

WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Optical Metrology and NDT
ME-593L, B'2011

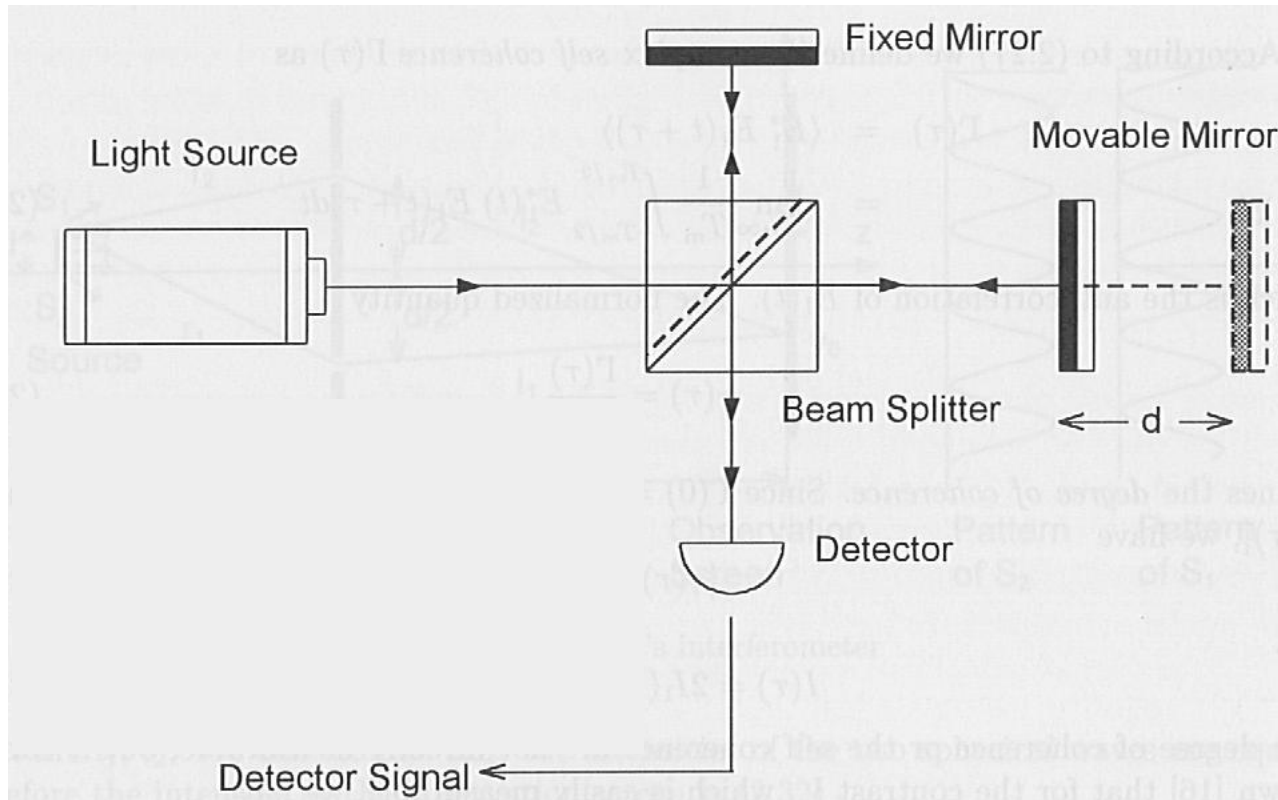
Introduction: Wave Optics
09 November 2011



Wave optics: coherence

Temporal coherence

- Review interference equation: [Lecture 04](#)
- Michelson interferometer to test for coherence length (temporal)

