WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, A'2016

Lecture 03

29 August 2016

Review on your own

Announcements
(also in the website of our course)

- Office hours:

  Prof. Furlong, Everyday 11:00-11:50 AM
  or by appointment (HL-151)

  TA: N/A

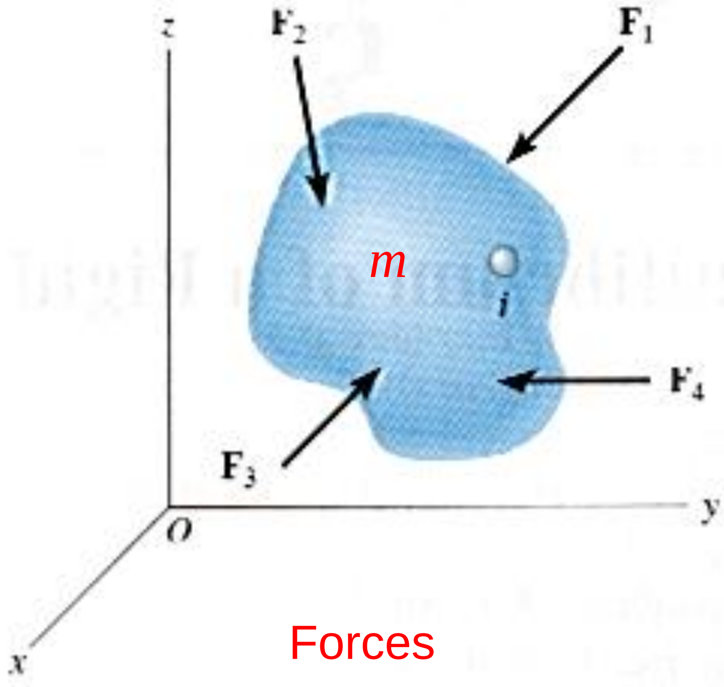
- Help sessions (attendance will be recorded):

  Fridays, 2:00-2:50 PM (HL-230)

- Design Project: list of members (due Today)
- Exams: please review website for details

Force analysis. Free-body diagrams

Static equilibrium: forces



Forces

Static equilibrium condition (vectorial representation) :

$$\sum \mathbf{F} = \mathbf{0}$$

Force analysis. Free-body diagrams

Static equilibrium: forces

Forces acting on a particle or body (vectorial representation):

$$\sum \mathbf{F} = \mathbf{0} = \sum F_x\,\mathbf{i} + \sum F_y\,\mathbf{j} + \sum F_z\,\mathbf{k}$$

Null vector !!

Individual orthogonal components (scalar representation):

$$\sum F_x = 0$$

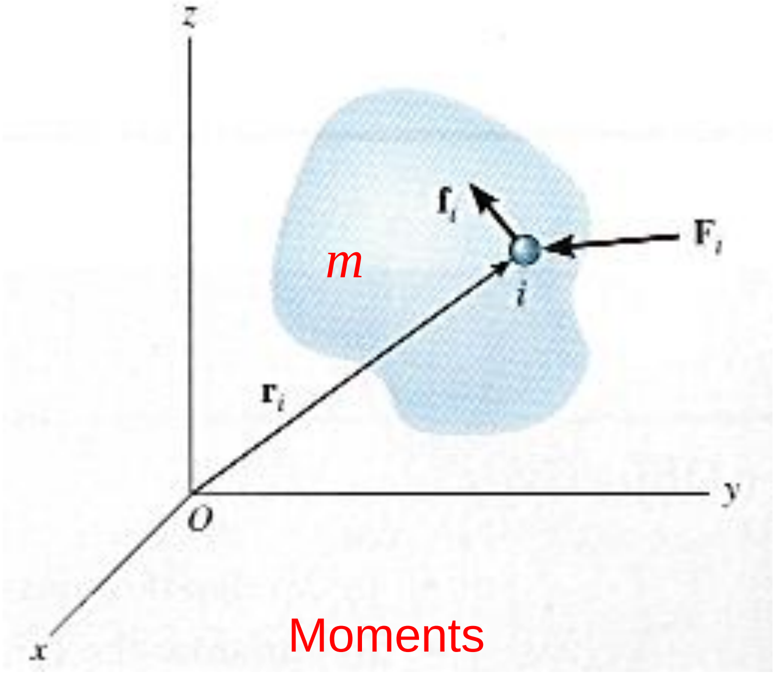
$$\sum F_y = 0$$

$$\sum F_z = 0$$

Null - Scalar !!

Force analysis. Free-body diagrams

Static equilibrium: moments



Static equilibrium condition (vectorial representation):

$$\sum \mathbf{M}_O = \mathbf{0}$$

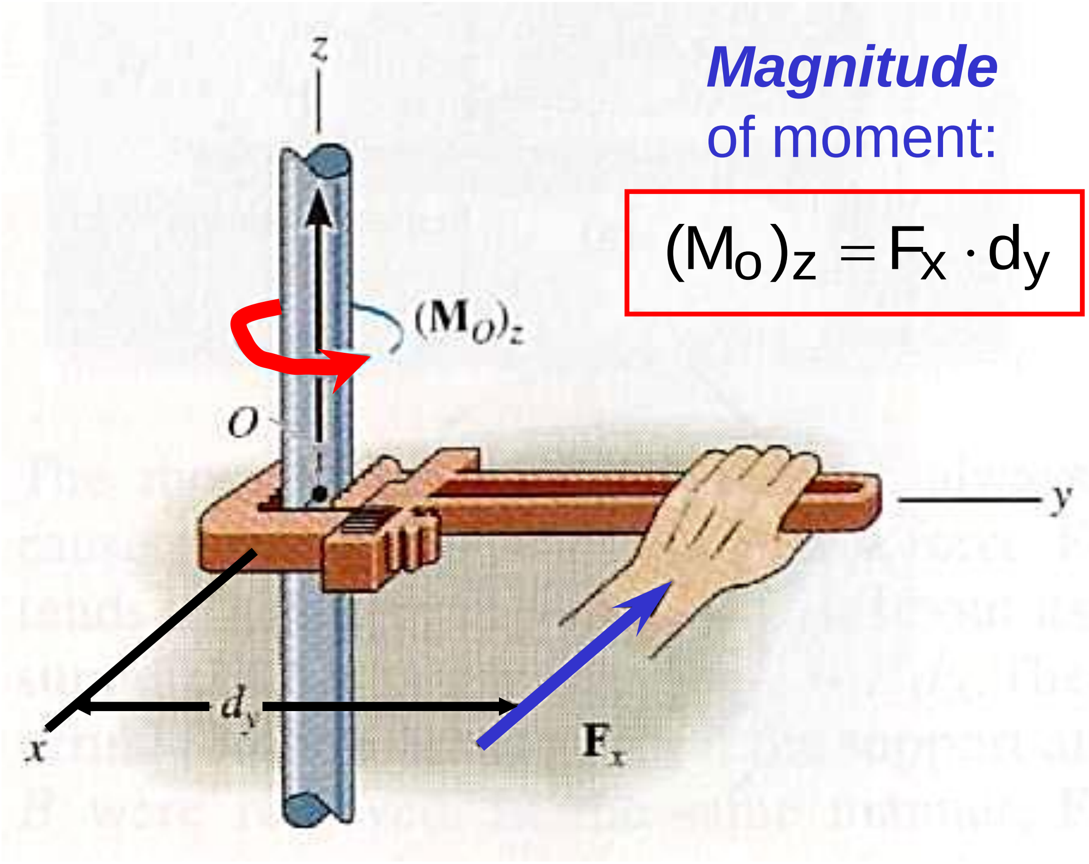
Orthogonal components:

