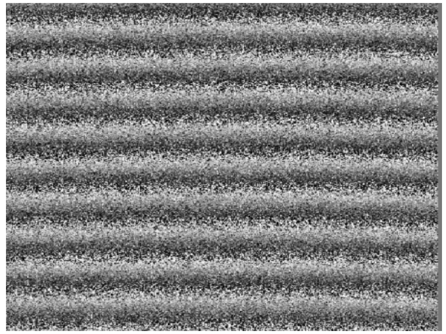
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

Optical Metrology and NDT
ME-593L, B'2014

Introduction: Phase Unwrapping
03 December 2014

Fringe-locus function mod[2π]: actual distributions

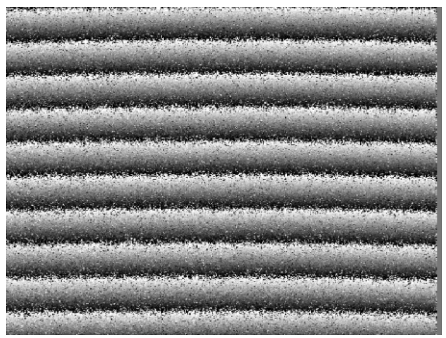
4-frames:



(a)

$$\Omega(u,v) = \operatorname{atan}\left[\frac{I_4(u,v) - I_2(u,v)}{I_1(u,v) - I_3(u,v)}\right]$$

8-frames:



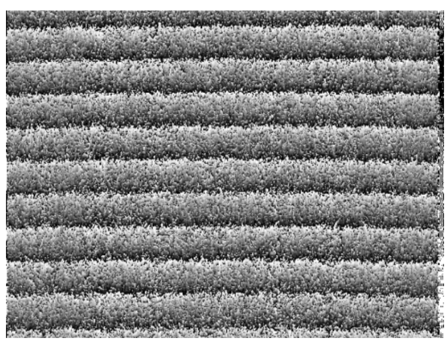
(b)

$$\Omega(u,v) = \operatorname{atan}\left[-\frac{N(u,v)}{D(U,v)}\right]$$

$$N(u,v) = (I_1 - I_3)(I_2' - I_4') - (I_2 - I_4)(I_1' - I_3')$$

$$D(u,v) = (I_1 - I_3)(I_1' - I_3') + (I_2 - I_4)(I_2' - I_4')$$

1-frame, FFT:



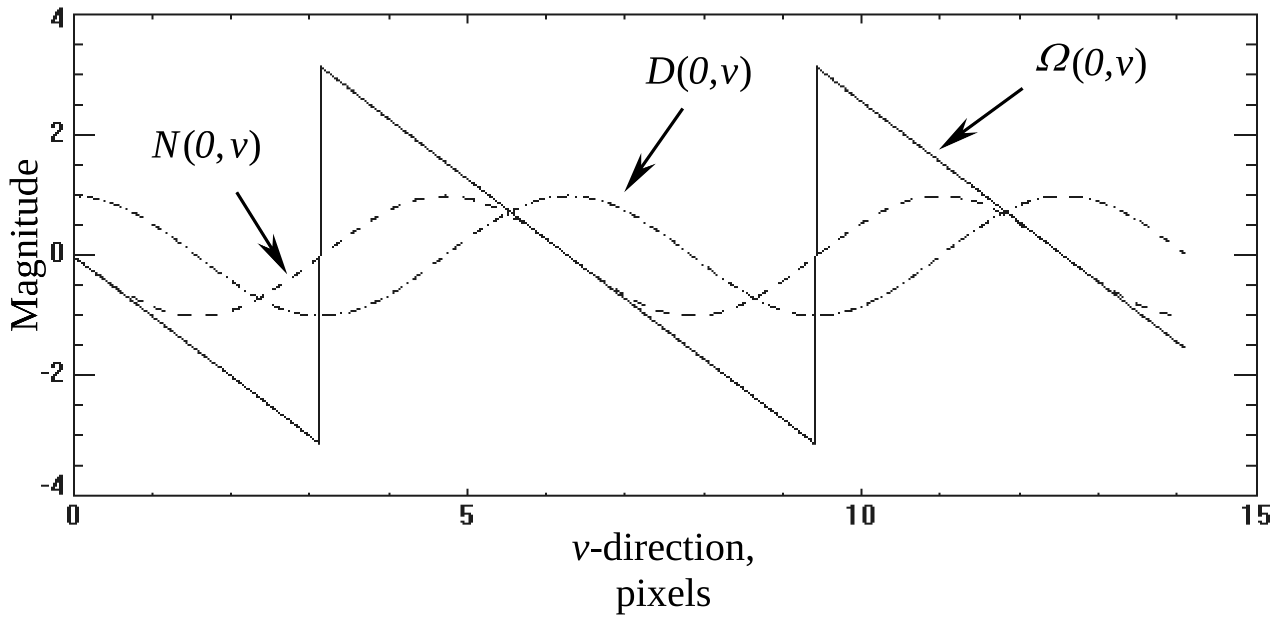
(c)

$$\Omega(u,v) = \operatorname{atan}\left\{\frac{\operatorname{Im}[I_c(u,v)]}{\operatorname{Re}[I_c(u,v)]}\right\}$$

Recovered optical phase

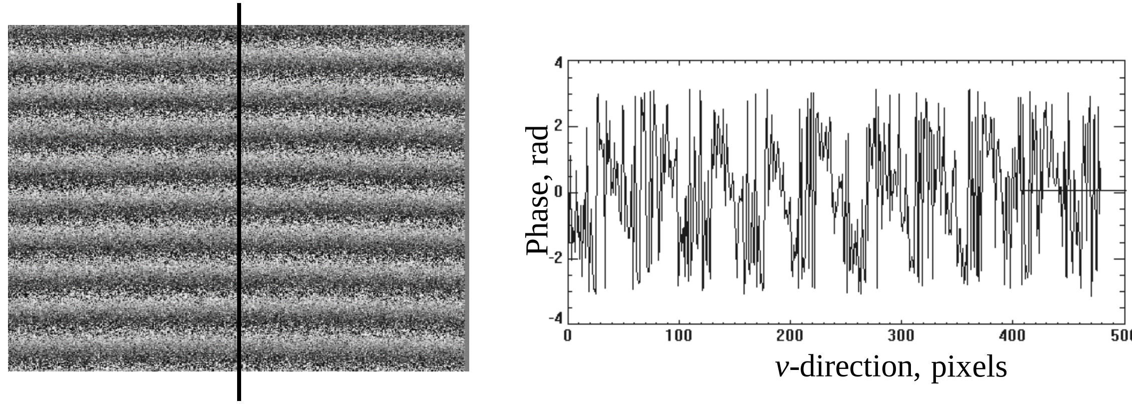
Fringe-locus function mod[2π]