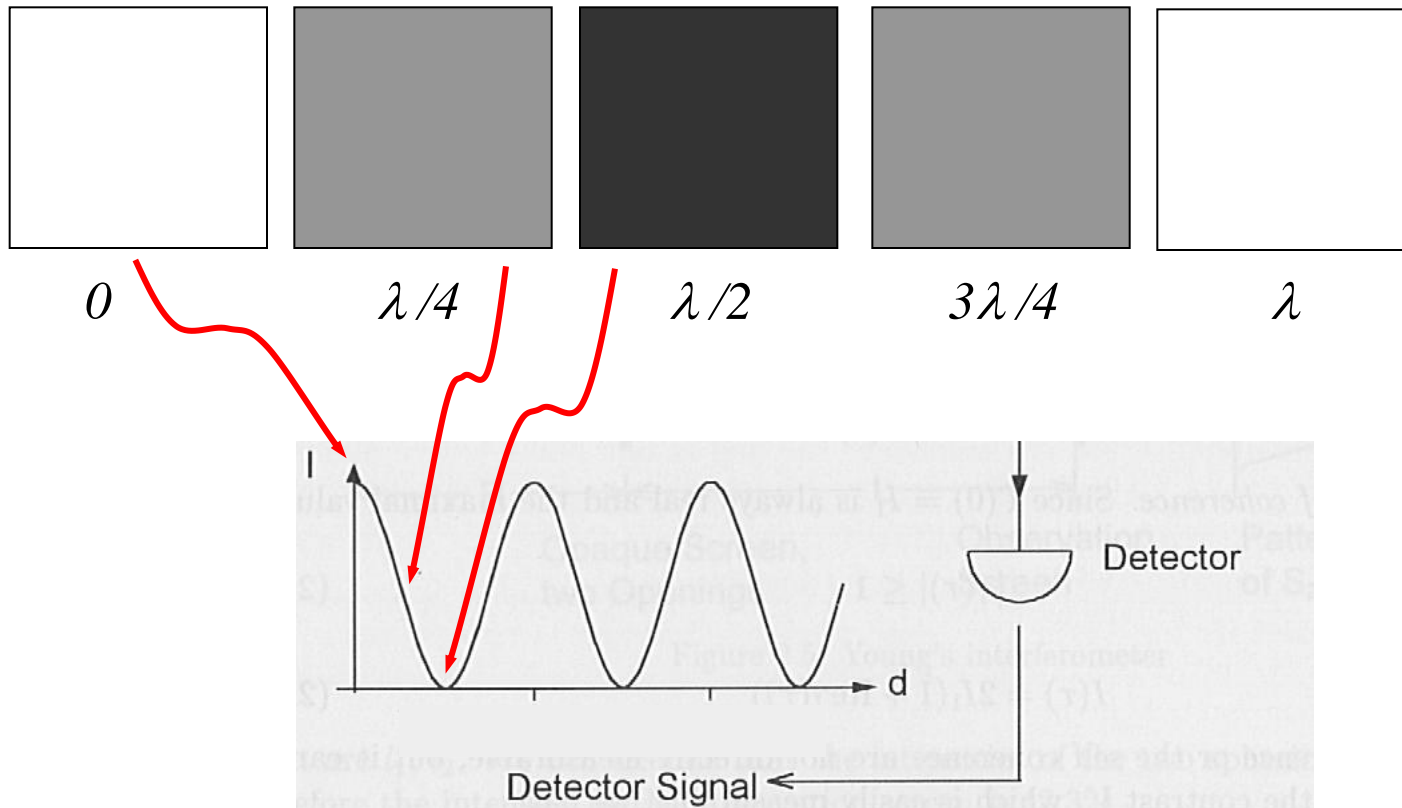


Wave optics: coherence

Temporal coherence

- Assume 100% flat and orthogonal mirrors

Detected intensities as a function of position of movable mirror



Wave optics: coherence

Temporal coherence

- Temporal coherence describes the self-correlation of a wave. Consider the normalized autocorrelation function

$$g(\tau) = \frac{G(\tau)}{G(0)} = \frac{\langle U^*(t) U(t + \tau) \rangle}{\langle U^*(t) U(t) \rangle}, \quad (1)$$

where τ is the time delay, related optical path length changes introduced by the movable mirror.

