

# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

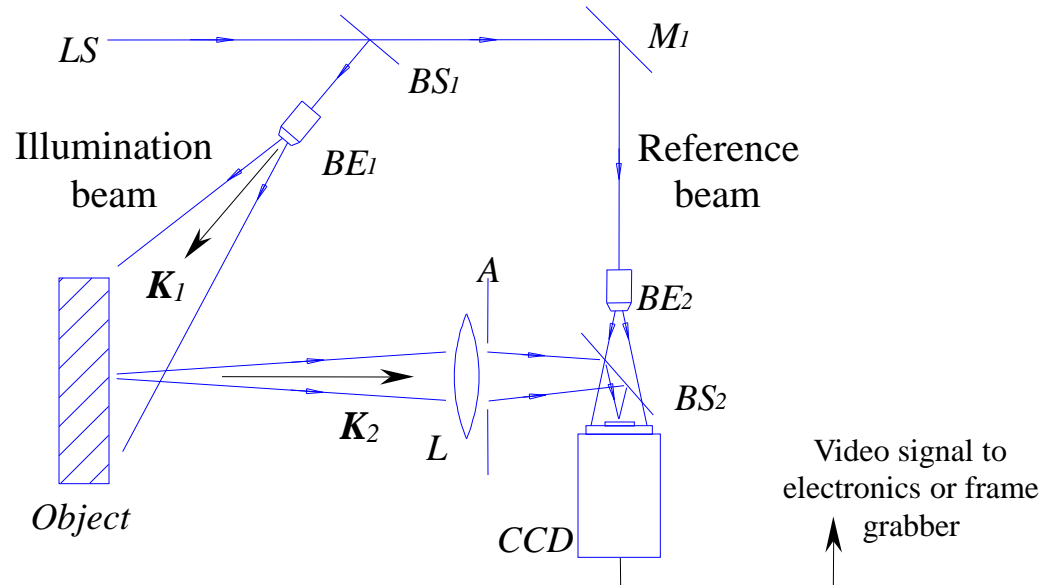
Optical Metrology and NDT  
ME-593L, B'2011

Introduction: Phase Sampling  
28 November 2011



# Interference

## Setup for recording of electronic interferograms



Intensity **before** deformations:  $I(u, v) = I_B(u, v) + I_M(u, v) \cos[\Delta\phi(u, v)]$

Intensity **after** deformations:  $I'(u, v) = I_B(u, v) + I_M(u, v) \cos[\Delta\phi(u, v) + \Omega(u, v)]$

Review, *in class*, unknowns and known variables

# Interference

Phase sampling by phase shifting

Intensity of the form:

$$I(x, y) = I_B + I_M \cos(\Delta\phi)$$

↑  
Known

↑  
Unknown

↑  
Unknown

↑  
Unknown

Solution for the unknowns: introduction of optical phase shifts

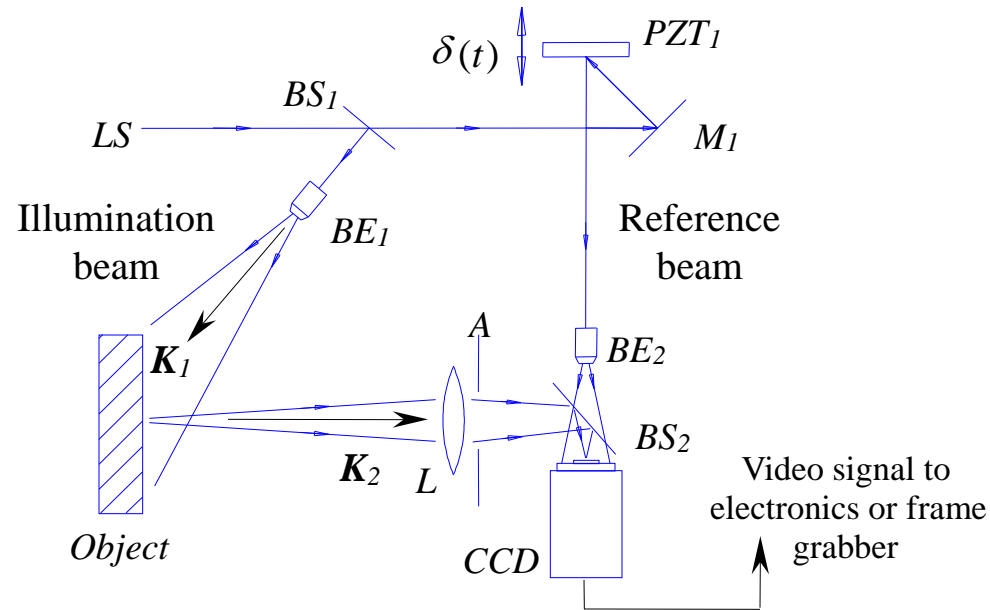
$$I_n(u, v) = I_B(u, v) + I_M(u, v) \cos[\Delta\phi(u, v) + \theta_n]$$