Recovered wrapped phase

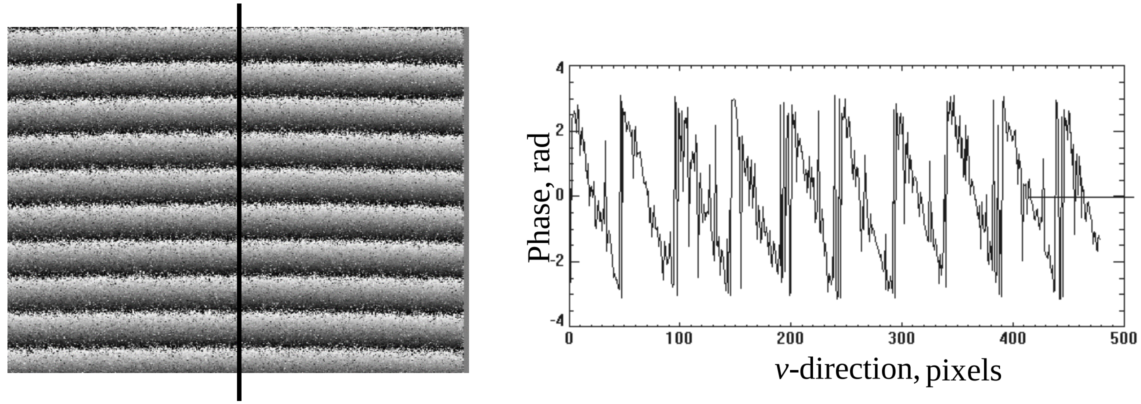


Fringe-locus function $\mathrm{mod}[2\pi]$: actual distributions

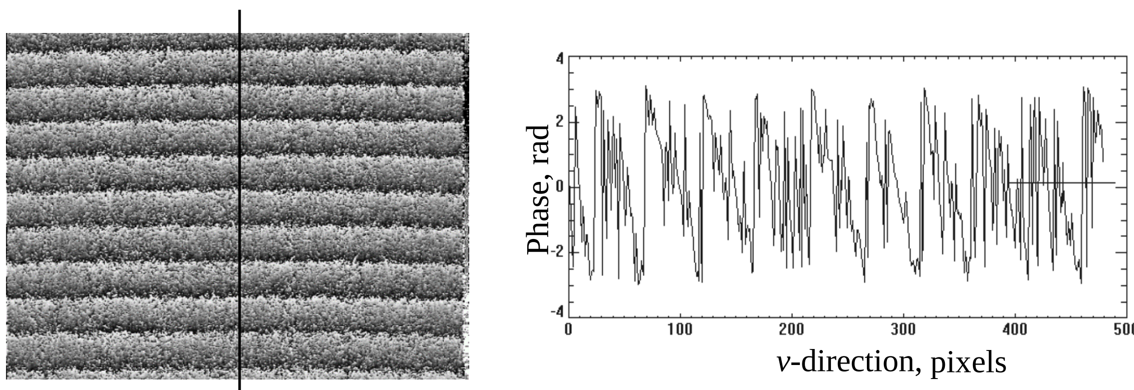
4-frames:



8-frames:



1-frame,
FFT:



Pre-processing: filtering (recommended but not necessary in phase unwrapping)

4-frames: $$\Omega(u,v)=\operatorname{atan}\left[\frac{I_4(u,v)-I_2(u,v)}{I_1(u,v)-I_3(u,v)}\right]$$

8-frames: $$\Omega(u,v)=\operatorname{atan}\left[-\frac{N(u,v)}{D(U,v)}\right]$$

$$N(u,v)=(I_1-I_3)(I_2'-I_4')-(I_2-I_4)(I_1'-I_3')$$
$$D(u,v)=(I_1-I_3)(I_1'-I_3')+(I_2-I_4)(I_2'-I_4')$$

1-frame, FFT: $$\Omega(u,v)=\operatorname{atan}\left\{\frac{\operatorname{Im}[I_c(u,v)]}{\operatorname{Re}[I_c(u,v)]}\right\}$$

Spatial filtering:

- Separate numerator (sine)
- Separate denominator (cosine)
- Filter numerator and denominator separately:
  - Low-pass filter
  - Median filter
  - Adaptive
  - Fourier
  - Other
- Phase re-evaluation