$$\sum M_{x_O}\,\mathbf{i} + \sum M_{y_O}\,\mathbf{j} + \sum M_{z_O}\,\mathbf{k} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

Vectorial evaluation:

$$\sum \mathbf{r}_i \times \mathbf{f}_i + \sum \mathbf{r}_i \times \mathbf{F}_i = \mathbf{0}$$

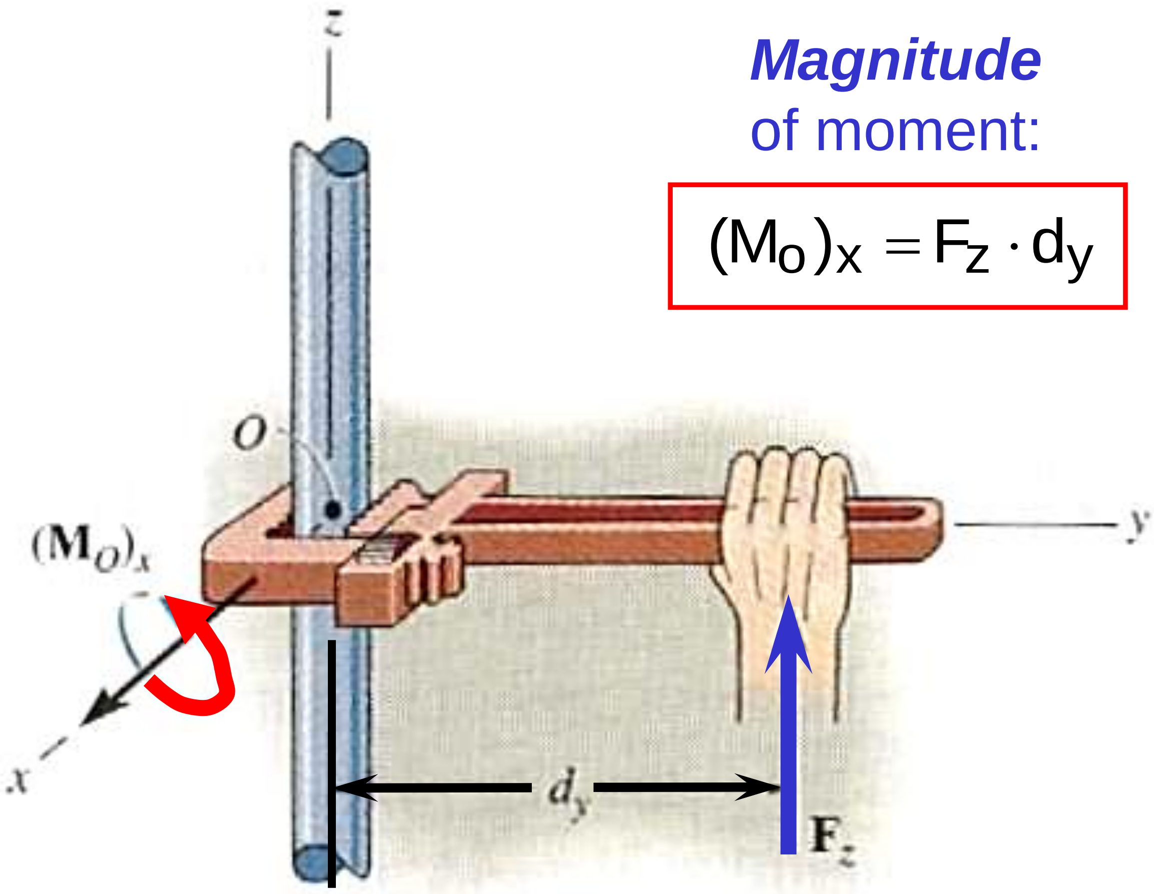
Moment of a force: about z-axis



Magnitude of moment:

$$(M_O)_z = F_x \cdot d_y$$

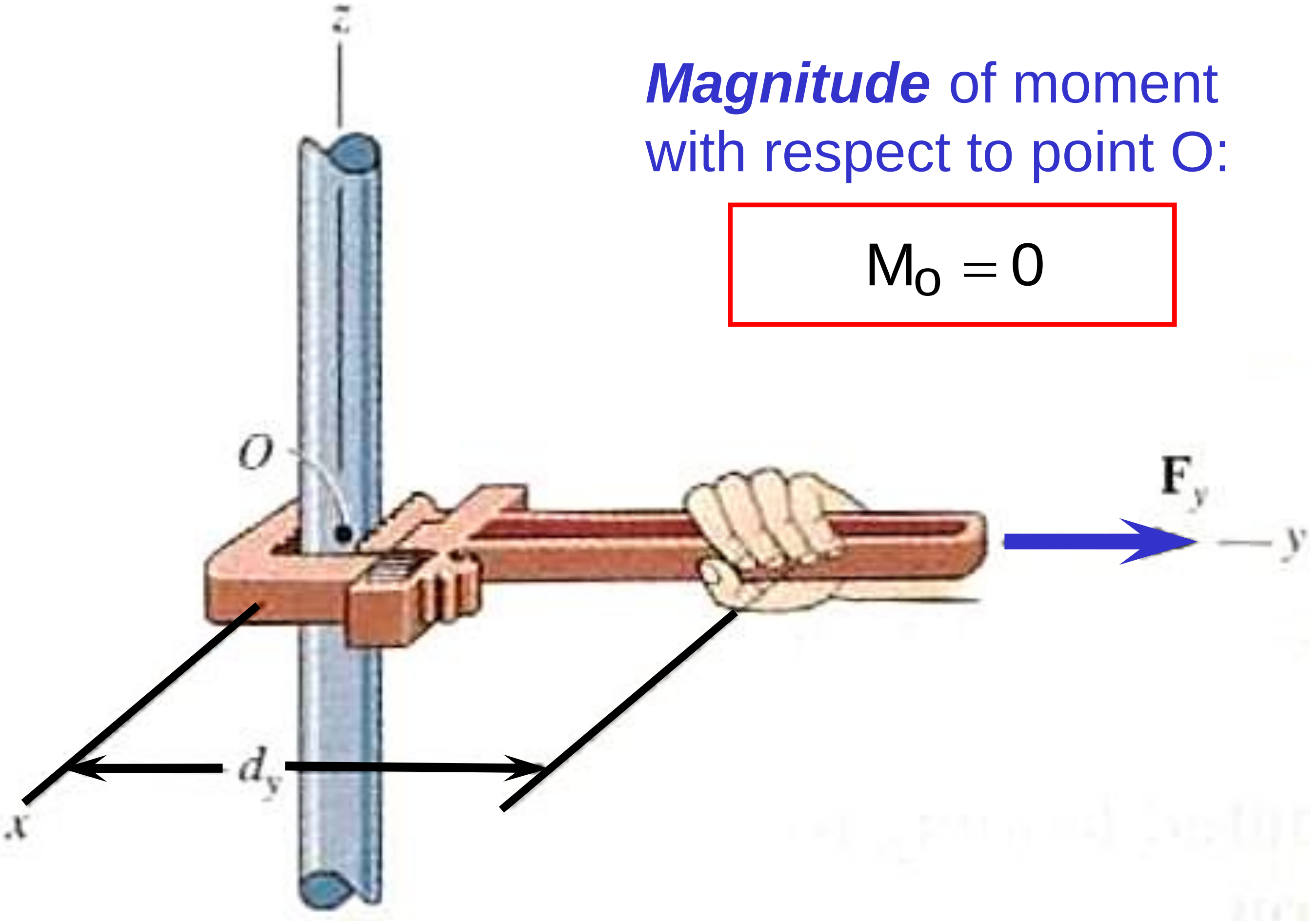
Moment of a force: about x-axis



Magnitude
of moment:

$$(M_O)_x = F_z \cdot d_y$$

Moment of a force: null

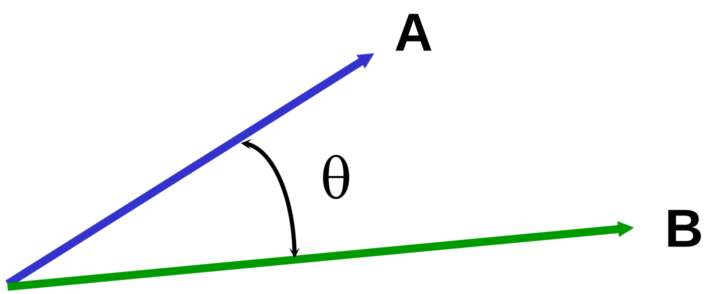


Magnitude of moment with respect to point O:

$$M_O = 0$$

Force analysis. Free-body diagrams

Dynamic equilibrium: scalar product



Dot (scalar) product of vectors **A** and **B** is:

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos(\theta) = AB\cos(\theta)$$

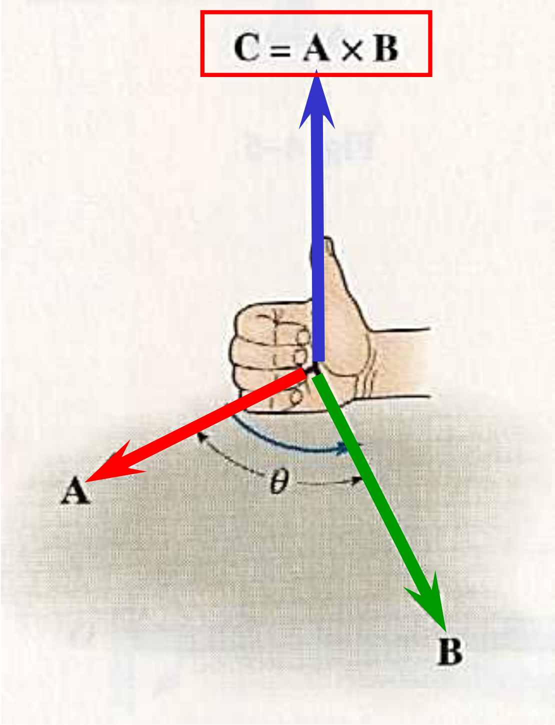
Scalar !!

$$C = \mathbf{A} \bullet \mathbf{B}$$

Vectors

Force analysis. Free-body diagrams

Dynamic equilibrium: cross-product

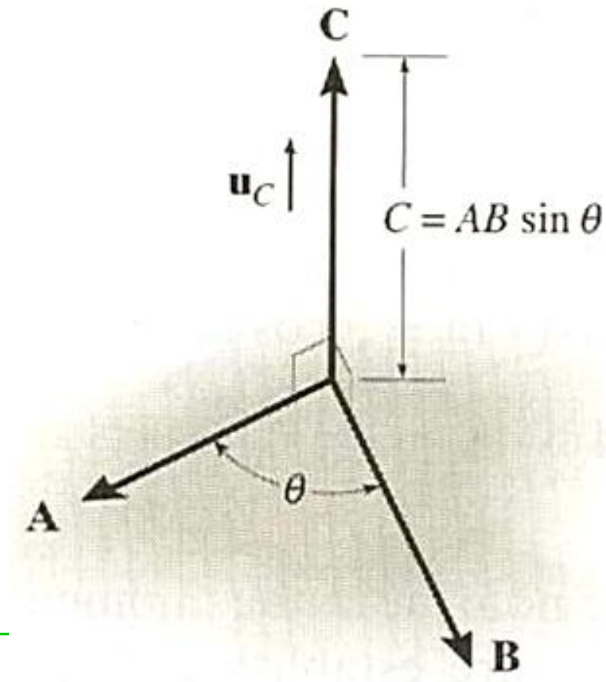


$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = A\,B\,\sin(\theta)\,\mathbf{u}_C$$

