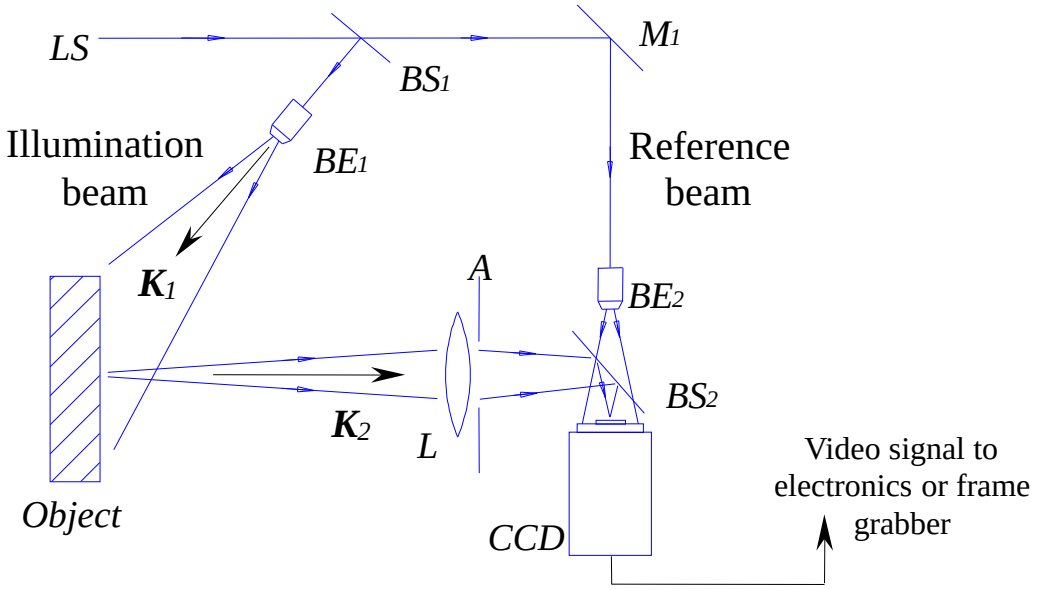
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

Optical Metrology and NDT
ME-593L, C'2018

Introduction: Phase Sampling
February 2018

Interference

Setup for recording of electronic interferograms



Intensity before deformations: $I(u,v) = I_B(u,v) + I_M(u,v)\cos[\Delta\phi(u,v)]$

Intensity after deformations: $I'(u,v) = I_B(u,v) + I_M(u,v)\cos[\Delta\phi(u,v) + \Omega(u,v)]$

Review, *in class*, unknowns and known variables

Interference

Phase sampling by phase shifting

Intensity of the form: $I(x,y) = I_B + I_M \cos(\Delta\phi)$

Known — I_B

Unknown — I_M

Unknown — $\cos(\Delta\phi)$

Unknown — $\Delta\phi$

Solution for the unknowns: introduction of optical phase shifts

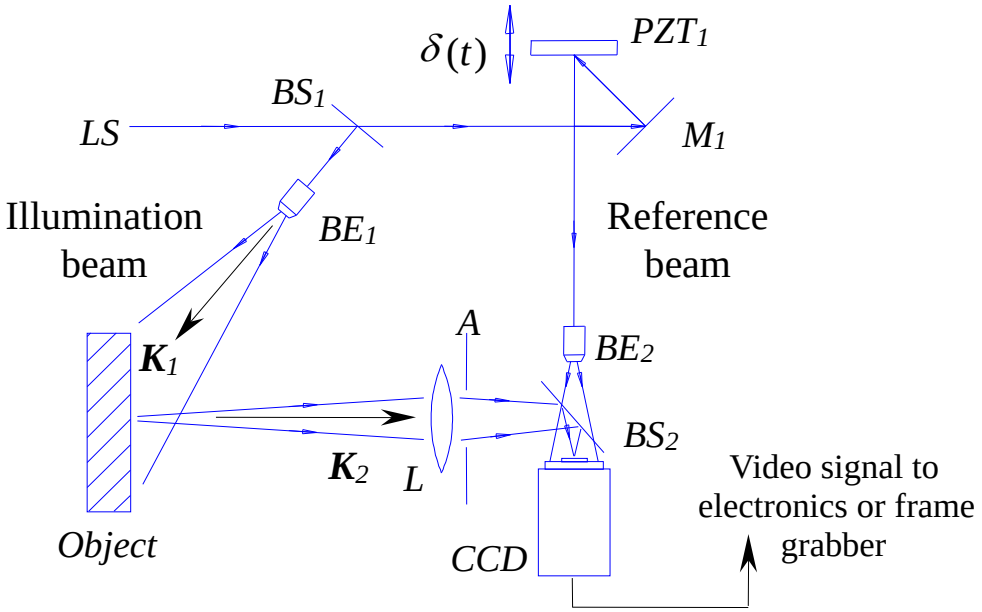
$$I_n(u,v) = I_B(u,v) + I_M(u,v)\cos\left[\Delta\phi(u,v) + \theta_n\right]$$