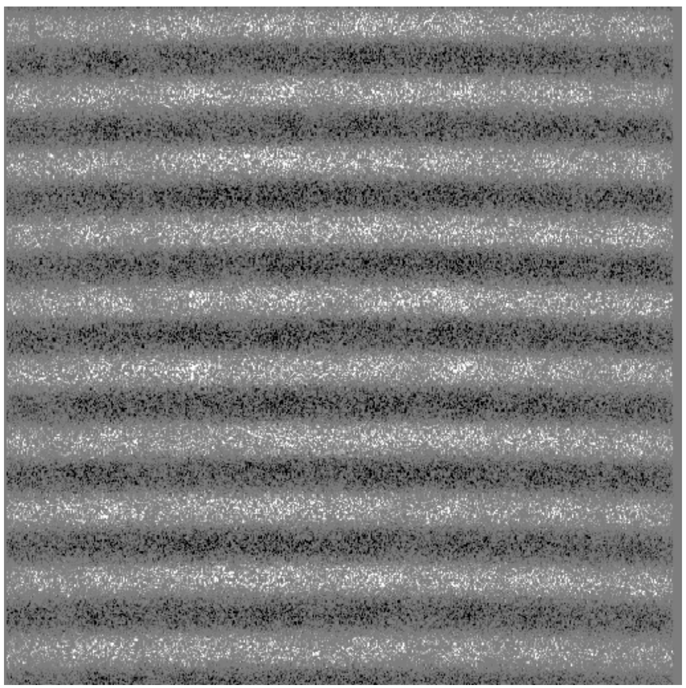
Pre-processing: filtering

Fourier filtering of sine/cosine images

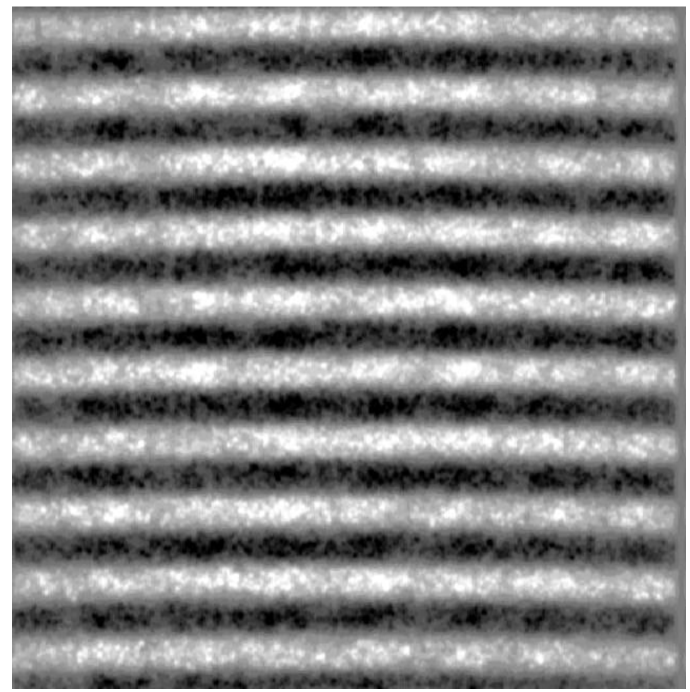
Convolution in the frequency domain: $Q_D^T(f_u, f_v) = Q_D(f_u, f_v) \cdot W_f(f_u, f_v)$

Inverse Fourier transformation: $Q_D^{'}(m,n) = \mathcal{F}^{-1}\{Q_D^T(f_u, f_v)\}$

Original interferogram



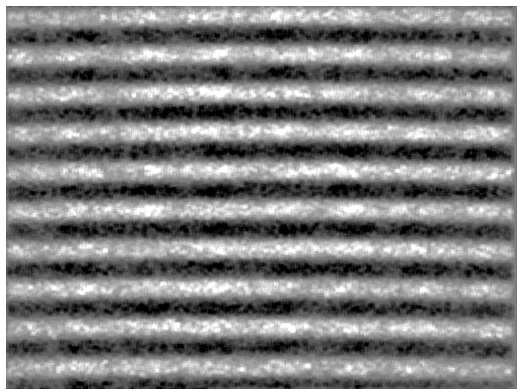
Filtered interferogram



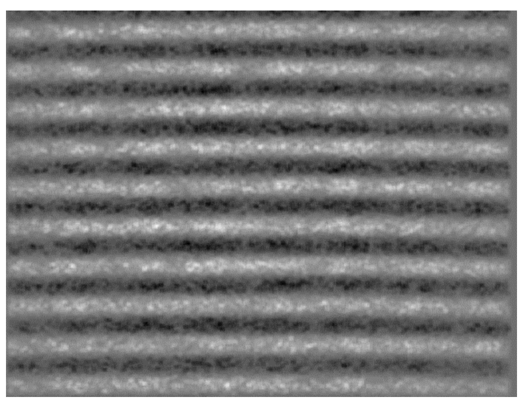
Pre-processing: filtering

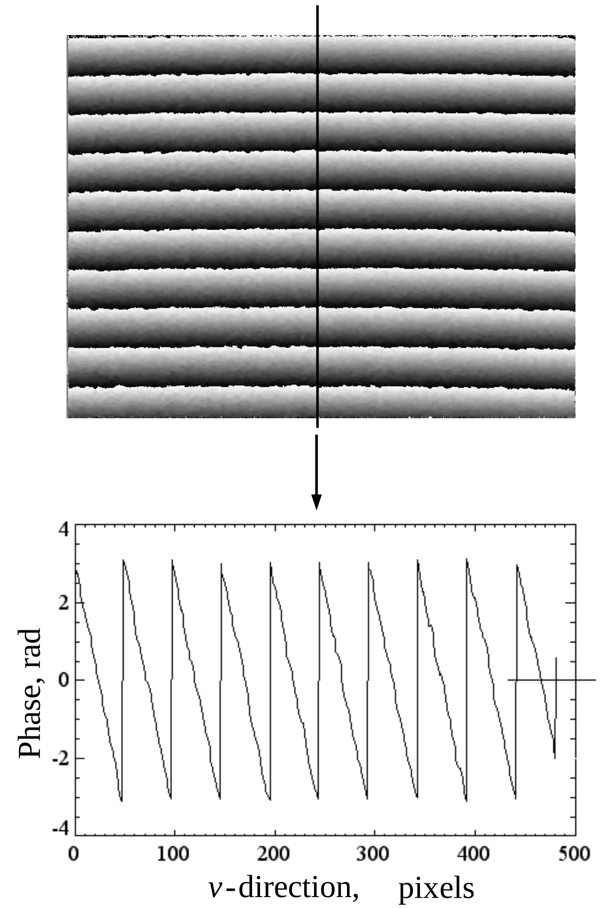
Fourier filtering of sine/cosine images

Filtered **sine** image:



Filtered **cosine** image:





Re-evaluated phase

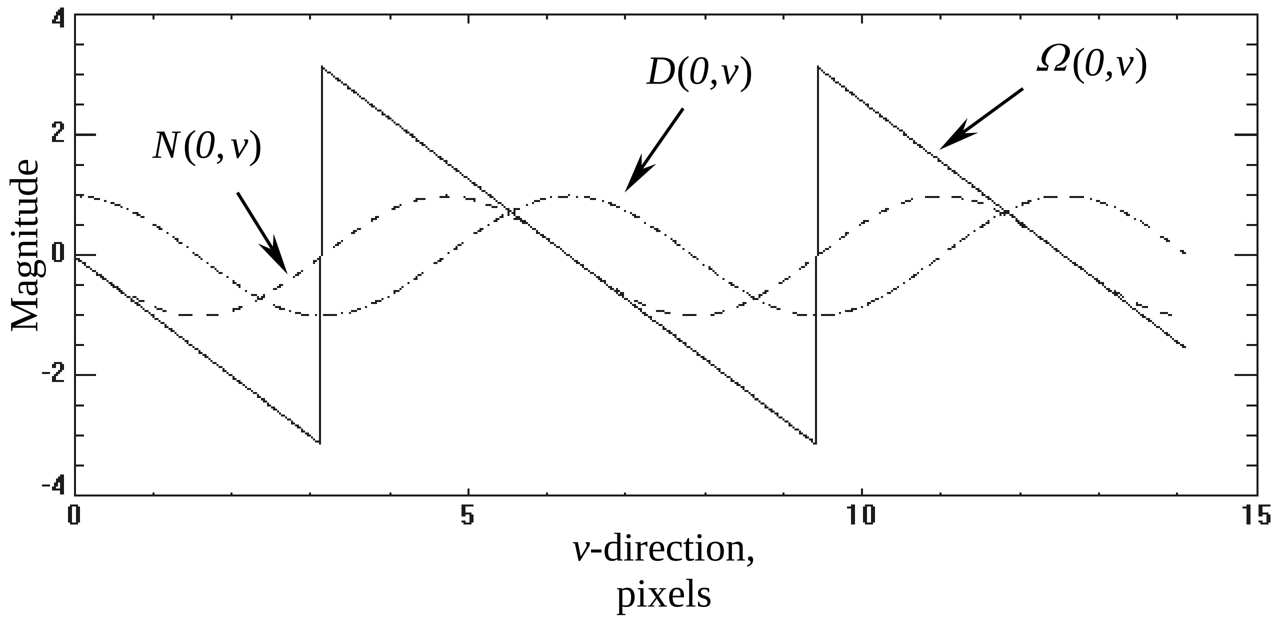
Phase unwrapping of phase $\mathrm{mod}[2\pi]$

Wrapped phase: $\Omega = \Omega(u,v)$

Unwrapped phase: $U\{\Omega(u,v)\} = \Omega(u,v) + 2\pi N(u,v)$

$N(u,v) =$ Round(fringe orders)

Wrapped phase



Row-by-row / Column-by-column algorithm

Evaluate finite difference $\Delta\ell(u,v)$:

$$\Delta\ell(u,v) = \begin{cases} \Omega(u,v) - \Omega(u-1,v), \ \text{row,} \\ \text{or} \\ \Omega(u,v) - \Omega(u,v-1), \ \text{column} \end{cases}$$

Line unwrapping:

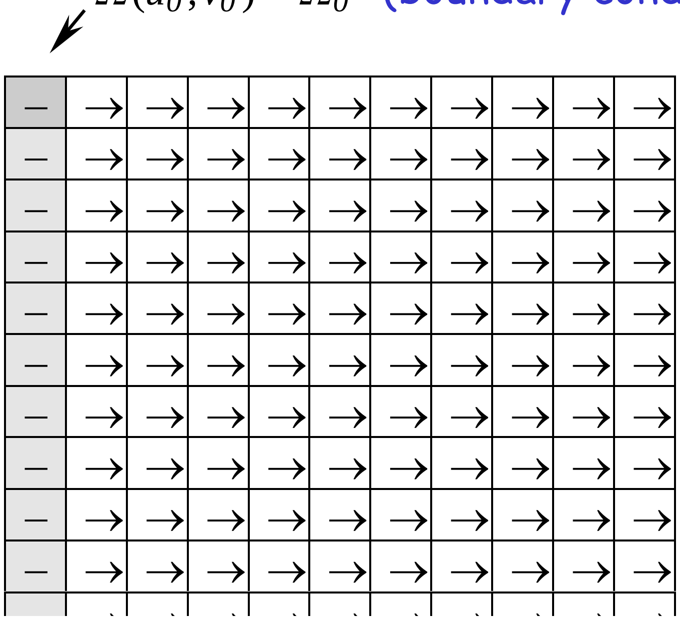
$$\Omega_c(u,v) = \begin{cases} \Omega(u,v) - C_d \cdot 2\pi & \text{if } \Delta\ell > \kappa \cdot 2\pi \\ \Omega(u,v) + C_d \cdot 2\pi & \text{if } \Delta\ell < -\kappa \cdot 2\pi \end{cases}$$

$C_d =$ signed integer

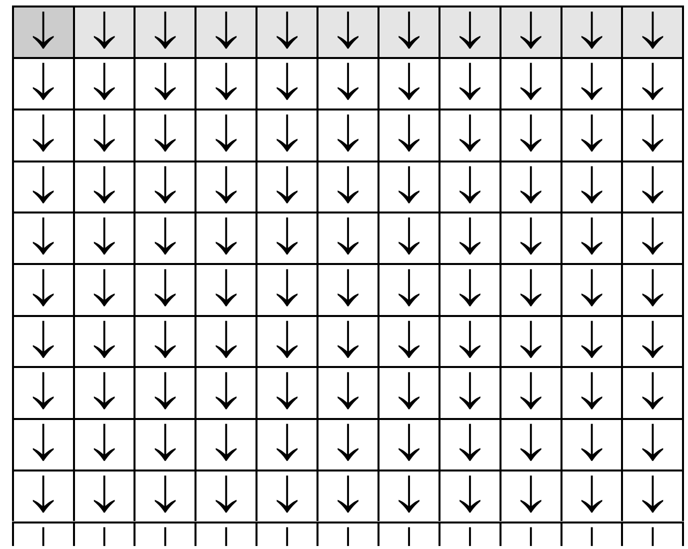
$\kappa < 1,$ unwrapping threshold

Row-by-row / Column-by-column algorithm

$\Omega(u_0, v_0) = \Omega_0$ (boundary condition)