Number of phase shifts:  $n = 1, 2, \dots, N$

$n$ -th phase shift:  $\theta_n$

# Interference

Setup for recording of phase shifted electronic interferograms

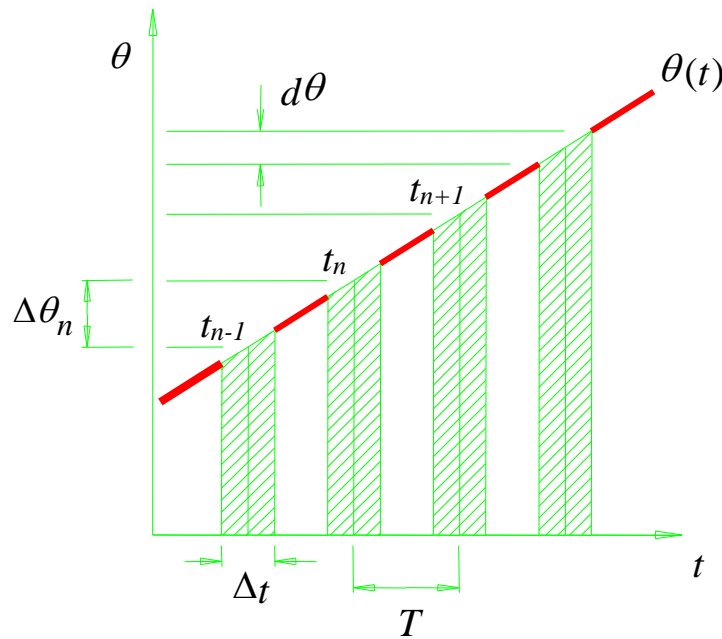


$$n\text{-th phase shift: } \theta_n = \theta(t_n) = \frac{2\pi}{\lambda} \delta(t_n)$$

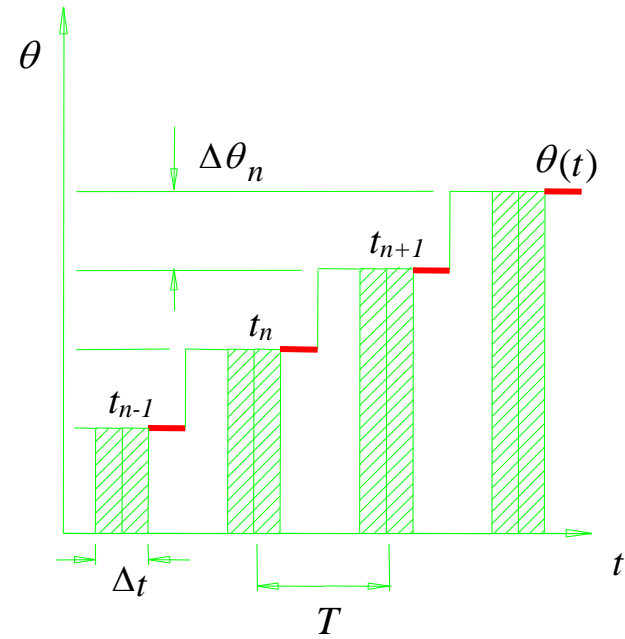
# Phase shifting

## Driving functions

### Continuous phase shifting



### Phase stepping



# Phase shifting

## Driving functions

Phase stepping is preferred:

- Minimum phase drifting during image acquisition
- Simple to implement
- Require detailed calibration, for the wavelength used



# Phase sampling equations

Least-squares minimization of residuals,  $R_d$

Residuals:  $R_d = I_B(u, v) + I_M(u, v) \cos[\Delta\phi(u, v) + \theta_n] - I_n(u, v)$

$$\text{Summation of residuals: } R_d = \sum_{n=1}^N [I_B(u, v) + A \cos(\theta_n) + B \sin(\theta_n) - I_n(u, v)]$$

$$\text{with } A = I_M(u, v) \cos[\Delta\phi(u, v)], \quad B = -I_M(u, v) \sin[\Delta\phi(u, v)]$$



# Phase sampling equations

Least-squares minimization of residuals,  $R_d$

Minimization of residuals :

$$\frac{\partial}{\partial I_B} \left\{ \sum_{n=1}^N [I_B(u, v) + A \cos(\theta_n) + B \sin(\theta_n) - I_n(u, v)]^2 \right\} = 0$$

$$\frac{\partial}{\partial A} \left\{ \sum_{n=1}^N [I_B(u, v) + A \cos(\theta_n) + B \sin(\theta_n) - I_n(u, v)]^2 \right\} = 0$$

$$\frac{\partial}{\partial B} \left\{ \sum_{n=1}^N [I_B(u, v) + A \cos(\theta_n) + B \sin(\theta_n) - I_n(u, v)]^2 \right\} = 0$$