Number of phase shifts: $n = 1,2,\ldots N$

n-th phase shift: θ_n

Interference

Setup for recording of phase shifted electronic interferograms

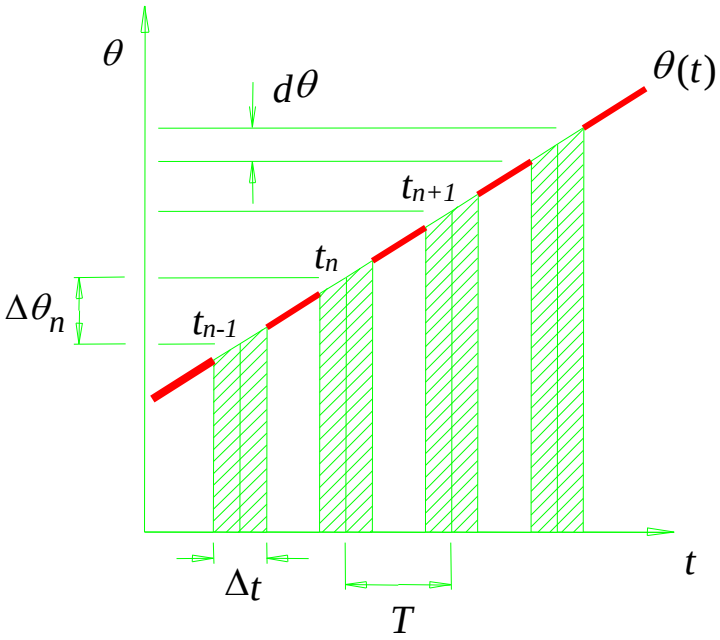


n-th phase shift: $\theta_n = \theta(t_n) = \frac{2\pi}{\lambda}\delta(t_n)$

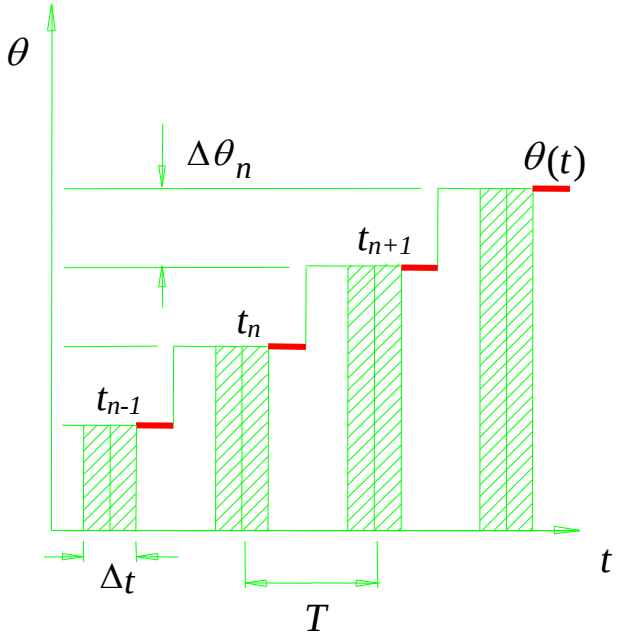
Phase shifting

Driving functions

Continuous phase shifting



Phase stepping



Phase shifting

Driving functions

Phase stepping is preferred:

- Minimum phase drifting during image acquisition
- Simple to implement
- Require detailed calibration, for the wavelength used

Phase sampling equations

Least-squares minimization of residuals, R_d

Residuals: $$R_d = I_B(u,v) + I_M(u,v)\cos\left[\Delta\phi(u,v) + \theta_n\right] - I_n(u,v)$$

Summation of residuals: $$R_d = \sum_{n=1}^{N}\left[I_B(u,v) + A\cos(\theta_n) + B\sin(\theta_n) - I_n(u,v)\right]$$

with $$A = I_M(u,v)\cos\left[\Delta\phi(u,v)\right],\ B = -I_M(u,v)\sin\left[\Delta\phi(u,v)\right]$$

Phase sampling equations

Least-squares minimization of residuals, R_d

Minimization of residuals :

$$\frac{\partial}{\partial I_B}\left\{\sum_{n=1}^{N}\left[I_B(u,v)+A\cos(\theta_n)+B\sin(\theta_n)-I_n(u,v)\right]^2\right\}=0$$

$$\frac{\partial}{\partial A}\left\{\sum_{n=1}^{N}\left[I_B(u,v)+A\cos(\theta_n)+B\sin(\theta_n)-I_n(u,v)\right]^2\right\}=0$$

$$\frac{\partial}{\partial B}\left\{\sum_{n=1}^{N}\left[I_B(u,v)+A\cos(\theta_n)+B\sin(\theta_n)-I_n(u,v)\right]^2\right\}=0$$