Vector

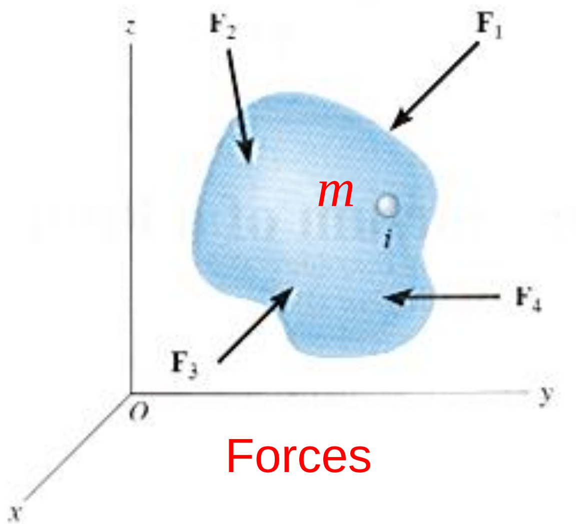
Scalar

Unit vector



Force analysis. Free-body diagrams

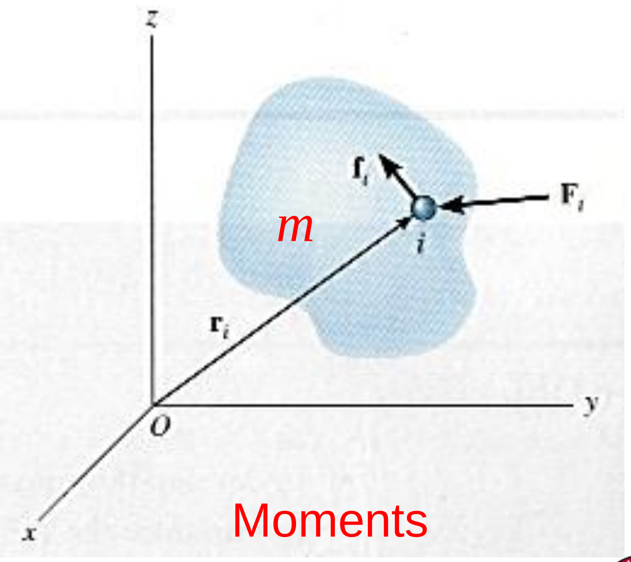
Dynamic equilibrium: forces and moments



Dynamic equilibrium condition (vectorial representation) :

$$\sum \mathbf{F} = m\mathbf{a}$$

($\mathbf{H}_G$ = angular momentum)



Dynamic equilibrium condition (vectorial representation) :

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G$$

with

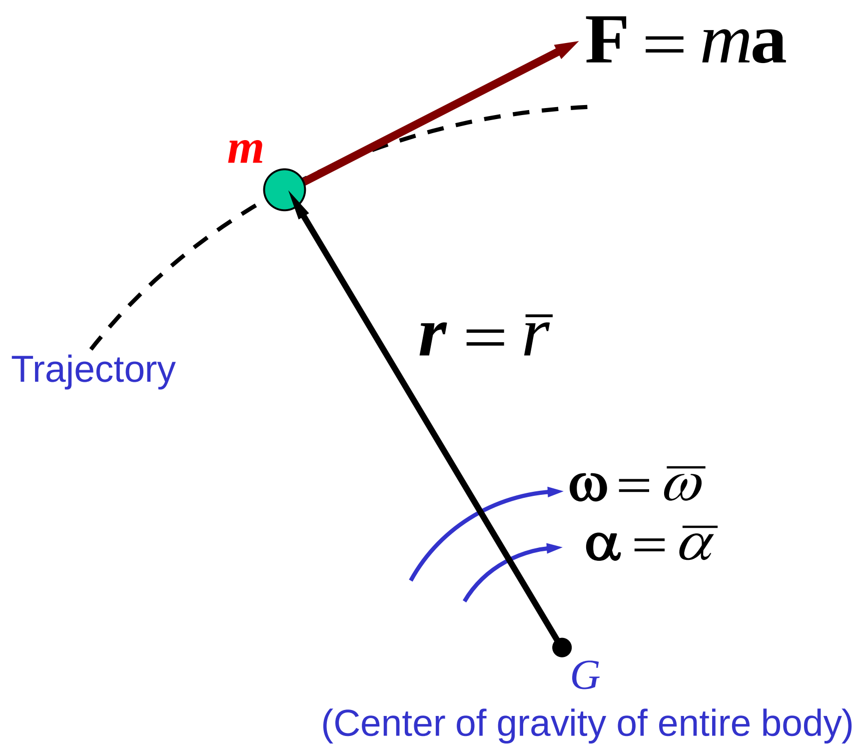
$$\dot{\mathbf{H}}_G = I_x\alpha_x\mathbf{i} + I_y\alpha_y\mathbf{j} + I_z\alpha_z\mathbf{k}$$