# Phase sampling equations

Least-squares minimization of residuals,  $R_d$

Resultant system of equations :

$$\begin{bmatrix} N & \sum_{n=1}^N \cos(\theta_n) & \sum_{n=1}^N \sin(\theta_n) \\ \sum_{n=1}^N \cos(\theta_n) & \sum_{n=1}^N \cos^2(\theta_n) & \sum_{n=1}^N \sin(\theta_n) \cos(\theta_n) \\ \sum_{n=1}^N \sin(\theta_n) & \sum_{n=1}^N \sin(\theta_n) \cos(\theta_n) & \sum_{n=1}^N \sin^2(\theta_n) \end{bmatrix} \begin{pmatrix} I_B \\ A \\ B \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^N I_n \\ \sum_{n=1}^N I_n \cos(\theta_n) \\ \sum_{n=1}^N I_n \sin(\theta_n) \end{pmatrix}$$

with  $A = I_M(u, v) \cos[\Delta\phi(u, v)]$ ,  $B = -I_M(u, v) \sin[\Delta\phi(u, v)]$

# Phase sampling equations

Four phase shifting algorithm,  $n = 4$

$n$ -th phase shifts:  $\theta_n = 0, \pi/2, \pi, 3\pi/2$

Resultant system of equations,  $n = 4$ :

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{pmatrix} I_B \\ A \\ B \end{pmatrix} = \begin{pmatrix} I_1 + I_2 + I_3 + I_4 \\ I_1 - I_3 \\ I_2 - I_4 \end{pmatrix}$$



# Phase sampling equations

Four phase shifting algorithm,  $n = 4$

Resultant background intensity:

$$I_B(u, v) = \frac{1}{4} [I_1(u, v) + I_2(u, v) + I_3(u, v) + I_4(u, v)]$$

Resultant modulation intensity:

$$\begin{aligned} I_M(u, v) &= \sqrt{[A(u, v)]^2 + [B(u, v)]^2} \\ &= \frac{1}{2} \sqrt{[I_1(u, v) - I_3(u, v)]^2 + [I_2(u, v) - I_4(u, v)]^2} \end{aligned}$$



# Phase sampling equations

Four phase shifting algorithm,  $n = 4$

Fringe visibility, or fringe contrast:

$$V(u, v) = \frac{I_M(u, v)}{I_B(u, v)}$$

Optical phase:

$$\Delta\phi(u, v) = \text{atan}\left[-\frac{B(u, v)}{A(u, v)}\right] = \text{atan}\left[\frac{\sin[\Delta\phi(u, v)]}{\cos[\Delta\phi(u, v)]}\right] = \text{atan}\left[\frac{I_4(u, v) - I_2(u, v)}{I_1(u, v) - I_3(u, v)}\right]$$



# Recovered optical phase

Before and after deformations or changes in OPL

Before deformations:

$$\Delta\phi(u, v) = \text{atan}\left[-\frac{B(u, v)}{A(u, v)}\right] = \text{atan}\left[\frac{\sin[\Delta\phi(u, v)]}{\cos[\Delta\phi(u, v)]}\right] = \text{atan}\left[\frac{I_4(u, v) - I_2(u, v)}{I_1(u, v) - I_3(u, v)}\right]$$

After deformations:

$$\Delta\phi(u, v) + \Omega(u, v) = \text{atan}\left[\frac{\sin[\Delta\phi(u, v) + \Omega(u, v)]}{\cos[\Delta\phi(u, v) + \Omega(u, v)]}\right] = \text{atan}\left[\frac{I'_4(u, v) - I'_2(u, v)}{I'_1(u, v) - I'_3(u, v)}\right]$$



# Recovered optical phase

## Fringe-locus function

Phase difference:

$$[\Delta\phi(u, v) + \Omega(u, v)] - \Delta\phi(u, v) = \text{atan} \left[ \frac{I'_4(u, v) - I'_2(u, v)}{I'_1(u, v) - I'_3(u, v)} \right] - \text{atan} \left[ \frac{I_4(u, v) - I_2(u, v)}{I_1(u, v) - I_3(u, v)} \right]$$

Fringe-locus function:

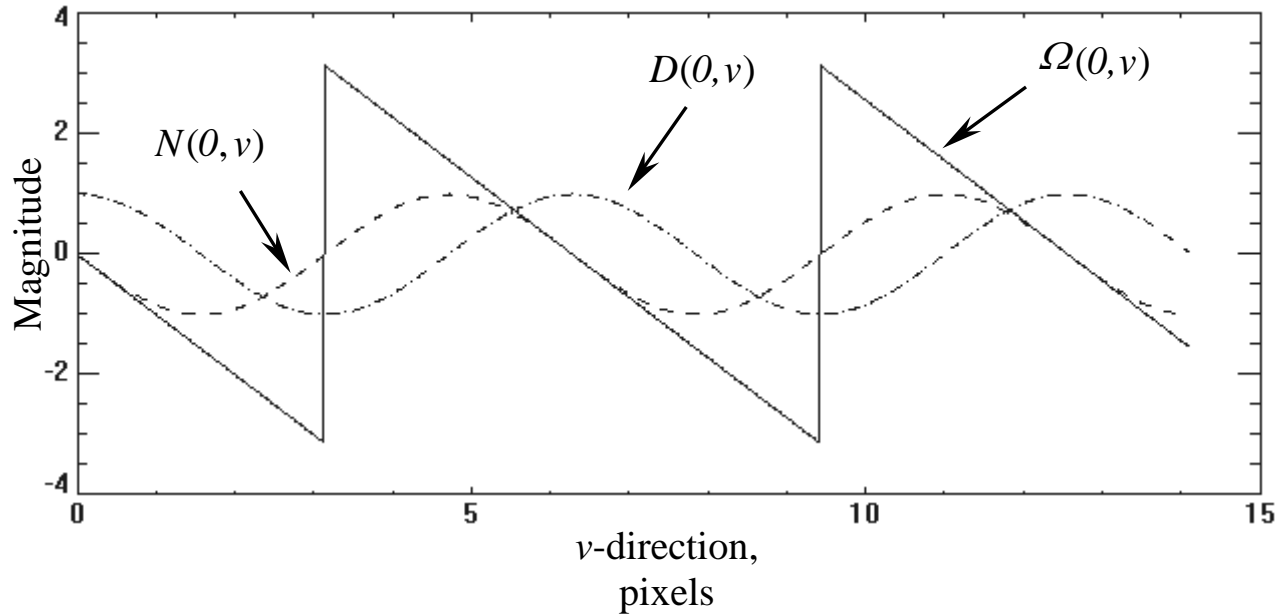
$$\Omega(u, v) = \text{atan} \left[ - \frac{(I_1 - I_3)(I'_2 - I'_4) - (I_2 - I_4)(I'_1 - I'_3)}{(I_1 - I_3)(I'_1 - I'_3) + (I_2 - I_4)(I'_2 - I'_4)} \right] = \text{atan} \left[ \frac{N(u, v)}{D(u, v)} \right]$$



# Recovered optical phase

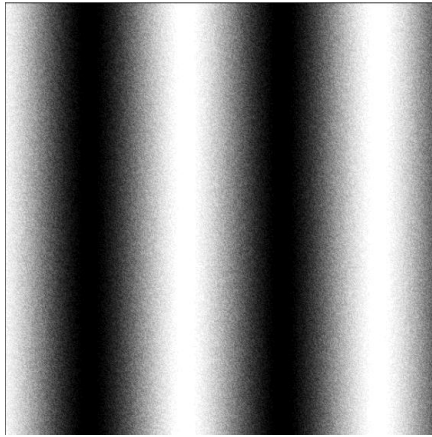
Fringe-locus function  $\text{mod}[2\pi]$