Equation 1 is called the **degree of temporal coherence** with

$$0 \leq |g(\tau)| \leq 1. \quad (2)$$

Usually, $|g(\tau)|$ drops from its largest values $|g(\tau)| = 1$ as τ increases and the fluctuations become uncorrelated for sufficiently large time delay τ .

Wave optics: coherence

Temporal coherence

- Coherence length is defined as

$$l_c = c \tau_c , \quad (3)$$

where c is the speed of light and τ_c is the power equivalent width of the function $|g(\tau)|$ given as

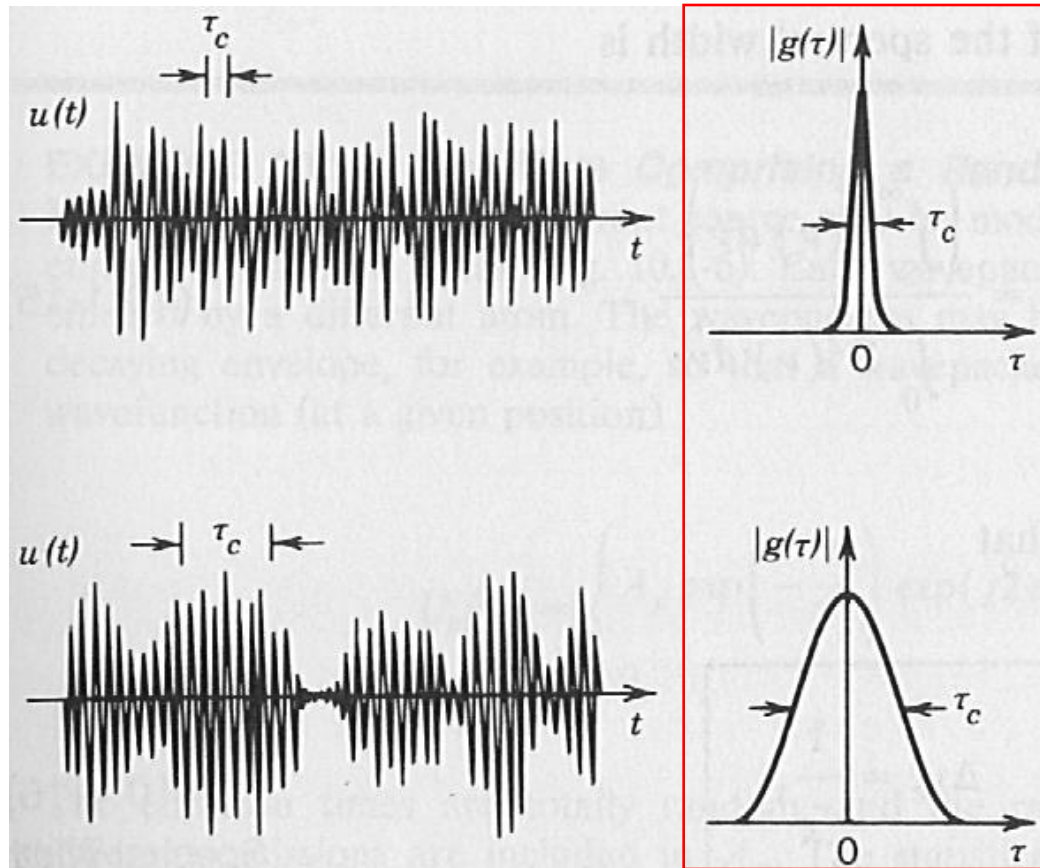
$$\tau_c = \int_{-\infty}^{\infty} |g(\tau)|^2 d\tau . \quad (4)$$