Force analysis. Free-body diagrams

Dynamic equilibrium

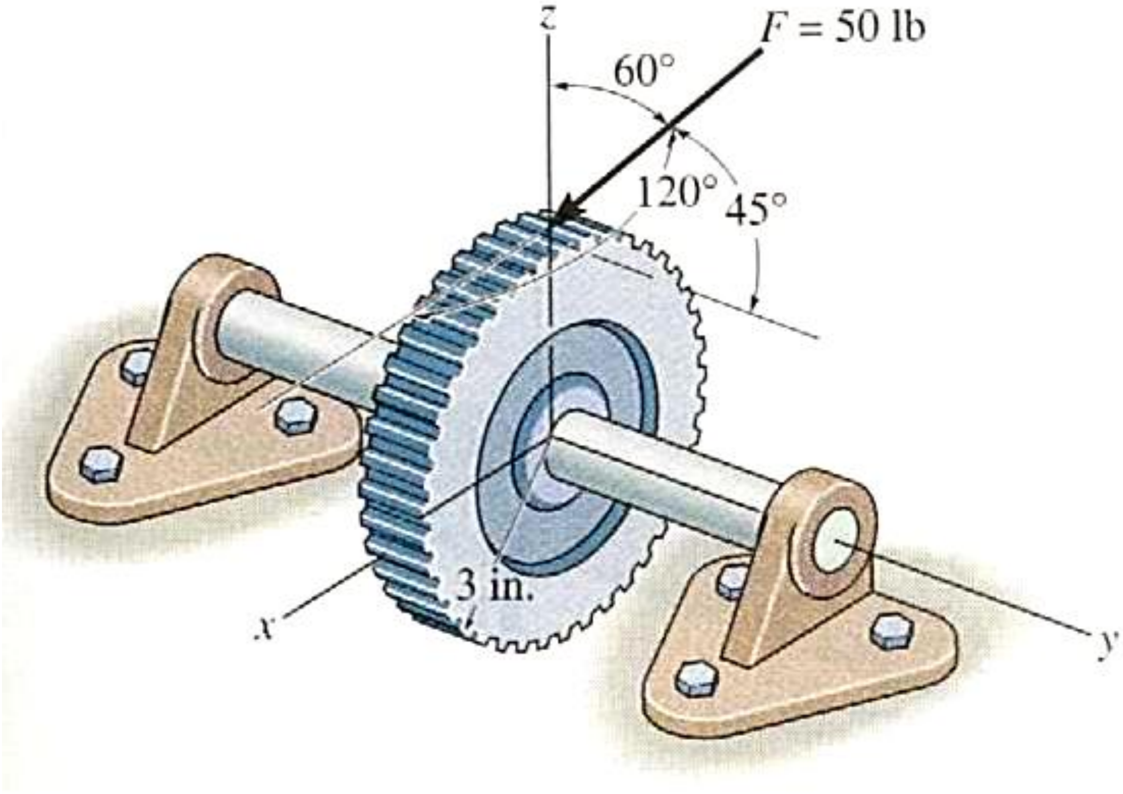
Demonstration (in class; review of scalar and cross-products) of:

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G \quad \text{with} \quad \dot{\mathbf{H}}_G = I_x \alpha_x \mathbf{i} + I_y \alpha_y \mathbf{j} + I_z \alpha_z \mathbf{k}$$



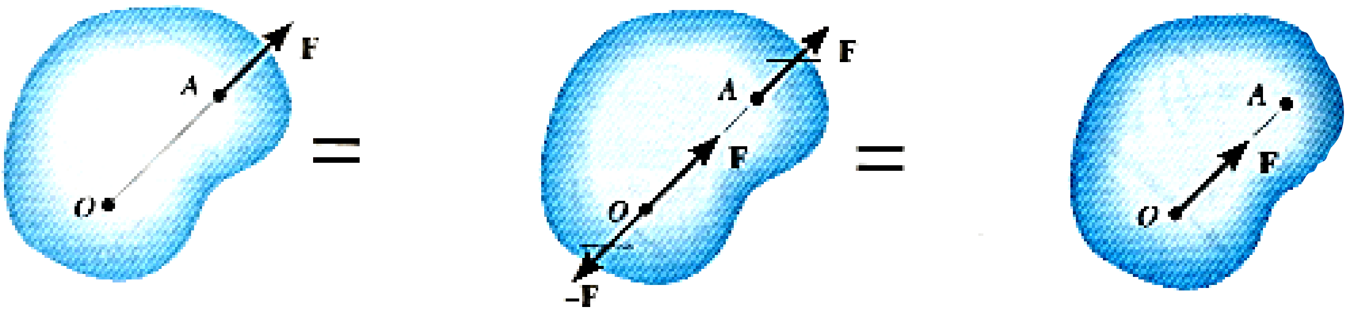
Force analysis: example

The 50 lb force acts on the gear in the direction shown. Determine the moment of this force about the *y*-axis.

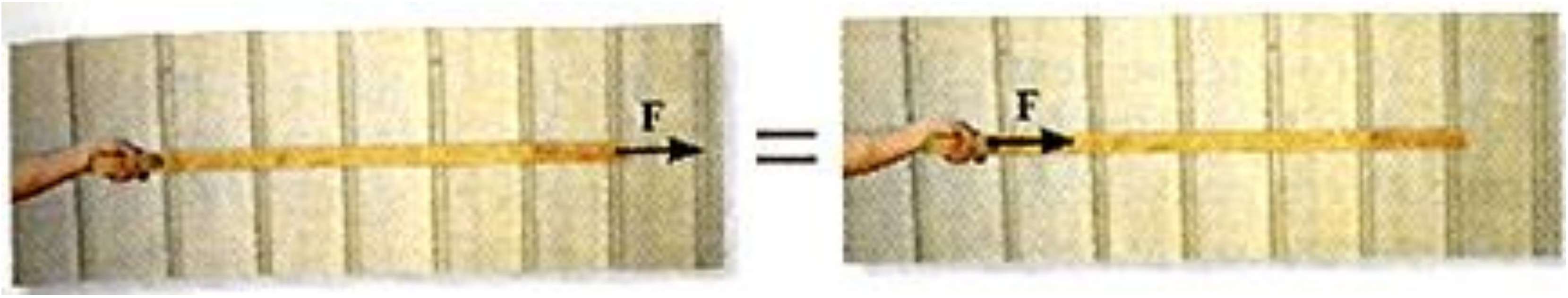


Equivalent systems

Force translated along its line of application:



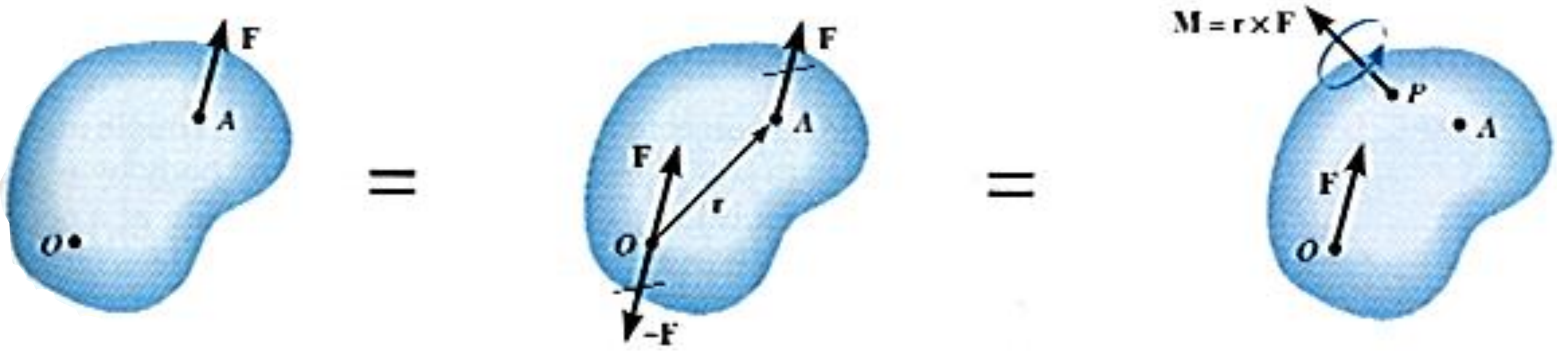
Example:



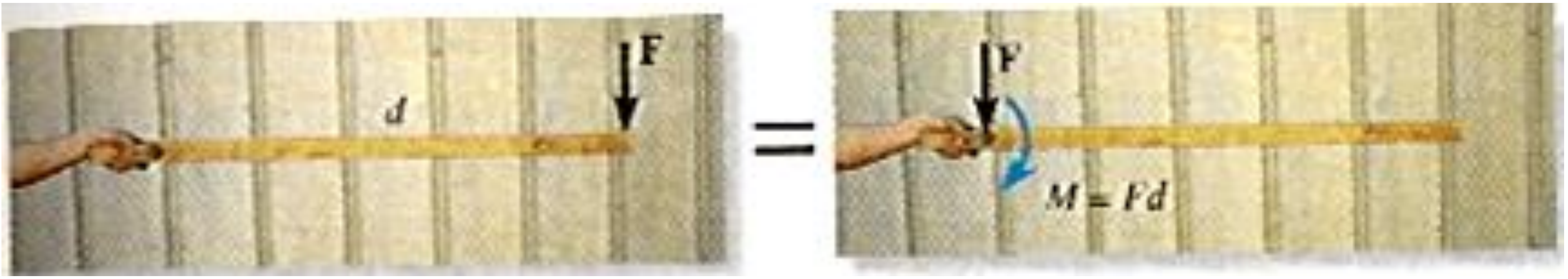
Idealization: similar effects at the support

Equivalent systems

Equivalent force and couple/moment system:

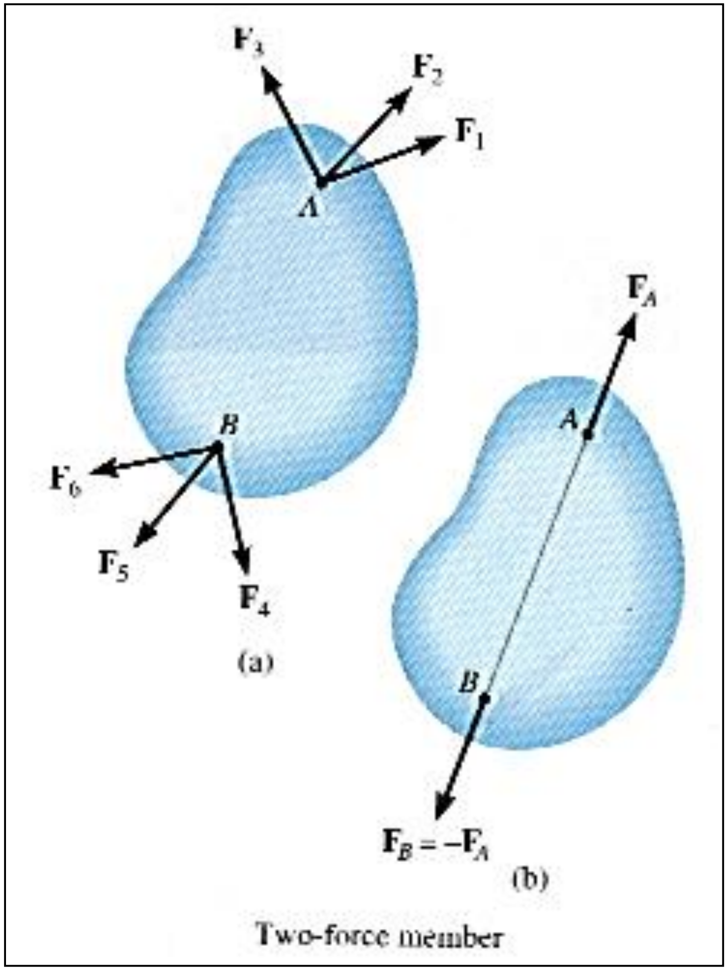


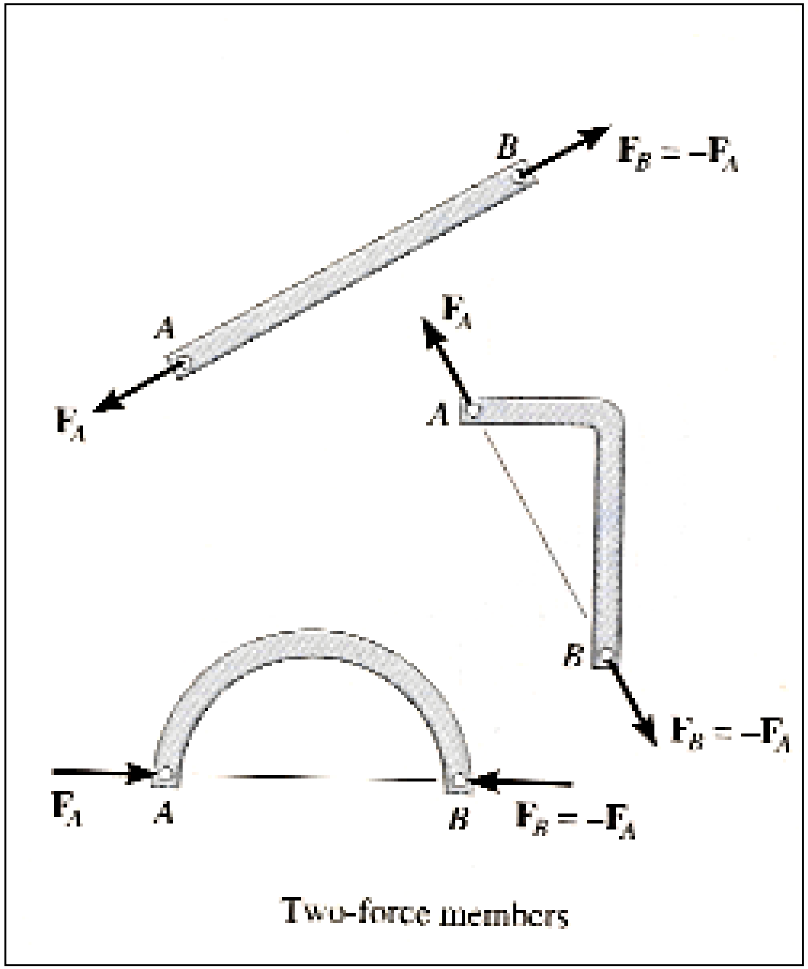
Example:



Idealization: similar effects at the support

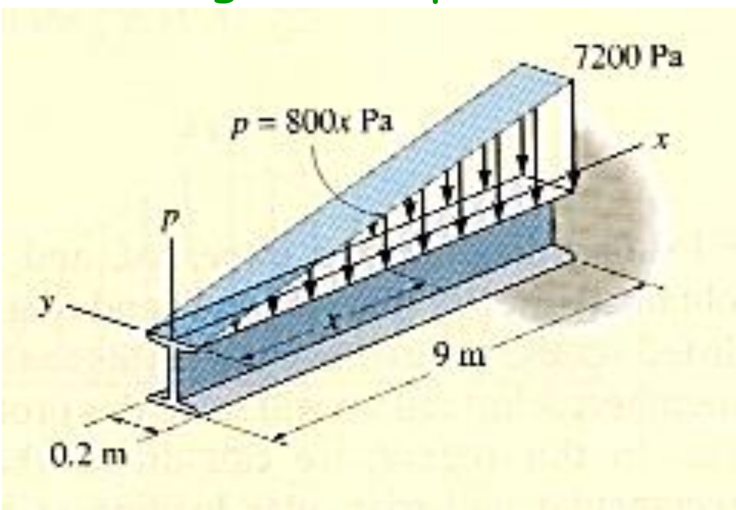
Equilibrium of rigid bodies: equivalent systems

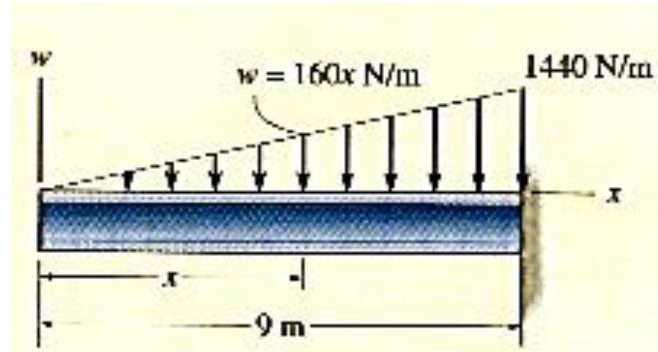




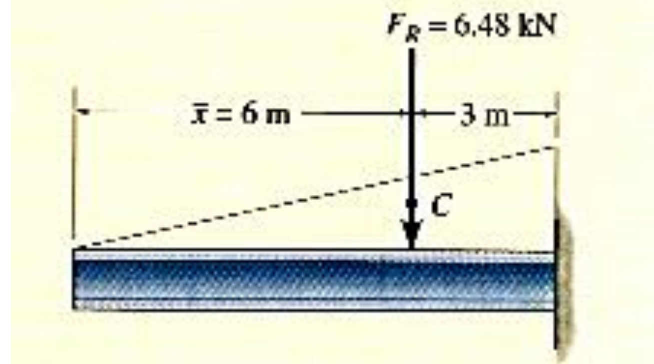
Distributed loads
Equivalent systems

Original 3D problem





2D projection



Equivalent system

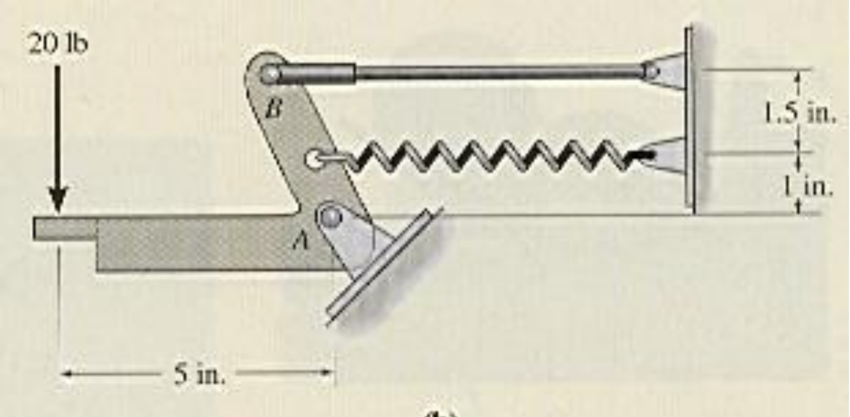
Free-body diagrams

Schematic representation:

Operator applies 20-lb to pedal stretching spring by 1.5 in.

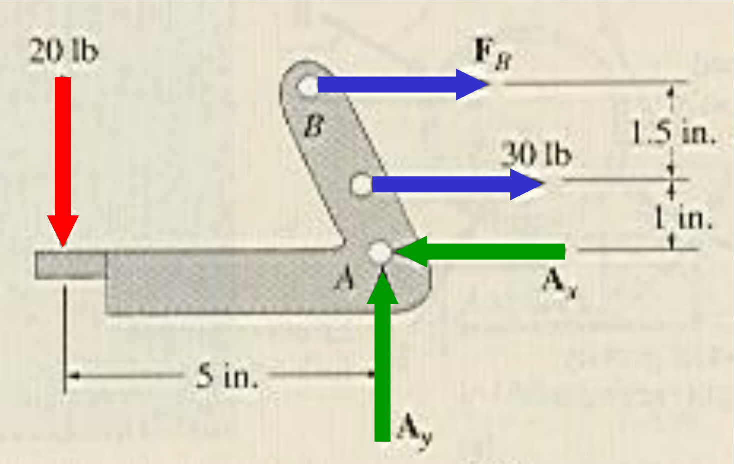
(a)

Actual mechanism



(b)