Recovered wrapped phase

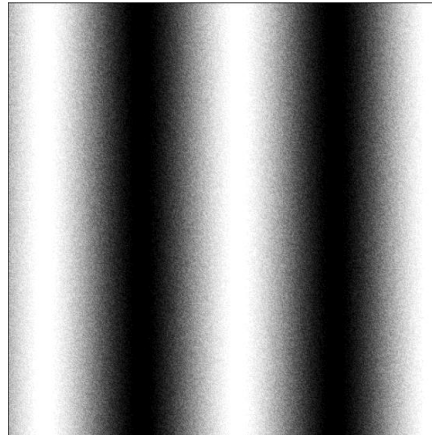


# Example: interferogram with open-fringes

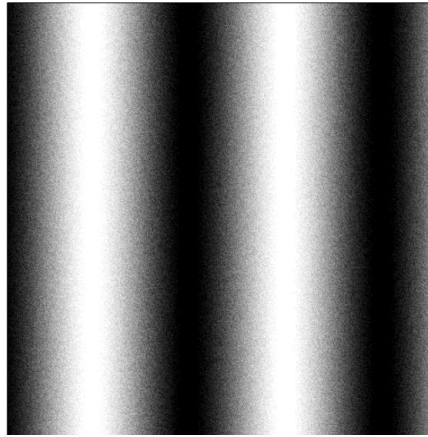
$I_1$  at  $\theta_1 = 0$



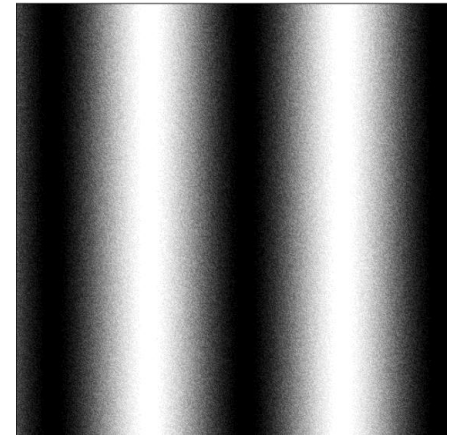
$I_2$  at  $\theta_2 = \frac{\pi}{2}$



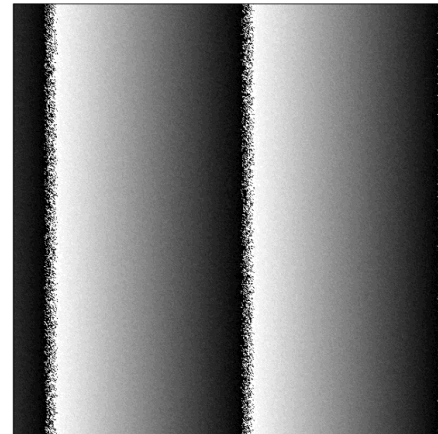
$I_3$  at  $\theta_3 = \pi$



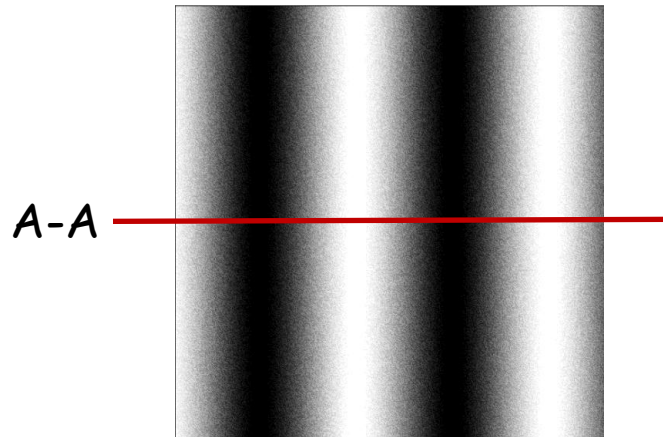
$I_4$  at  $\theta_4 = \frac{3\pi}{2}$



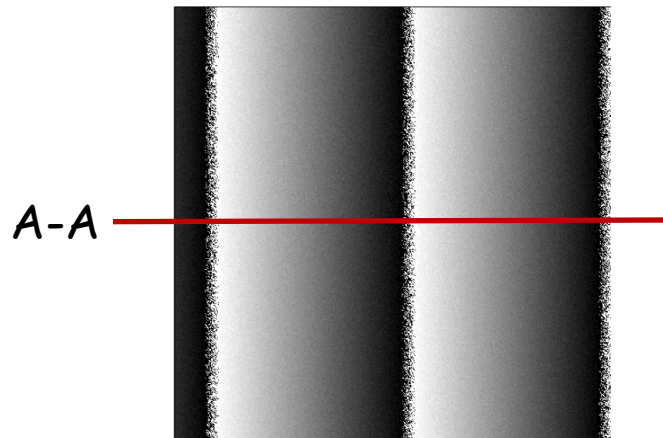
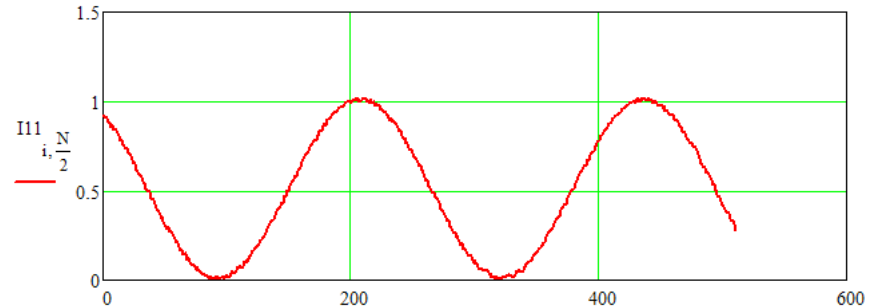
$$\Omega(u, v) = \text{atan} \left[ \frac{I_4(u, v) - I_2(u, v)}{I_1(u, v) - I_3(u, v)} \right], \text{ mod}[2\pi]$$



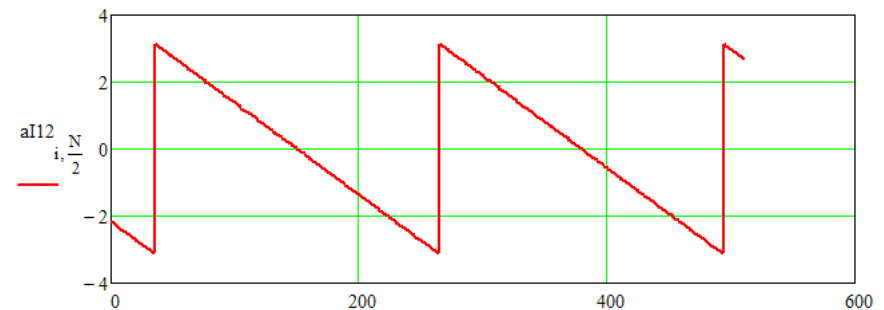
# Example: interferogram with open-fringes