Phase sampling equations

Least-squares minimization of residuals, R_d

Resultant system of equations :

$$\begin{bmatrix} N & \sum_{n=1}^{N}\cos(\theta_n) & \sum_{n=1}^{N}\sin(\theta_n) \\ \sum_{n=1}^{N}\cos(\theta_n) & \sum_{n=1}^{N}\cos^2(\theta_n) & \sum_{n=1}^{N}\sin(\theta_n)\cos(\theta_n) \\ \sum_{n=1}^{N}\sin(\theta_n) & \sum_{n=1}^{N}\sin(\theta_n)\cos(\theta_n) & \sum_{n=1}^{N}\sin^2(\theta_n) \end{bmatrix} \begin{pmatrix} I_B \\ A \\ B \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^{N} I_n \\ \sum_{n=1}^{N} I_n\cos(\theta_n) \\ \sum_{n=1}^{N} I_n\sin(\theta_n) \end{pmatrix}$$

with $A = I_M(u,v)\cos\left[\Delta\phi(u,v)\right]$, $B = -I_M(u,v)\sin\left[\Delta\phi(u,v)\right]$

Phase sampling equations

Four phase shifting algorithm, $n = 4$

n-th phase shifts: $\theta_n = 0, \pi/2, \pi, 3\pi/2$

Resultant system of equations, $n = 4$:

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{pmatrix} I_B \\ A \\ B \end{pmatrix} = \begin{pmatrix} I_1 + I_2 + I_3 + I_4 \\ I_1 - I_3 \\ I_2 - I_4 \end{pmatrix}$$

Phase sampling equations

Four phase shifting algorithm, $n = 4$

Resultant background intensity:

$$I_B(u,v) = \frac{1}{4}[I_1(u,v) + I_2(u,v) + I_3(u,v) + I_4(u,v)]$$

Resultant modulation intensity:

$$I_M(u,v) = \sqrt{[A(u,v)]^2 + [B(u,v)]^2}$$

$$= \frac{1}{2}\sqrt{[I_1(u,v) - I_3(u,v)]^2 + [I_2(u,v) - I_4(u,v)]^2}$$

Phase sampling equations

Four phase shifting algorithm, $n = 4$

Fringe visibility, or fringe contrast:

$$V(u,v) = \frac{I_M(u,v)}{I_B(u,v)}$$

Optical phase:

$$\Delta\phi(u,v) = \operatorname{atan}\left[-\frac{B(u,v)}{A(u,v)}\right] = \operatorname{atan}\left[\frac{\sin[\Delta\phi(u,v)]}{\cos[\Delta\phi(u,v)]}\right] = \operatorname{atan}\left[\frac{I_4(u,v) - I_2(u,v)}{I_1(u,v) - I_3(u,v)}\right]$$

Recovered optical phase

Before and after deformations or changes in OPL

Before deformations:

$$\Delta\phi(u,v) = \operatorname{atan}\left[-\frac{B(u,v)}{A(u,v)}\right] = \operatorname{atan}\left[\frac{\sin[\Delta\phi(u,v)]}{\cos[\Delta\phi(u,v)]}\right] = \operatorname{atan}\left[\frac{I_4(u,v)-I_2(u,v)}{I_1(u,v)-I_3(u,v)}\right]$$

After deformations:

$$\Delta\phi(u,v)+\Omega(u,v) = \operatorname{atan}\left[\frac{\sin[\Delta\phi(u,v)+\Omega(u,v)]}{\cos[\Delta\phi(u,v)+\Omega(u,v)]}\right] = \operatorname{atan}\left[\frac{I'_4(u,v)-I'_2(u,v)}{I'_1(u,v)-I'_3(u,v)}\right]$$

Recovered optical phase

Fringe-locus function

Phase difference:

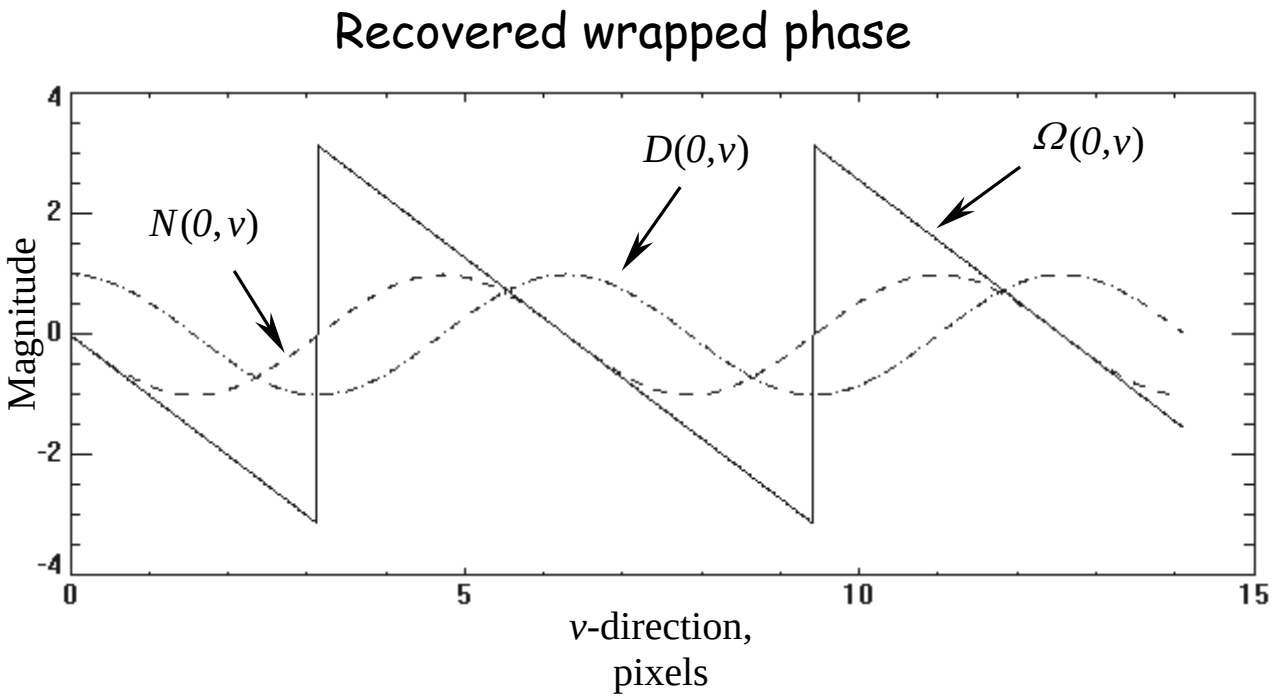
$$[\Delta\phi(u,v)+\Omega(u,v)]-\Delta\phi(u,v)=\operatorname{atan}\left[\frac{I'_4(u,v)-I'_2(u,v)}{I'_1(u,v)-I'_3(u,v)}\right]-\operatorname{atan}\left[\frac{I_4(u,v)-I_2(u,v)}{I_1(u,v)-I_3(u,v)}\right]$$

Fringe-locus function:

$$\Omega(u,v)=\operatorname{atan}\left[-\frac{(I_1-I_3)(I'_2-I'_4)-(I_2-I_4)(I'_1-I'_3)}{(I_1-I_3)(I'_1-I'_3)+(I_2-I_4)(I'_2-I'_4)}\right]=\operatorname{atan}\left[\frac{N(u,v)}{D(u,v)}\right]$$

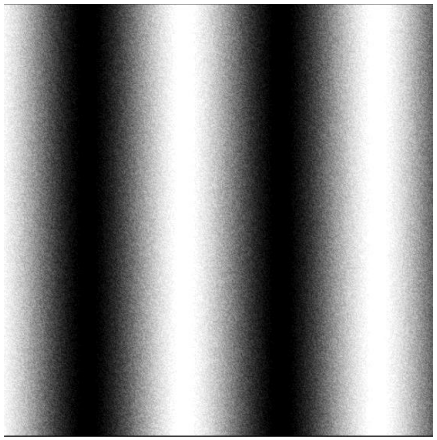
Recovered optical phase

Fringe-locus function mod[2π]