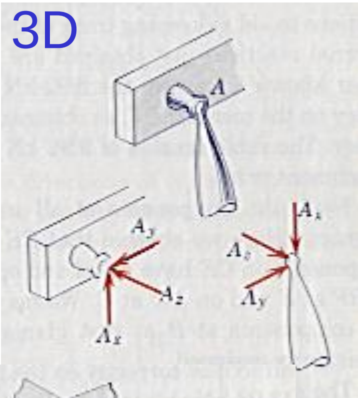
Free-body diagram:



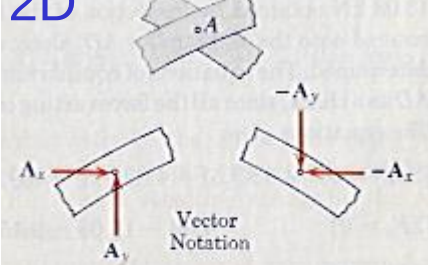
(c)

Force analysis. Free-body diagrams

3D

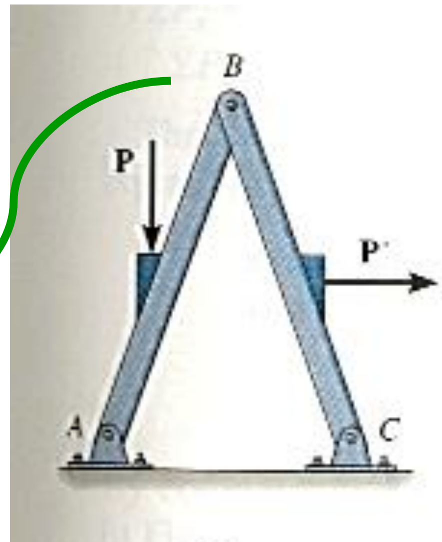


2D

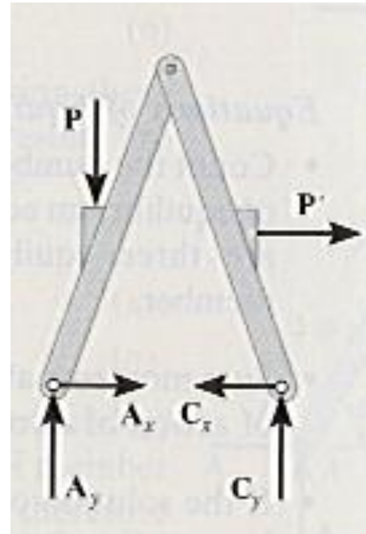


Number of unknowns ?

Equilib. Equations?



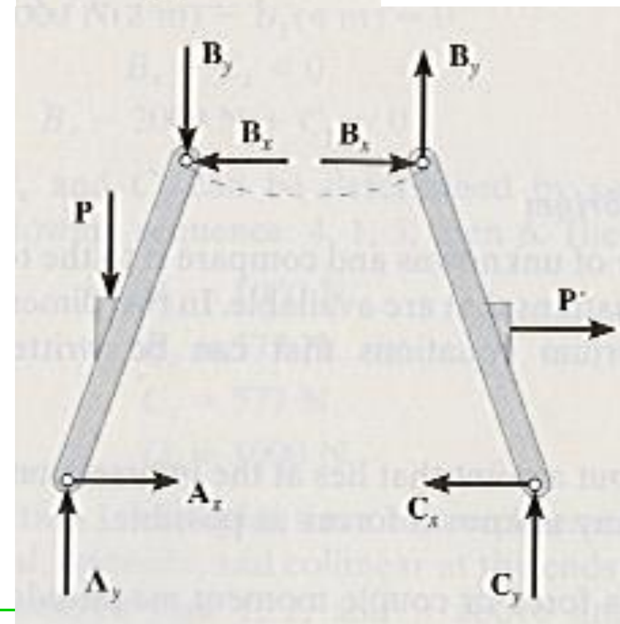
(a)



(c)

Overall FBD

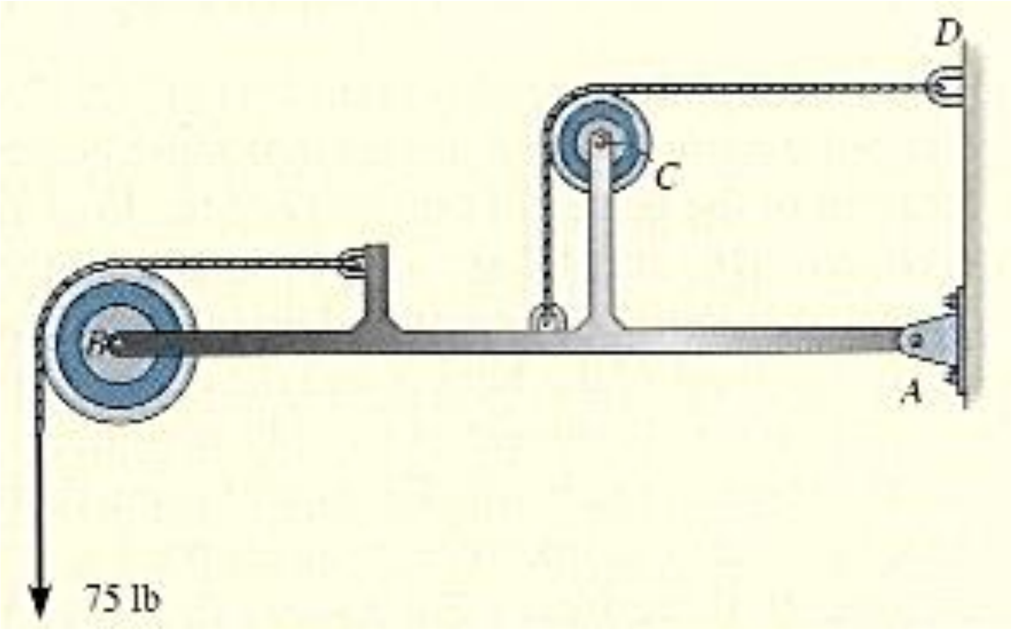
Individual FBD's



(b)

Force analysis. Free-body diagrams

Draw free-body diagrams of (a) entire frame, and (b) each of the components



Reading assignment

- Chapter 3 of textbook
- Review notes and text: ES-2501, ES-2502

Homework assignment

- **Author's:** 3-4A, 3-4B
- **Solve:** 3-1, 3-4, 3-8, 3-10