Interferogram: A-A

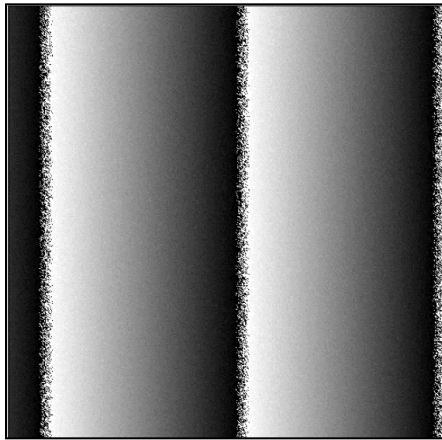


Recovered wrapped phase: A-A

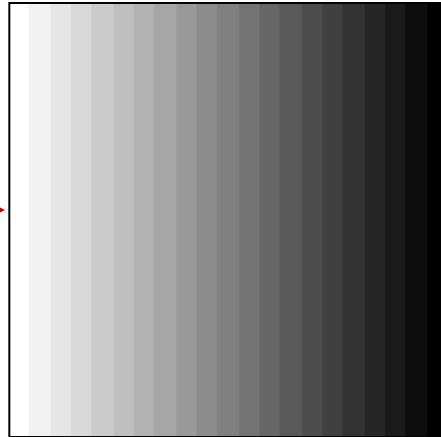


# Example: interferogram with open-fringes

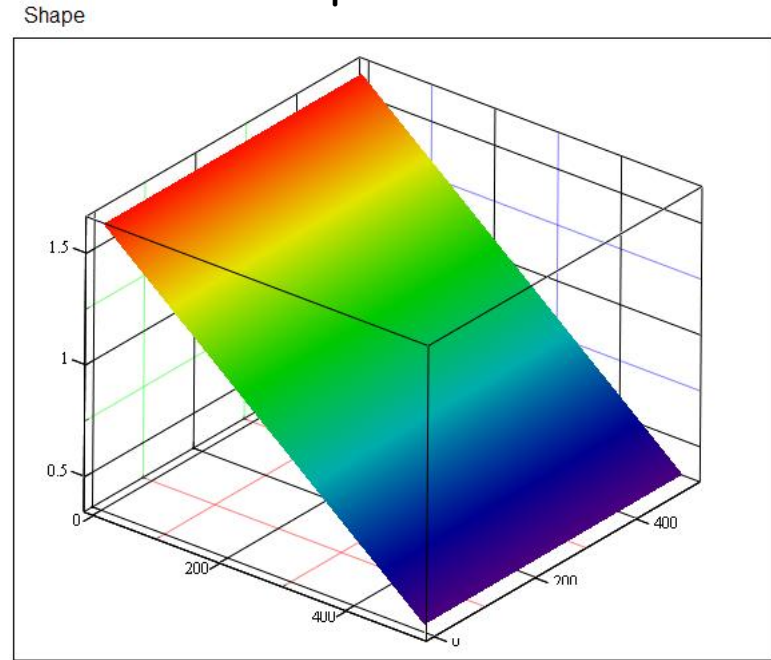
Wrapped phase



Unwrapped phase



Interpretation

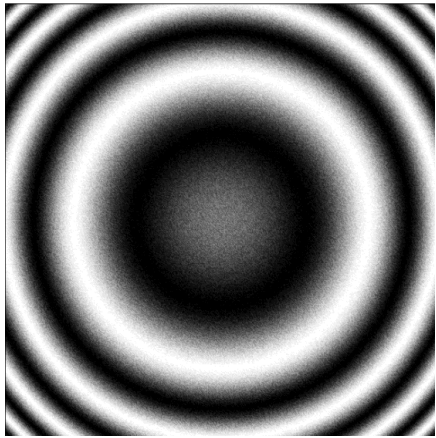


Need:

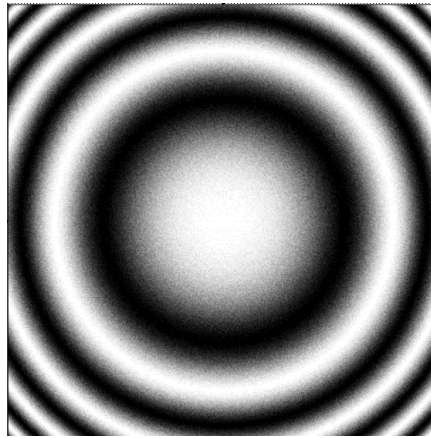
Camera calibration &  
Sensitivity Vector