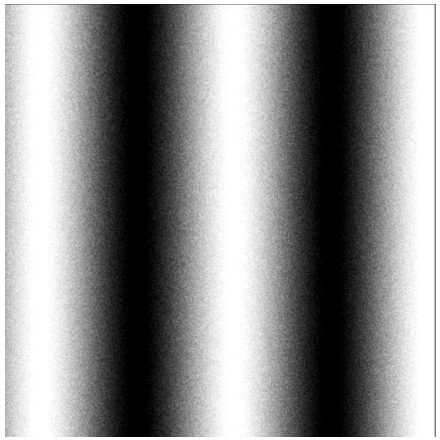


Example: interferogram with open-fringes

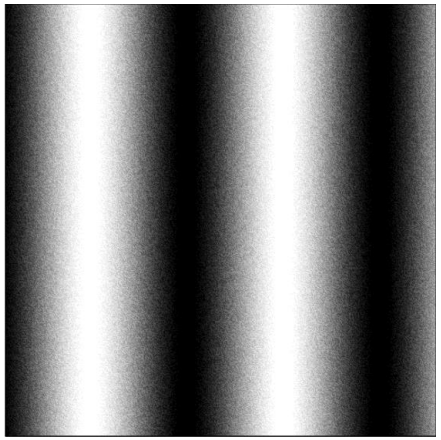
I_1 at $\theta_1 = 0$



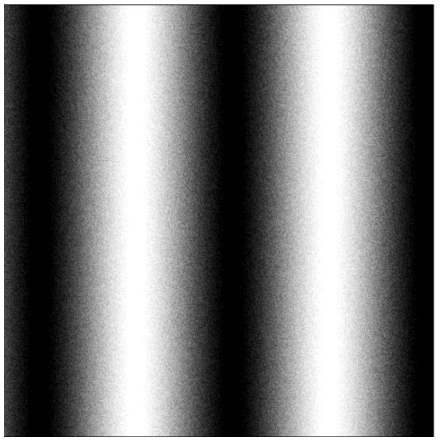
I_2 at $\theta_2 = \frac{\pi}{2}$



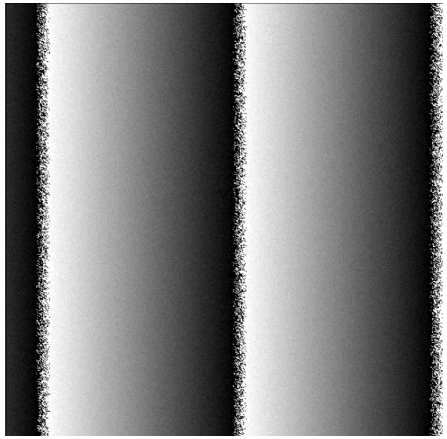
I_3 at $\theta_3 = \pi$



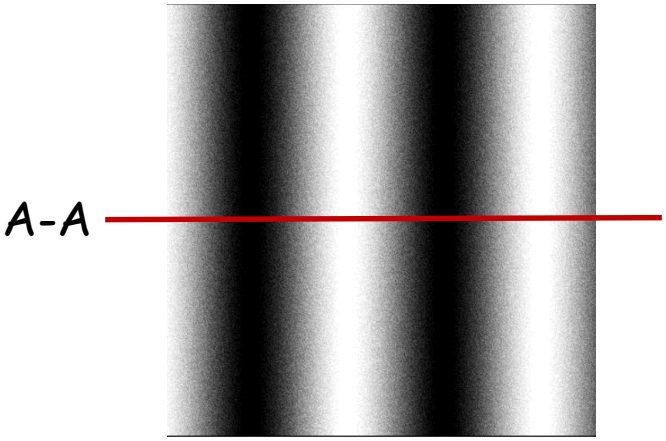
I_4 at $\theta_4 = \frac{3\pi}{2}$



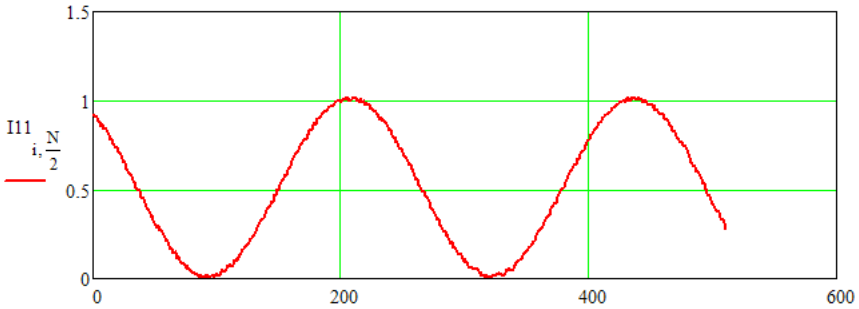
$$\Omega(u,v) = \text{atan}\left[\frac{I_4(u,v) - I_2(u,v)}{I_1(u,v) - I_3(u,v)}\right], \ \text{mod}[2\pi]$$