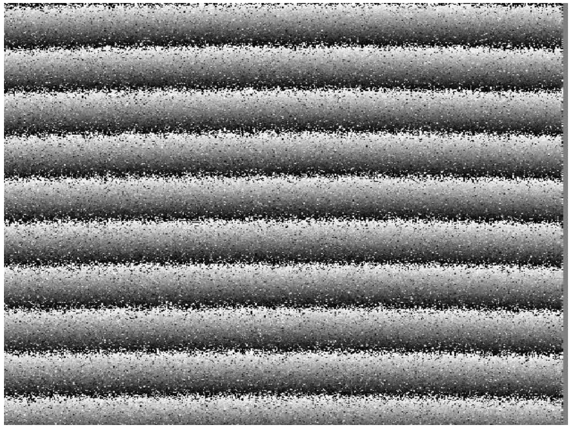
Row-by-row



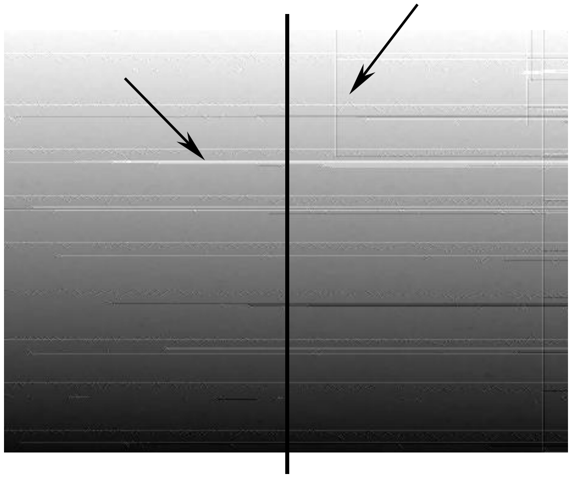
Column-by-column

Row-Column algorithm

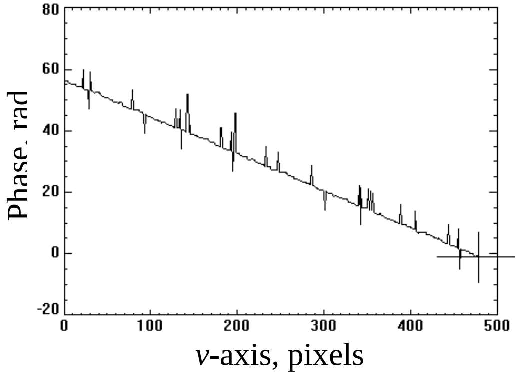
Unfiltered wrapped phase

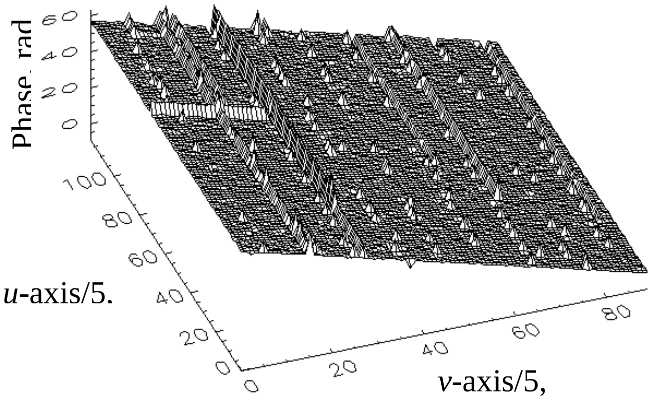


Unwrapped phase

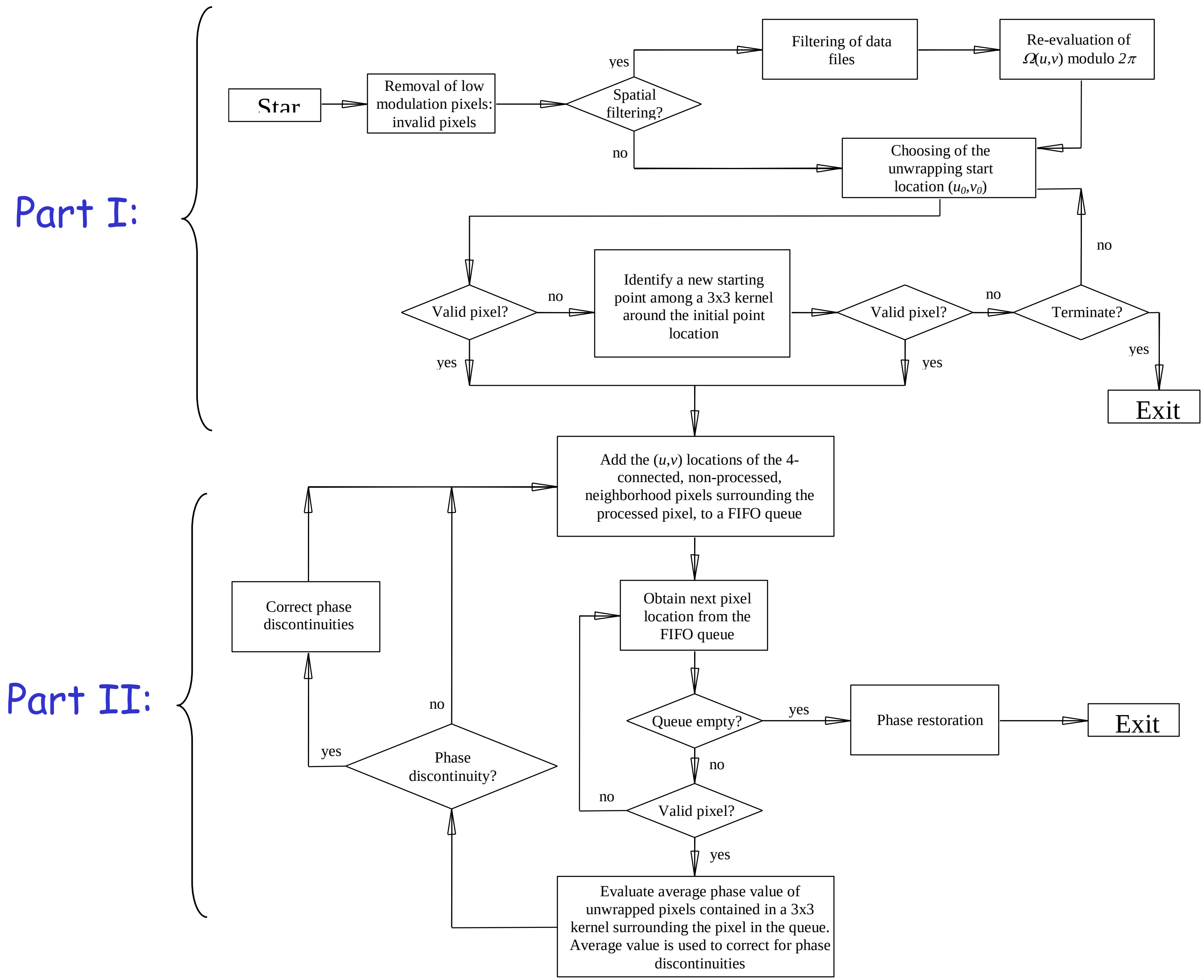