# Example: interferogram with closed-fringes

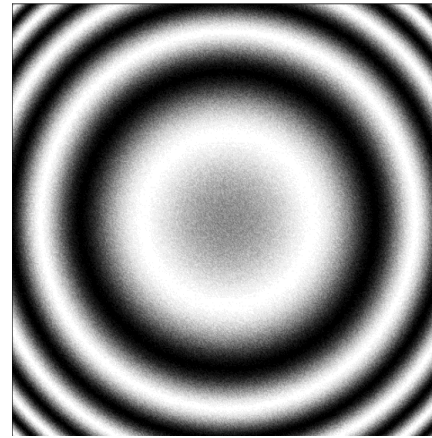
$I_1$  at  $\theta_1 = 0$



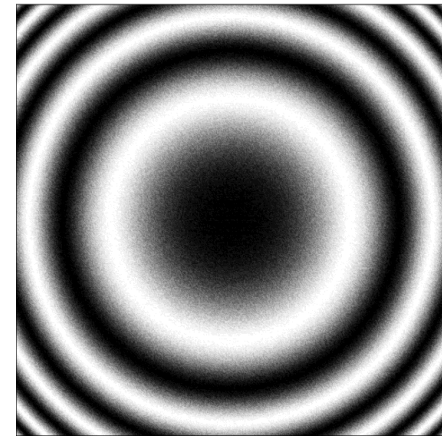
$I_2$  at  $\theta_2 = \frac{\pi}{2}$



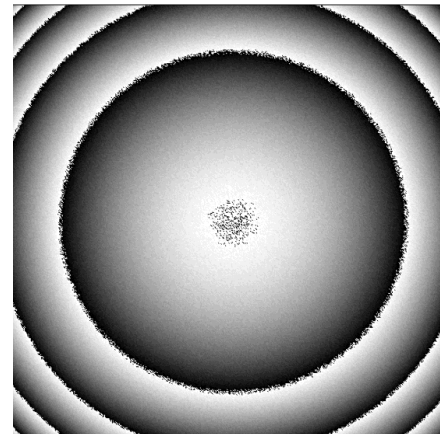
$I_3$  at  $\theta_3 = \pi$



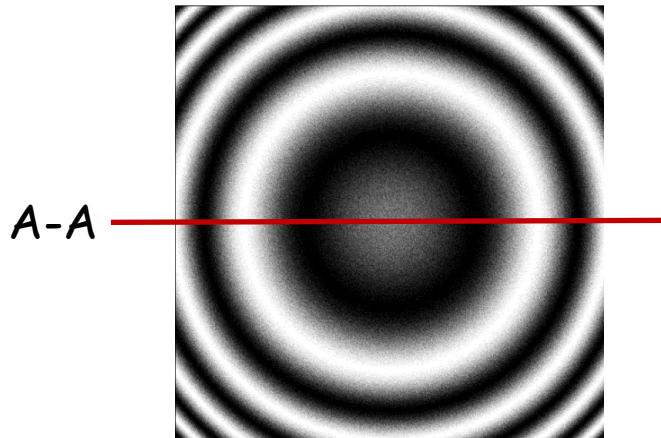
$I_4$  at  $\theta_4 = \frac{3\pi}{2}$



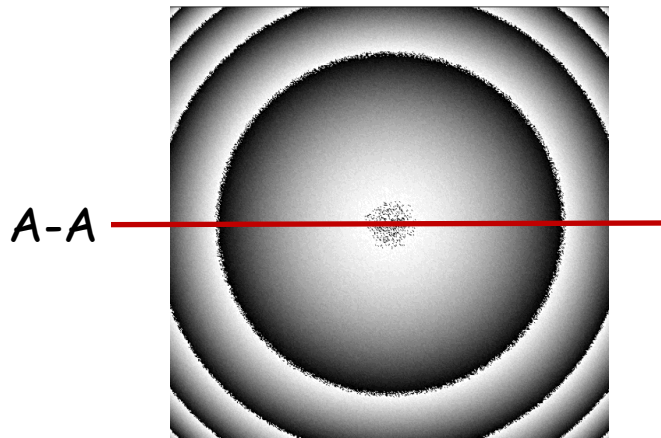
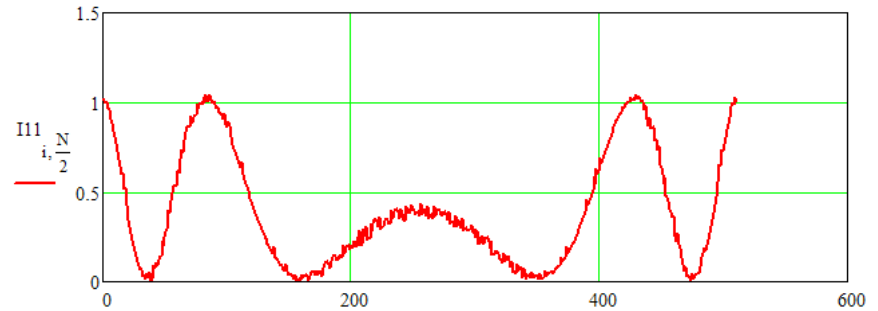
$$\Omega(u, v) = \text{atan} \left[ \frac{I_4(u, v) - I_2(u, v)}{I_1(u, v) - I_3(u, v)} \right], \text{ mod}[2\pi]$$