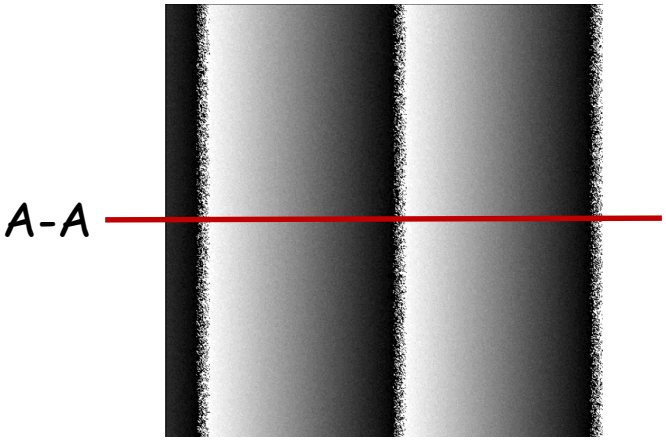


Example: interferogram with open-fringes

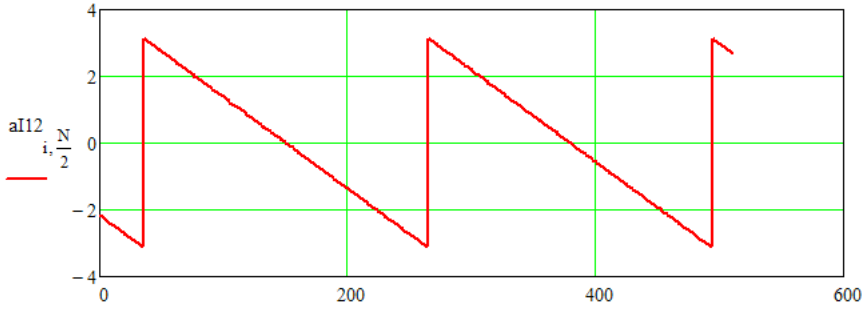


Interferogram: A-A



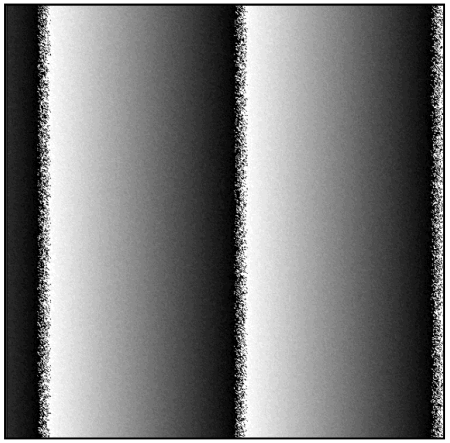


Recovered wrapped phase: A-A

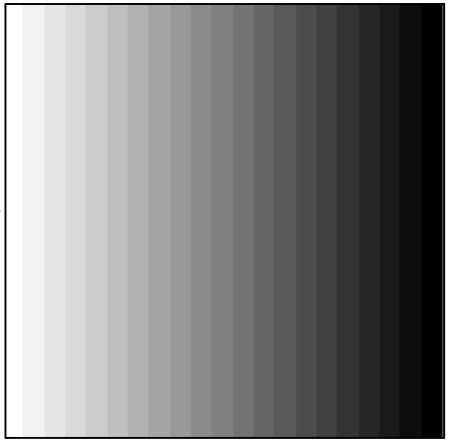


Example: interferogram with open-fringes

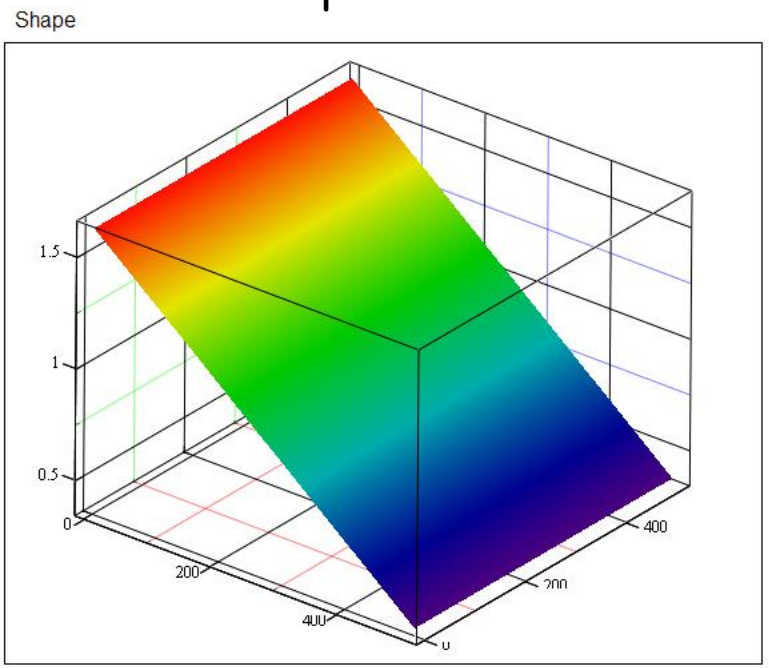
Wrapped phase



Unwrapped phase



Interpretation



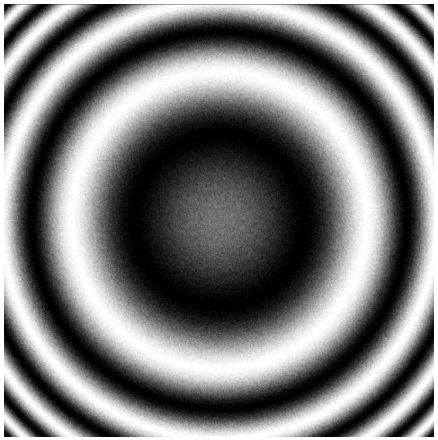
Camera calibration & Sensitivity Vectors, and

next, relate measurements to physical parameters of interest

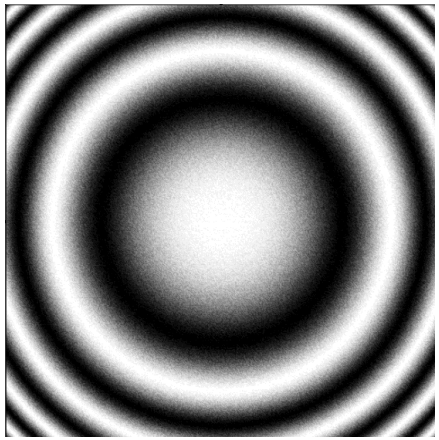
Now, to fully quantify we need:

Example: interferogram with closed-fringes

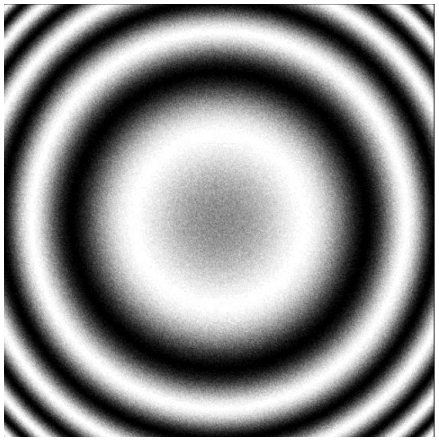
I_1 at $\theta_1 = 0$



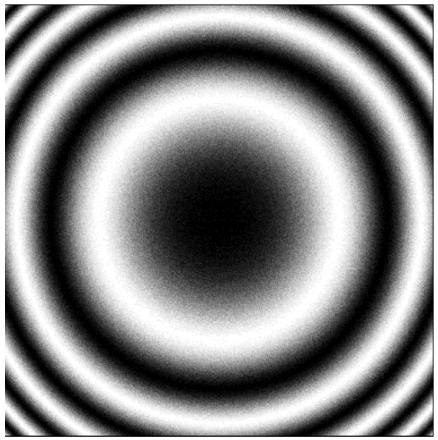
I_2 at $\theta_2 = \frac{\pi}{2}$



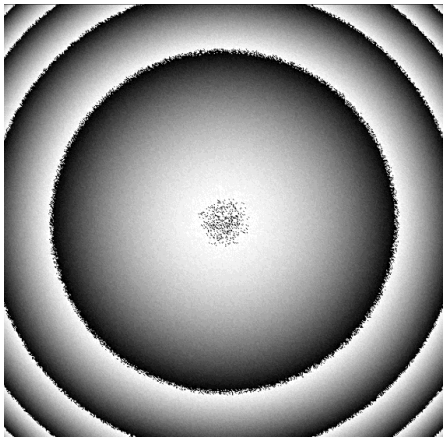
I_3 at $\theta_3 = \pi$



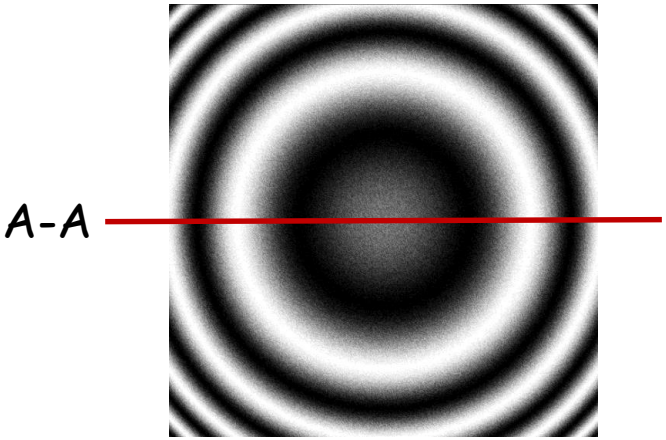
I_4 at $\theta_4 = \frac{3\pi}{2}$



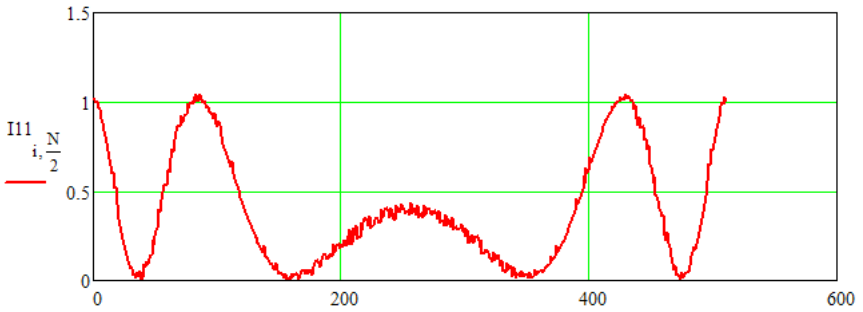
$$\Omega(u,v) = \text{atan}\left[\frac{I_4(u,v) - I_2(u,v)}{I_1(u,v) - I_3(u,v)}\right], \ \text{mod}[2\pi]$$