Phase discontinuities still present



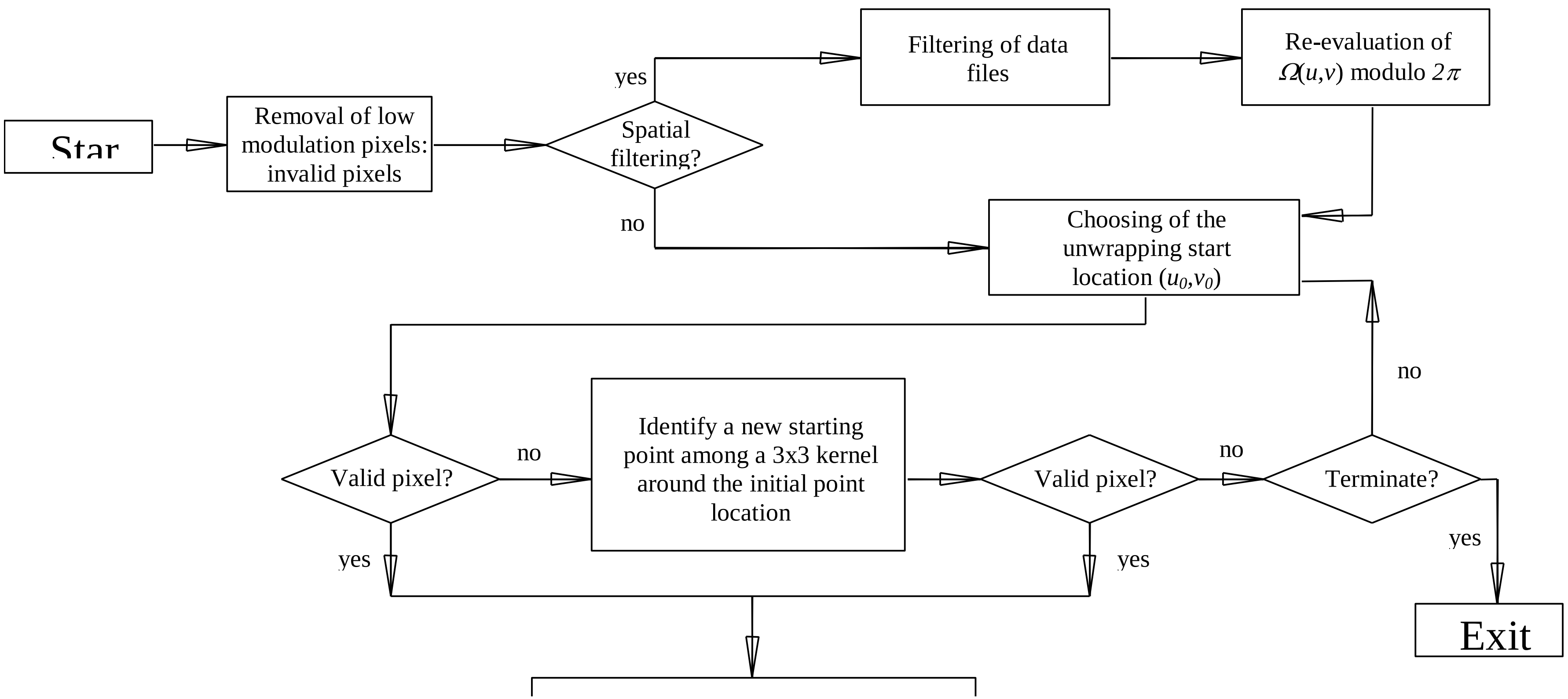


Edge following algorithm



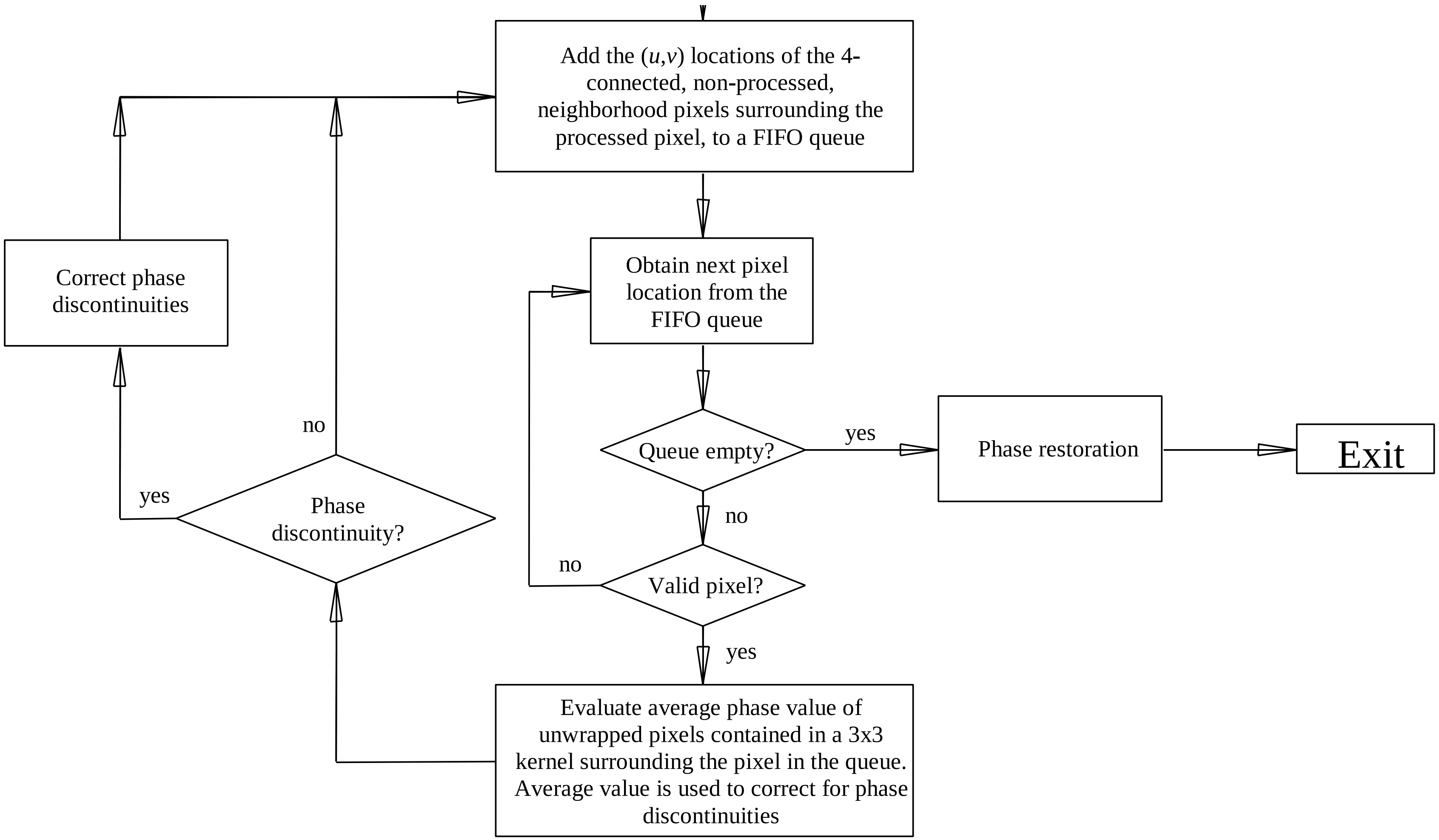
Edge following algorithm

Part I



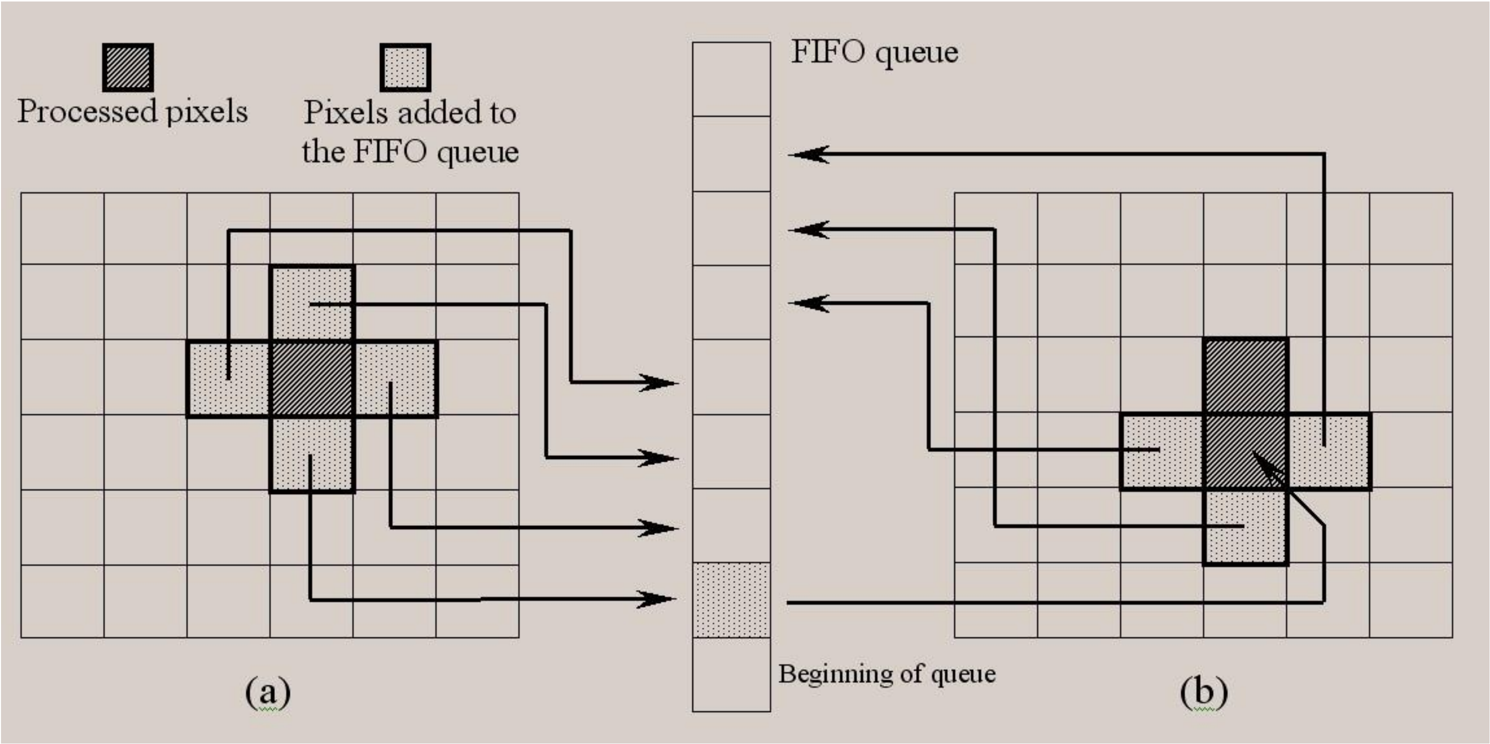
Edge following algorithm

Part II



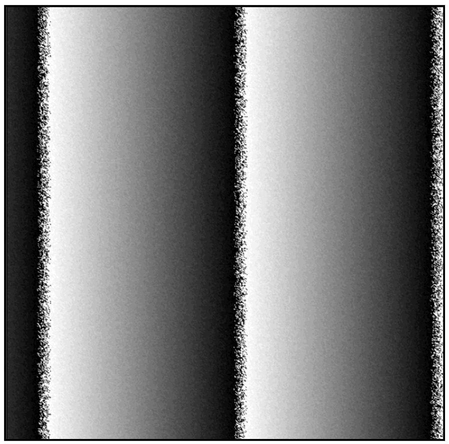
Edge following algorithm: FIFO queue

(a) 4 connected pixels to processed pixel are added to the queue, (b) the next pixel to be processed is taken from the queue and after processing, the unprocessed pixels contained in the 4 connected pixels surrounding the processed pixel are added to the queue.