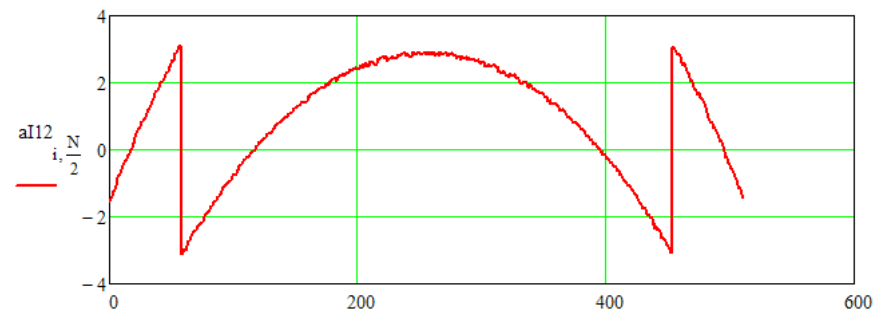
# Example: interferogram with closed-fringes



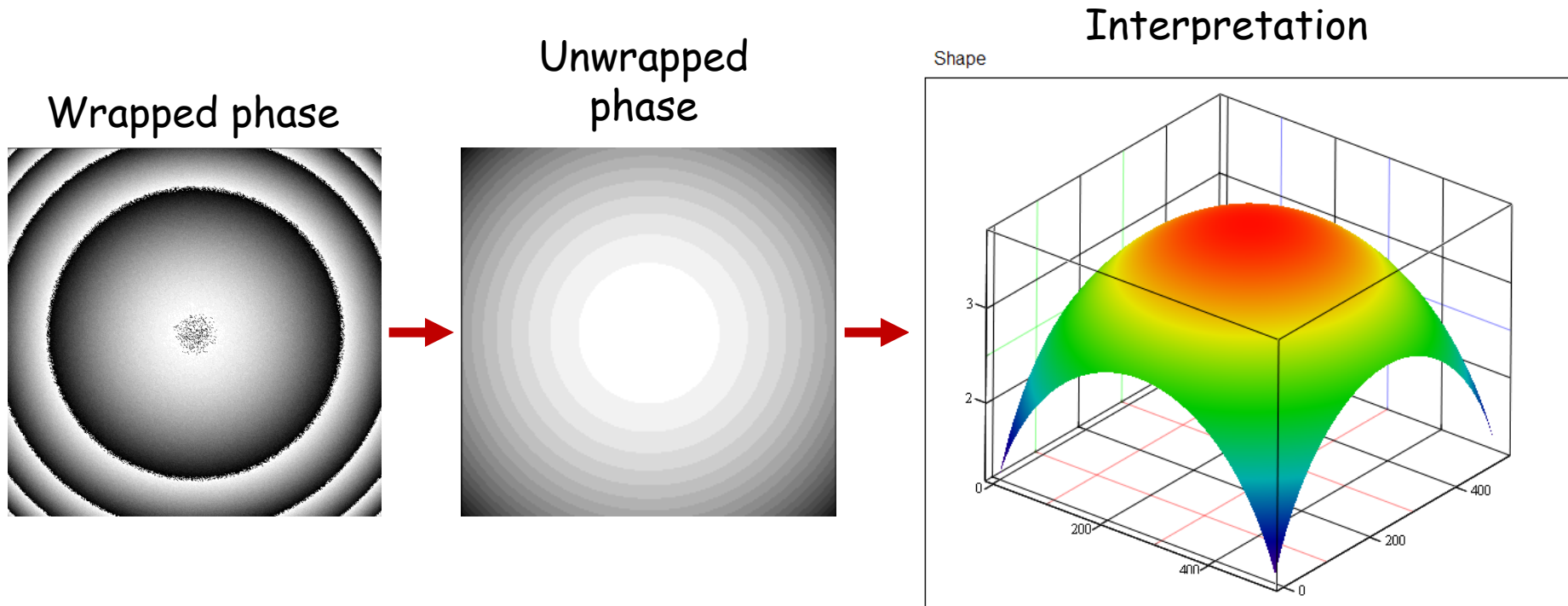
Interferogram: A-A



Recovered wrapped phase: A-A



# Example: interferogram with closed-fringes



Need:

Camera calibration &  
Sensitivity Vector