Announcements
(also in the website of our course)

✧ Office hours:

C. Furlong, M, Tu, W, and F from 10:00 AM to 11:00 PM
or by appointment (HL-151)

TA: Koohyar Pooladvand: M and W, 11:00 AM to 12:00 PM or
by appointment (HL-146)

✧ Two help sessions (attendance will be recorded):

M at 4:00 PM (HL-230) and Th at 3:00 PM (GH-012)

✧ Design Project: list of members (due this Tuesday)

✧ Exams: please review website for details
Topics for today

• Introduction to MathCAD: step functions

• Shear, moment, torsion diagrams: examples w/singularity functions

• Progress report specifications

• Questions to ask (design project): e.g., geometry of the designs, location of bearings?, distance between bearings?, types of fixtures, materials, height of the tower, load types and boundary conditions, etc.
Internal forces and moments
Shear and bending moments

Internal forces (determination of shear and moment diagrams)

Component

FBD

Sections

Internal: moments, shear, and normal forces at point C
Internal forces and moments
Shear and bending moments

Section at point C

- Shear force
- Normal force
- Bending moment

Bending moment components
Normal force
Shear force components
Torsional moment
Shear and bending-moment diagrams

A suspended bar supports a 600-lb engine. Plot the shear and moment diagrams for the bar.

Method of sections: plot using step functions + MathCad
Shear and bending-moment diagrams

Method of sections: *plot using step functions + MathCad*

ME-3320: more details in Thursday’s help session

A suspended bar supports a 600-lb engine.
Plot the shear and moment diagrams for the bar.

Input:

\[
\begin{align*}
L & := 3 \\
a & := 1.5 \\
x & := 0, 0.001L, L
\end{align*}
\]

Define unit step function:

\[
S(x, z) := \text{if}(x \geq z, 1, 0)
\]

Define shear function:

\[
V(x) = -300 \cdot S(x, 0) + 600 \cdot S(x, 1.5)
\]

Define moments function:

\[
\begin{align*}
M_1(x) & := -300 \cdot x \\
M_2(x) & := 300 \cdot x - 900 \\
M_T(x) & := S(x, 0) \cdot M_1(x) - S(x, 1.5) \cdot M_1(x) + S(x, 1.5) \cdot M_2(x)
\end{align*}
\]
Shear and bending-moment diagrams

Singularity functions:

• Definitions:

  • $n < 0^*$: $f_n(x) \equiv \langle x-a \rangle_n = \begin{cases} \infty & x = a \\ 0 & x \neq a \end{cases}$

  • $n \geq 0$: $f_n(x) \equiv \langle x-a \rangle^n = \begin{cases} (x-a)^n & x \geq a \\ 0 & x < a \end{cases}$

• Integration rules:

  • $n < 0$: $\int_{-\infty}^{x} \langle x-a \rangle_n dx = \langle x-a \rangle_{n+1}$

  • $n \geq 0$: $\int_{-\infty}^{x} \langle x-a \rangle^n dx = \frac{1}{n+1} \langle x-a \rangle^{n+1}$

*Remark: the subscript positioning of $n$ when $n < 0$ is sometimes used to emphasize the fact that the singularity function behaves differently from $n \geq 0$
# Shear and bending-moment diagrams

## Singularity functions

Main singularity functions and their use

<table>
<thead>
<tr>
<th>Singularity function</th>
<th>Graphical representation</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_-(x) = (x-a)_-$</td>
<td><img src="image1.png" alt="Graphical representation" /></td>
<td>$w(x) = -M_0(x-a)_-$</td>
</tr>
<tr>
<td>(couple)</td>
<td><img src="image2.png" alt="Loading" /></td>
<td></td>
</tr>
<tr>
<td>$f_-(x) = (x-a)_+$</td>
<td><img src="image3.png" alt="Graphical representation" /></td>
<td>$w(x) = -W_0'(x-a)_+$</td>
</tr>
<tr>
<td>(concentrated load)</td>
<td><img src="image4.png" alt="Loading" /></td>
<td></td>
</tr>
<tr>
<td>$f_0(x) = (x-a)^0$</td>
<td><img src="image5.png" alt="Graphical representation" /></td>
<td>$w(x) = -w_0(x-a)^0$</td>
</tr>
<tr>
<td>(uniformly distributed load)</td>
<td><img src="image6.png" alt="Loading" /></td>
<td></td>
</tr>
<tr>
<td>$f_1(x) = (x-a)^1$</td>
<td><img src="image7.png" alt="Graphical representation" /></td>
<td>$w(x) = -\frac{w_0}{b-a}(x-a)^1$</td>
</tr>
<tr>
<td>(linearly distributed load)</td>
<td><img src="image8.png" alt="Loading" /></td>
<td></td>
</tr>
<tr>
<td>$f_2(x) = (x-a)^2$</td>
<td><img src="image9.png" alt="Graphical representation" /></td>
<td>$w(x) = -\frac{w_0}{(b-a)^2}(x-a)^2$</td>
</tr>
<tr>
<td>(quadratic distributed load)</td>
<td><img src="image10.png" alt="Loading" /></td>
<td></td>
</tr>
</tbody>
</table>
Shear and bending-moment diagrams

Singularity functions

Loading function:  \( q(x) \)

Shear function:  \( V(x) = \int q(x) \, dx \)

Moment function:  \( M(x) = \int V(x) \, dx \)
Shear and bending-moment diagrams

Singularity functions: in-class examples (loading functions)

(a) Simply supported beam with uniformly distributed loading
Shear and bending-moment diagrams

Singularity functions: in-class examples (loading functions)

(a) Simply supported beam with uniformly distributed loading

(b) Cantilever beam with concentrated loading

(c) Overhanging beam with moment and linearly distributed loading

(d) Statically indeterminate beam with uniformly distributed loading
Shear and bending-moment diagrams

Singularity functions: example E1

Determine and plot the shear and moment functions for the simply supported beam shown:
Shear and bending-moment diagrams

Singularity functions: example E1 - MathCad

To generate the shear and moment functions over the length of the beam, equations (a) and (c) must be evaluated for a range of values of \( x \) from 0 to \( l \), after substituting the above values of \( C_1, C_2, R_1, \) and \( R_2 \) in them. For a Mathcad solution, define a step function \( S \). This function will have a value of zero when \( x \) is less than the dummy variable \( z \), and a value of one when it is greater than or equal to \( z \). It will have the same effect as the singularity function.

Range of \( x \)
\[ x := 0 \text{ in}, 0.01:1..1 \]

Unit step function
\[ S(x, z) = \text{if}(x \geq z, 1, 0) \]

Write the shear and moment equations in Mathcad form, using the function \( S \) as a multiplying factor to get the effect of the singularity functions:

\[
V(x) := R_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 - w \cdot S(x, a) \cdot (x - a)^1 + R_2 \cdot S(x, l) \cdot (x - l)^0
\]

\[
M(x) := R_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^1 - \frac{w}{2} \cdot S(x, a) \cdot (x - a)^2 + R_2 \cdot S(x, l) \cdot (x - l)^1
\]

Plot the shear and moment diagrams.

(b) Shear Diagram

(c) Moment Diagram
Shear and bending-moment diagrams

Singularity functions: example E2

Determine and plot the shear and moment functions for the cantilever beam shown:

(b) Cantilever beam with concentrated loading
Shear and bending-moment diagrams

Singularity functions: example E2 - MathCad

To generate the shear and moment functions over the length of the beam, equations (b) and (c) must be evaluated for a range of values of $x$ from 0 to $l$, after substituting the above values of $C_1$, $C_2$, $R_1$, and $M_1$ in them. For a Mathcad solution, define a step function $S$. This function will have a value of zero when $x$ is less than the dummy variable $z$, and a value of one when it is greater than or equal to $z$. It will have the same effect as the singularity function.

Range of $x$  
\[ x := 0 \text{ in}, 0.01 \cdot l \ldots l \]

Unit step function  
\[ S(x, z) := \text{if}(x \geq z, 1, 0) \]

Write the shear and moment equations in Mathcad form, using the function $S$ as a multiplying factor to get the effect of the singularity functions.

\[ V(x) := R_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 - F \cdot S(x, a) \cdot (x - a)^0 \]

\[ M(x) := -M_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 + R_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^1 - F \cdot S(x, a) \cdot (x - a)^1 \]
Shear and bending-moment diagrams

Singularity functions: example E3

Determine and plot the shear and moment functions for the beam shown:

(c) Overhung beam with moment and linearly distributed loading

(a) Loading Diagram
Shear and bending-moment diagrams

Singularity functions: example E3 - MathCad

To generate the shear and moment functions over the length of the beam, equations (b) and (c) must be evaluated for a range of values of x from 0 to l, after substituting the above values of C1, C2, R1, and R2 in them. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

Range of \( x \)  
\[ x := 0 \text{ in}, 0.005 \text{ in} \ldots l \]

Unit step function  
\[ S(x, z) := \text{if}(x \geq z, 1, 0) \]

Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

\[
V(x) := R_1 \cdot S(x, a) \cdot (x - a)^0 - \frac{w}{2} \cdot S(x, a) \cdot (x - a)^2 + R_2 \cdot S(x, l) \cdot (x - l)^0
\]

\[
M(x) := M_1 \cdot S(x, 0 \text{ in}) \cdot (x - 0)^0 + R_1 \cdot S(x, a) \cdot (x - a)^1 - \frac{w}{6} \cdot S(x, a) \cdot (x - a)^3 \ldots + R_2 \cdot S(x, l) \cdot (x - l)^1
\]
Design project: movie clip + handout

Discussions and tasks to be performed

Source: http://www.mecal.nl
Design project: Schedule
(refer to our entire course outline – in course webpage)

- Thursday, Jan 28. Progress Report #1
  - Background research. Design constraints. FBD’s. Assumptions
- Thursday, Feb 11. Progress Report #2
  - Full static and dynamic analysis
- Thursday, Feb 25. Progress Report #3
  - Full stress analysis. Preliminary designs
- Friday, March 04. Final Report
  - Final proposed designs and detail report
Design project: Report Specifications

WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT
ME-3320, C’16. DESIGN OF MACHINE ELEMENTS
MEMORANDUM

DATE: 19 January 2016
TO: Worcester Precision Instruments, Design Engineers
FROM: Chief Design Engineer
SUBJECT: Progress Report Specifications

A progress report must be clear and "CONCISE". Webster’s dictionary defines concise as expressing much in few words. Your progress report should describe what you have done on the particular task since the last report. The maximum acceptable length for these reports is four (4) pages typed, including figures and/or tables. It should only include additional pages of illustrations if they are germane to the issue and if they are clearly labeled, titled, referred to and discussed in the document. It must not contain appendices. It should not be in a binder, but be stapled in the upper left corner. I strongly recommend that the entire text be 1000-2000 words maximum.

The format should be that of a memorandum (like this document), addressed as follows:

DATE: Date
TO: Chief Engineer
FROM: Your name, or group name and group number
SUBJECT: Progress Report on ‘X’

Body of report, including figures placed near where they are discussed.

Neither salutation nor closing is needed. Note that this report must be typed and spell-checked (which requires a word processor). Any computer graphics should be imported to your word processor file via the clipboard.
Design project
Discussions and tasks to be performed

• Your first report must include: (Consider visiting an actual site)
  o Your design objectives & constrains (specific to your designs).
  o Introduction and Background research on Wind Turbines (in general): e.g., historical perspectives, materials used, typical dimensions, efficiencies, design configurations/specifications, operation, etc.
  o FBD’s (specific to your designs): identification of geometrical configurations, loading conditions (static and dynamic), assumptions.
  o Remember: shear, bending, torsion diagrams (main shaft + tower) are next - leave this task as part of the next report.

Source: http://www.mecal.nl
Location: Danforth, Maine
### Design project

#### Discussions and tasks to be performed

Identify geometrical configurations & loading conditions (specific to your designs.)

Example mechanical configuration  
Source: [http://www.nordex-online.com](http://www.nordex-online.com)

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The Rotor Blades are made of glass fibre reinforced plastic. The rotor is pitch regulated.</td>
</tr>
<tr>
<td>2.</td>
<td>The Hub is made of cast iron.</td>
</tr>
<tr>
<td>3.</td>
<td>The Turbine Frame is made of ductile cast iron. A superior material with regards to strength, vibration and noise reduction features.</td>
</tr>
<tr>
<td>4.</td>
<td>The Rotor Bearing is a solid double spherical roller bearing with a ductile cast iron casing.</td>
</tr>
<tr>
<td>5.</td>
<td>The Rotor Shaft is made of ductile cast iron.</td>
</tr>
<tr>
<td>6.</td>
<td>The Gearbox is a custom design 2-stage planetary gear.</td>
</tr>
<tr>
<td>7.</td>
<td>The Disk Brake is equipped with two brake calipers and located on the high speed shaft of the gearbox.</td>
</tr>
<tr>
<td>8.</td>
<td>The Generator Coupling is a flexible coupling.</td>
</tr>
<tr>
<td>9.</td>
<td>The Generator is a 2600 kW liquid cooled double-fed asynchronous generator.</td>
</tr>
<tr>
<td>10.</td>
<td>The Cooling Radiator is a part of the gearbox cooling system.</td>
</tr>
<tr>
<td>11.</td>
<td>The Fan Coolers for the generator cooling.</td>
</tr>
<tr>
<td>12.</td>
<td>The Wind Measuring System consists of a redundant anemometer and wind vane, which measures the wind conditions and gives signal to the turbine control system.</td>
</tr>
<tr>
<td>13.</td>
<td>The Control System monitors and controls the operation of the wind turbine.</td>
</tr>
<tr>
<td>14.</td>
<td>The Hydraulic System maintains and controls the hydraulic pressure to the disc brakes and the yaw brake system.</td>
</tr>
<tr>
<td>15.</td>
<td>The Yaw Drive consists of 2 planetary yaw gears, driven by frequency controlled electrical motors.</td>
</tr>
<tr>
<td>16.</td>
<td>The Yaw Bearing is a 4-point ball-bearing with outer teething. In addition the turbine is equipped with an active yaw disc brake system.</td>
</tr>
<tr>
<td>17.</td>
<td>The Nacelle Cover is made of glass fibre reinforced plastic on a steel frame.</td>
</tr>
<tr>
<td>18.</td>
<td>The Tower is a tubular steel structure which can be delivered in various heights.</td>
</tr>
<tr>
<td>19.</td>
<td>The Pitch system consists of 3 independent pitch gears, driven by electrical motors.</td>
</tr>
</tbody>
</table>

Example: low-speed shaft configuration  
Source: ZENKUNG Co., China
Design project
Discussions and tasks to be performed

Identify geometrical configurations & loading conditions (specific to your designs.)

Example shaft /generator configuration
Source: Boston Science Museum
Reading assignment

• Chapters 1, 3, and 9 of textbook
• Review notes and text: ES-2501, ES-2502

Homework assignment

• Author’s: 3-5A, 3-5B, 3-6
• Solve: 1-7, 1-8(optional), 3-23(c,h), 3-24(c,h)