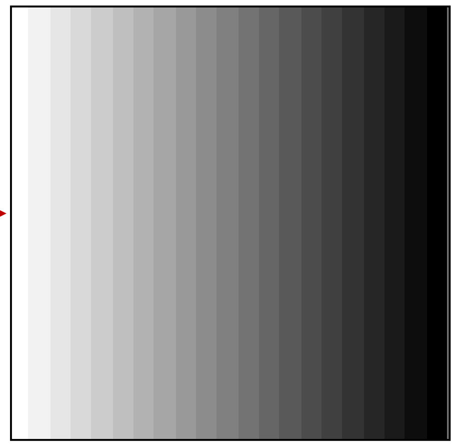


Example: interferogram with open-fringes

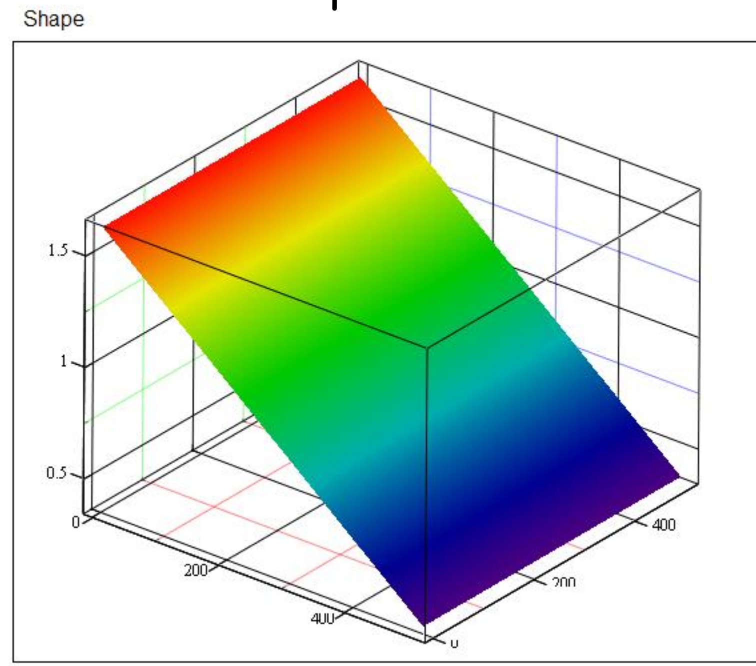
Wrapped phase



Unwrapped phase



Interpretation

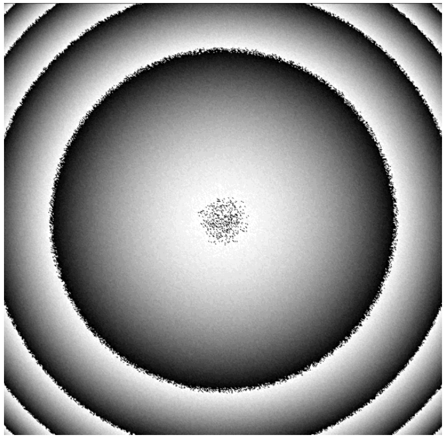


Need:

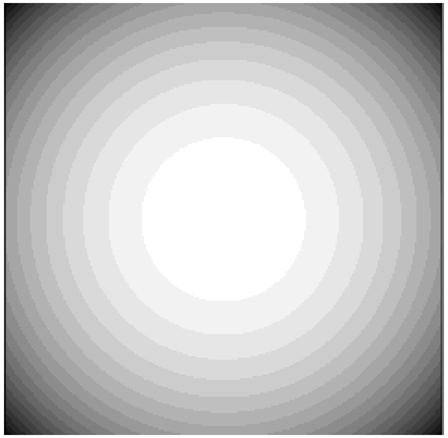
Camera calibration &
Sensitivity Vector

Example: interferogram with closed-fringes

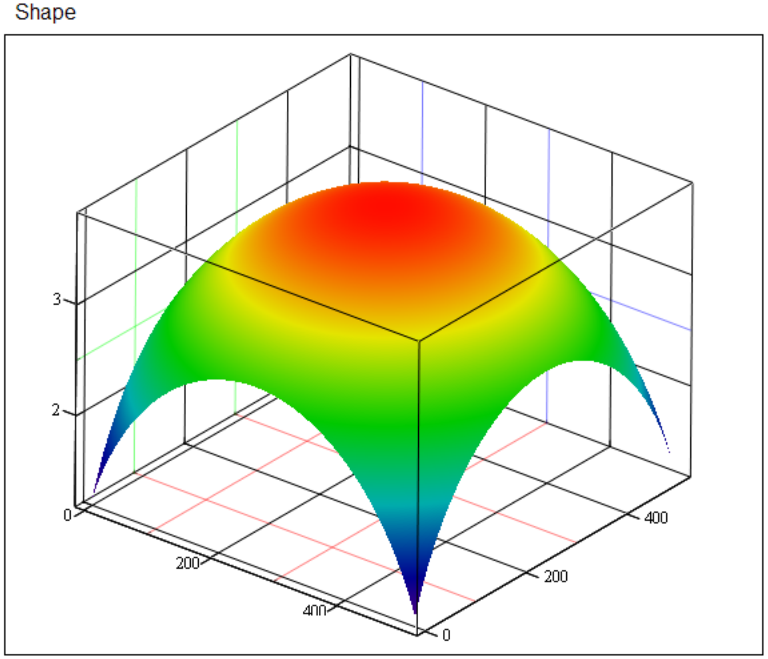
Wrapped phase



Unwrapped phase



Interpretation



Need:

Camera calibration & Sensitivity Vector

Recommended Reading Assignment

Papers by

1) Ge at al.; (2001). "Coordinate-transform technique for closed-fringe analysis by the Fourier-transform method

2) Takeda et al., (1982). "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry."