangular beam



$$\tau_{max} = \frac{4 V}{3 A}$$

(b) solid round beam



$$\tau_{max} \cong \frac{2V}{A}$$

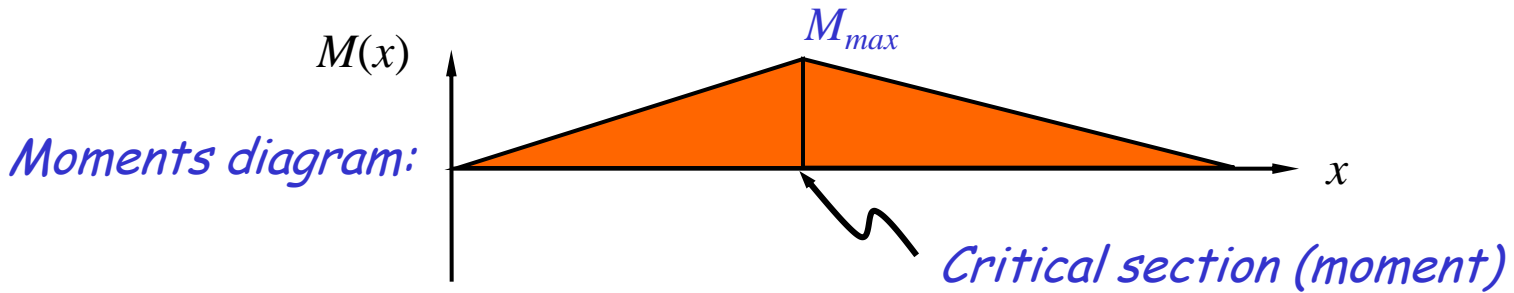
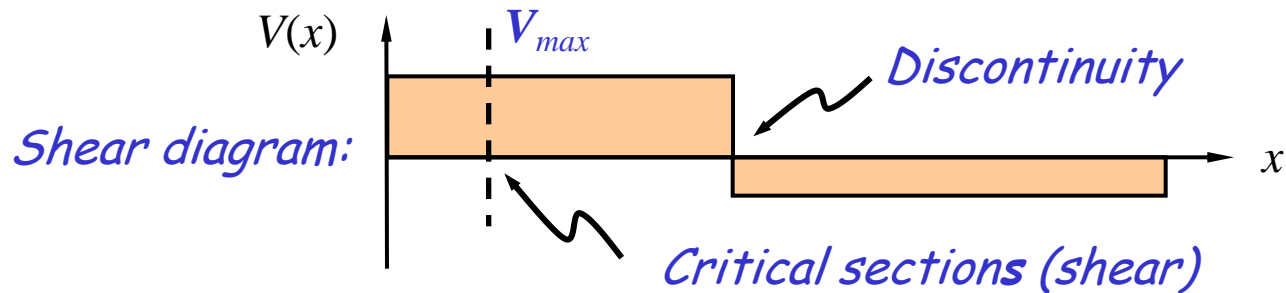
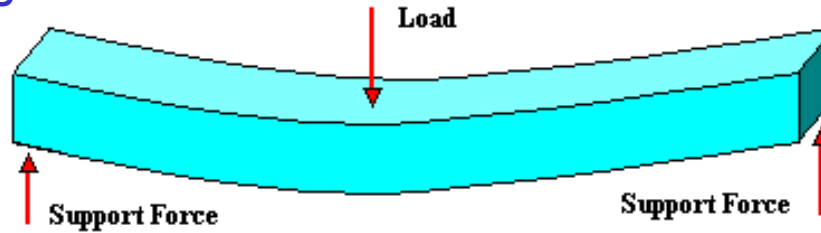
(c) hollow round beam



Stress distribution in cross-sections

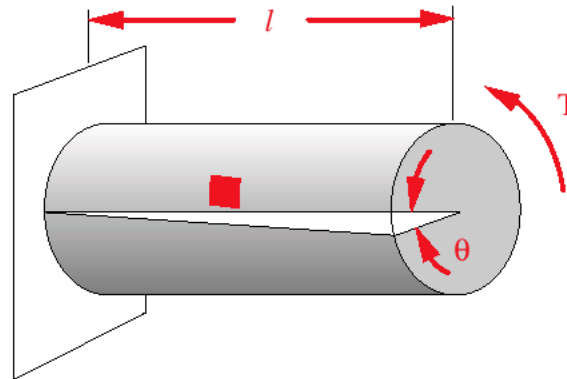
Transverse shear stress

Bending load:



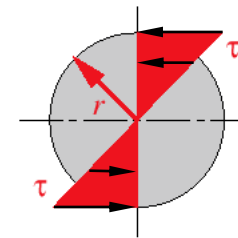
Stress distribution in cross-sections

Torsional stress



(a) Deflection θ

Pure shear stress



(b) Shear-Stress Distribution τ

Shear stress distribution:

$$\tau = \frac{T \rho}{J}, \quad \text{recall that } J = I_x + I_y$$

Angular deflection:

$$\theta = \frac{T l}{K G}, \quad \text{recall that the shear modulus is } G = \frac{E}{2(1+\nu)}$$

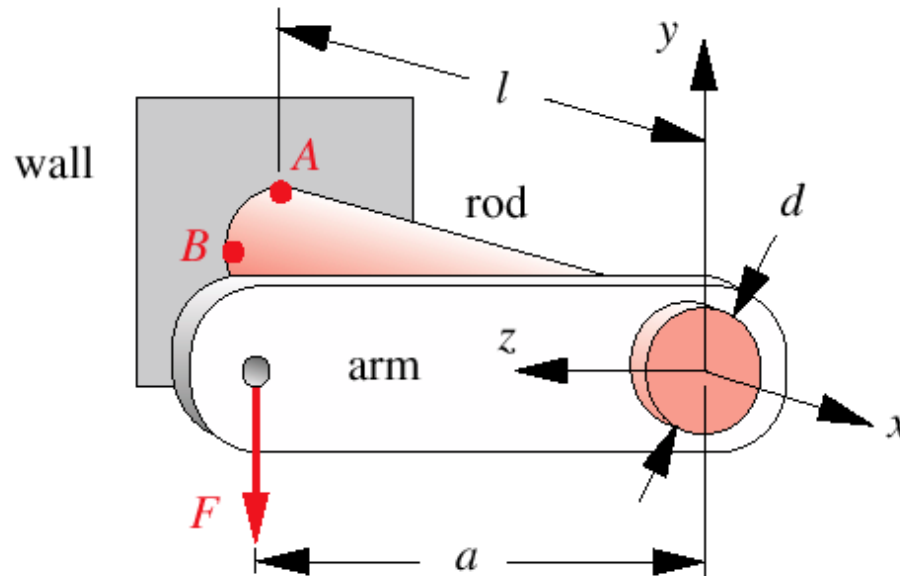
(K is a geometric factor -- see Table 4-3, p.176)



Stress distribution in cross-sections

Examples

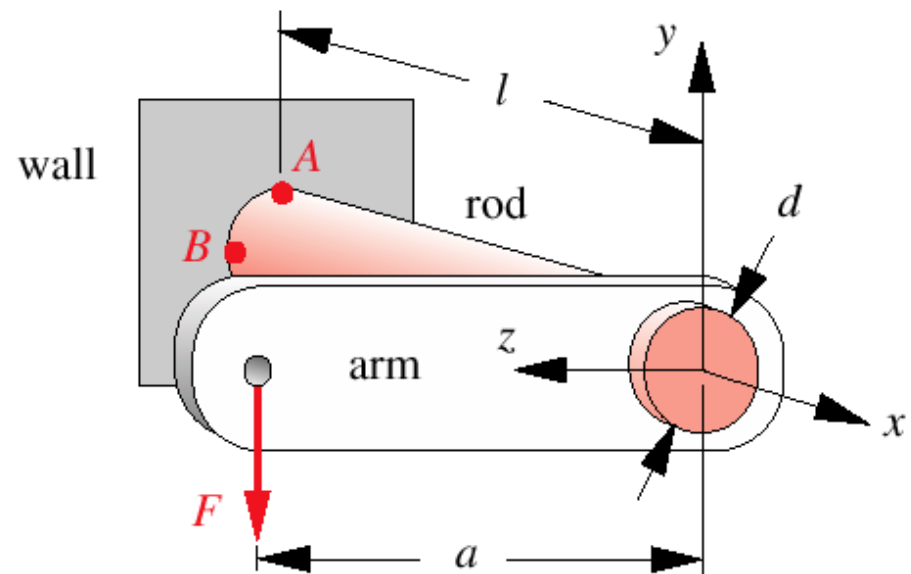
Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points A and B



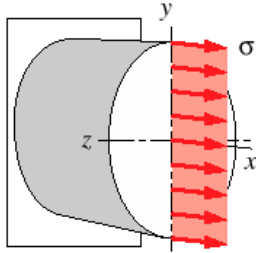
Stress distribution in cross-sections

Examples

Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points *A* and *B*

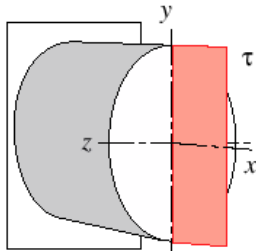


(a) Uniaxial tension, stress distribution across section



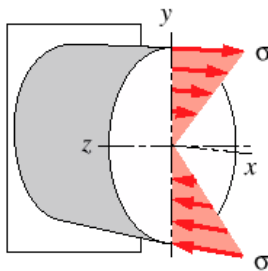
$$\sigma = \frac{P}{A}$$

(b) Direct shear, average-stress distribution across section



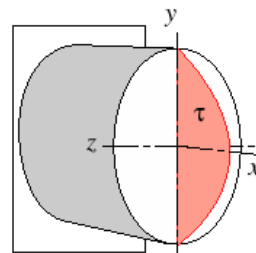
$$\tau = \frac{P}{A_{shear}}$$

(c) Bending, normal-stress distribution across section



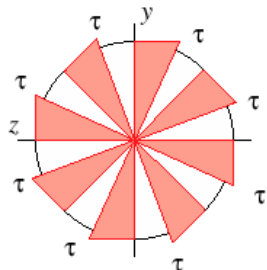
$$\sigma = \frac{My}{I}$$

(d) Bending, shear-stress distribution across section



$$\tau = \frac{VQ}{Ib}$$

(e) Torsion, shear-stress distribution across section



$$\tau = \frac{Tr}{J}$$

Deflection in beams

Example E1 - in class

Recall:

$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$

Load function - deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$

Shear function - deflection

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$

Moment function - deflection

$$\theta = \frac{dy}{dx}$$

Slope - deflection

$$y = f(x)$$

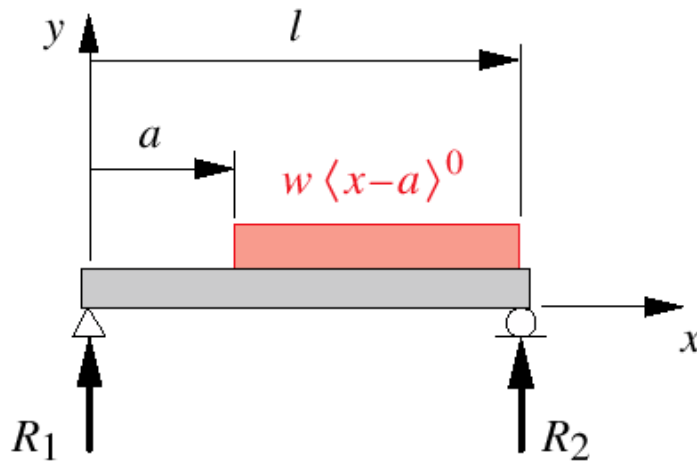
Deflection



Use of singularity functions: slope & deformations

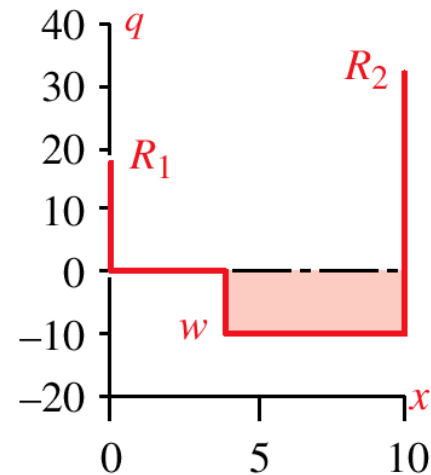
Example E1 (based on Norton's examples 3-2B and 4-4)

Determine: shear, moments, slope, and deflection functions for the simply supported beam shown:



(a) Simply supported beam with uniformly distributed loading

(a) Loading Diagram



Reading assignment

- Chapters 4 of textbook: Sections 4.7 to 4.11
- Review notes and text: ES2501, ES2502

Homework assignment

- Author's: 4-4, 4-6 to 4-9
- Solve: 4-18, 4-22