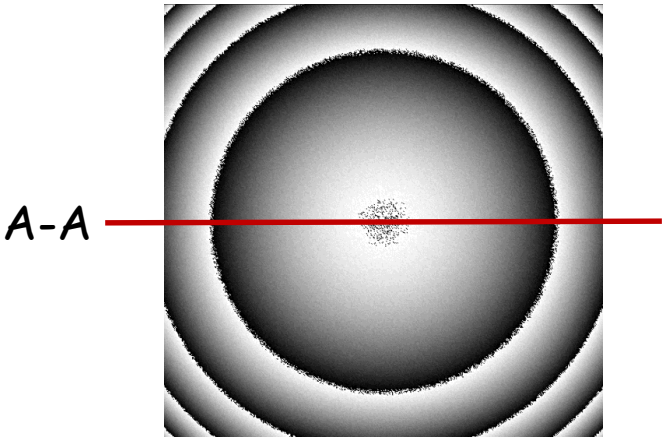


Example: interferogram with closed-fringes

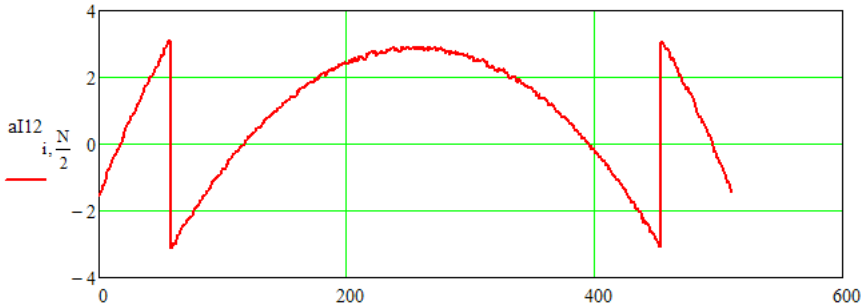


Interferogram: A-A



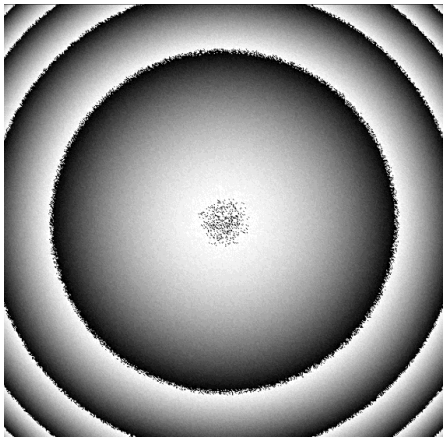


Recovered wrapped phase: A-A

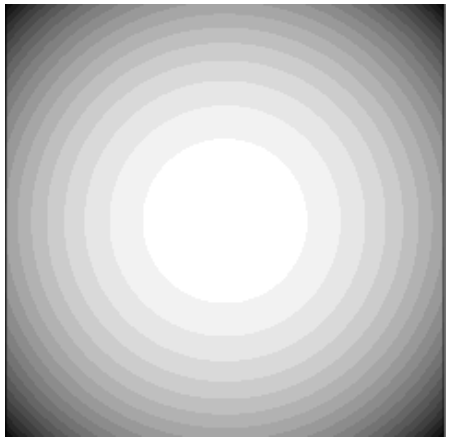


Example: interferogram with closed-fringes

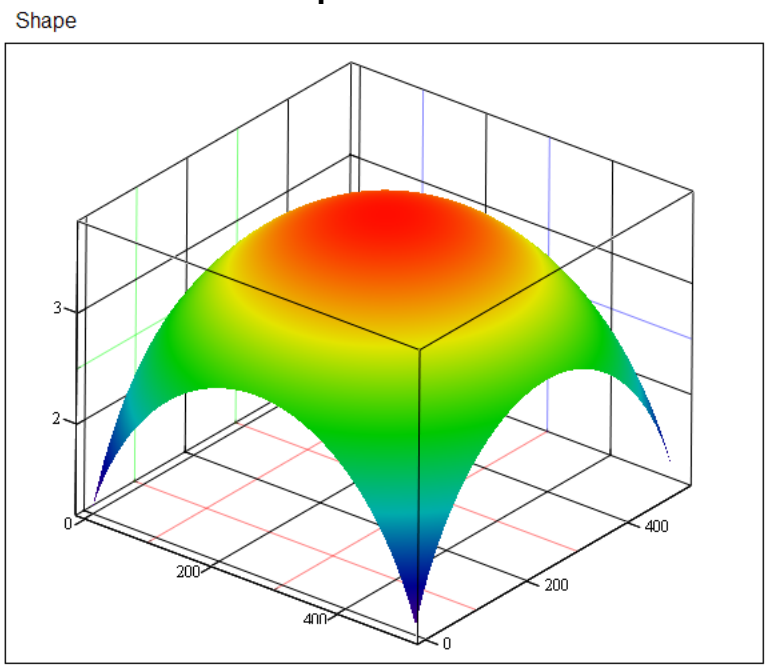
Wrapped phase



Unwrapped phase



Interpretation



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