Wave optics: coherence

Temporal coherence

Magnitude of the degree of temporal coherence $|g(\tau)|$



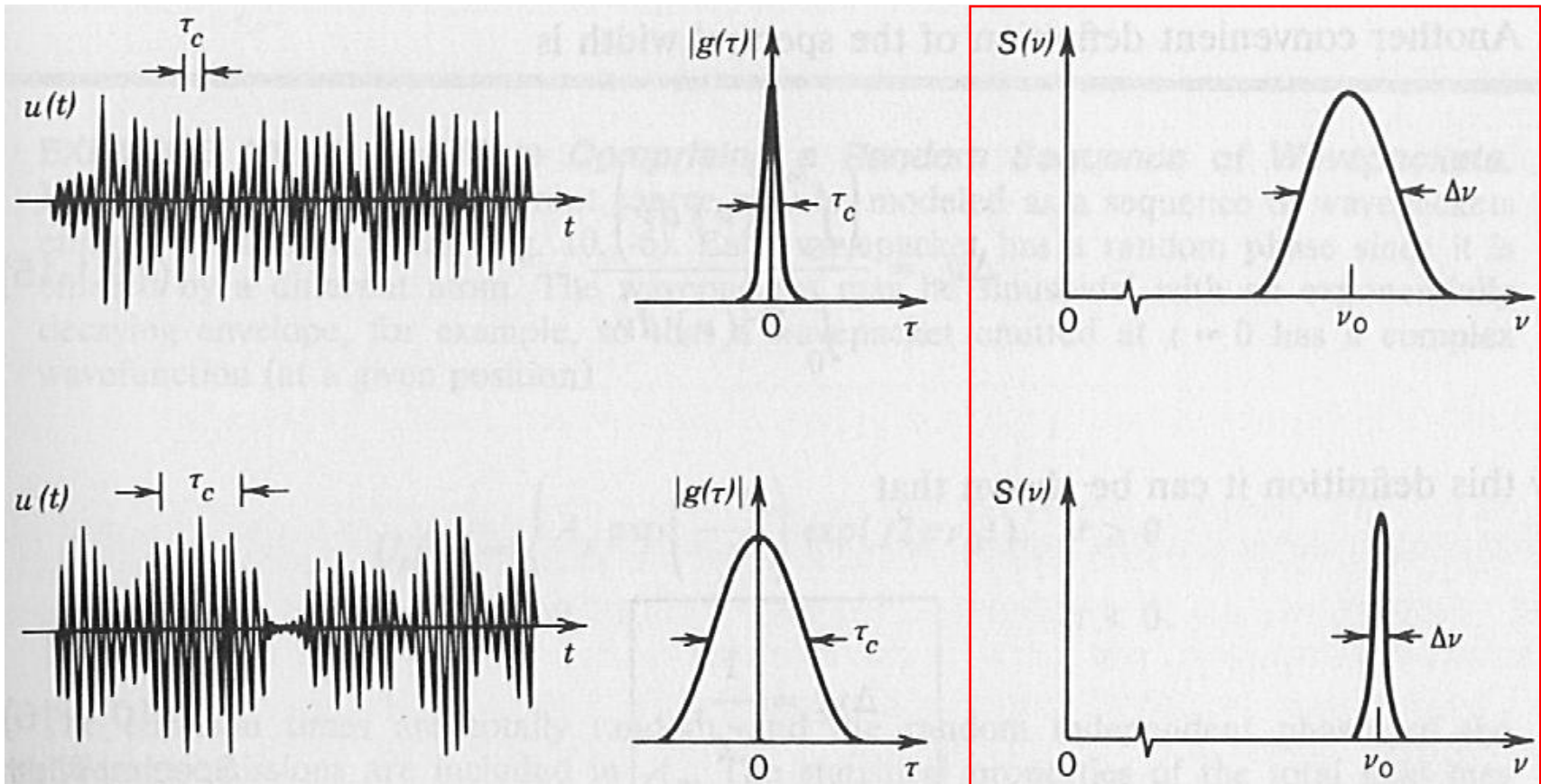
Short
coherence
time

Long
coherence
time

Wave optics: coherence

Temporal coherence

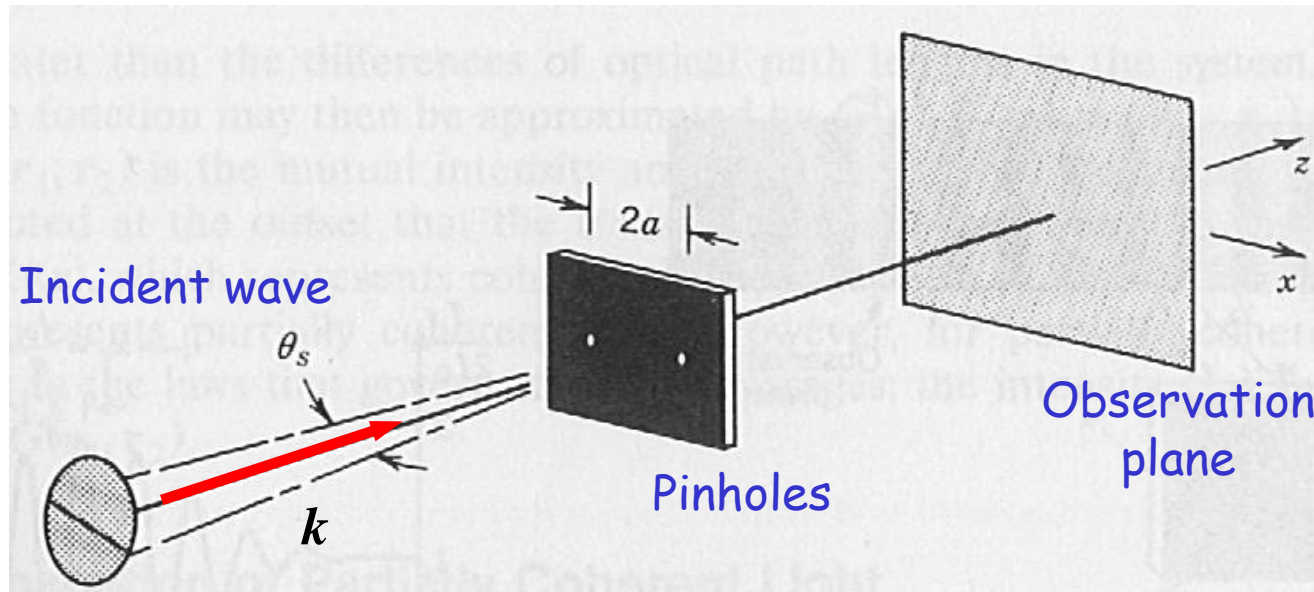
Power spectral density:
$$S(\nu) = \int_{-\infty}^{\infty} g(\tau) \exp(-j2\pi\nu\tau) d\tau$$



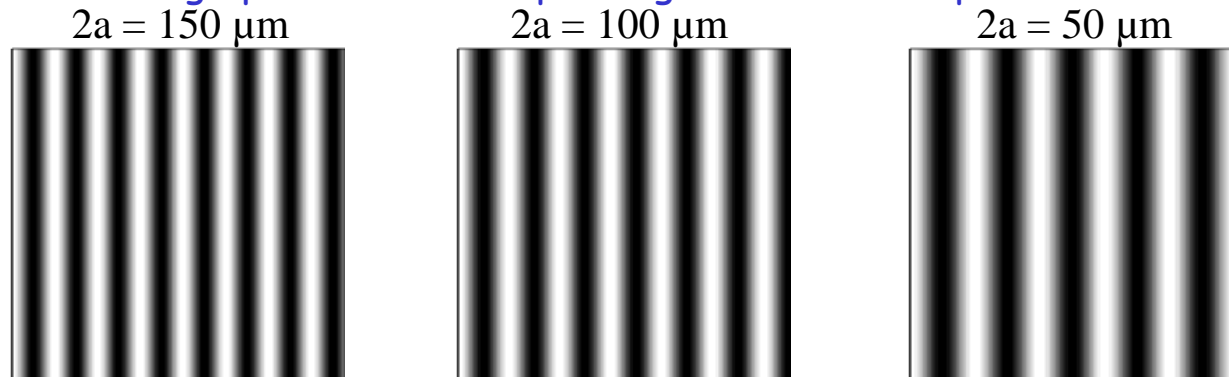
Wave optics: coherence

Spatial coherence

Young's double pinhole interferometer



Predicted fringe patterns corresponding to difference pinholes distances



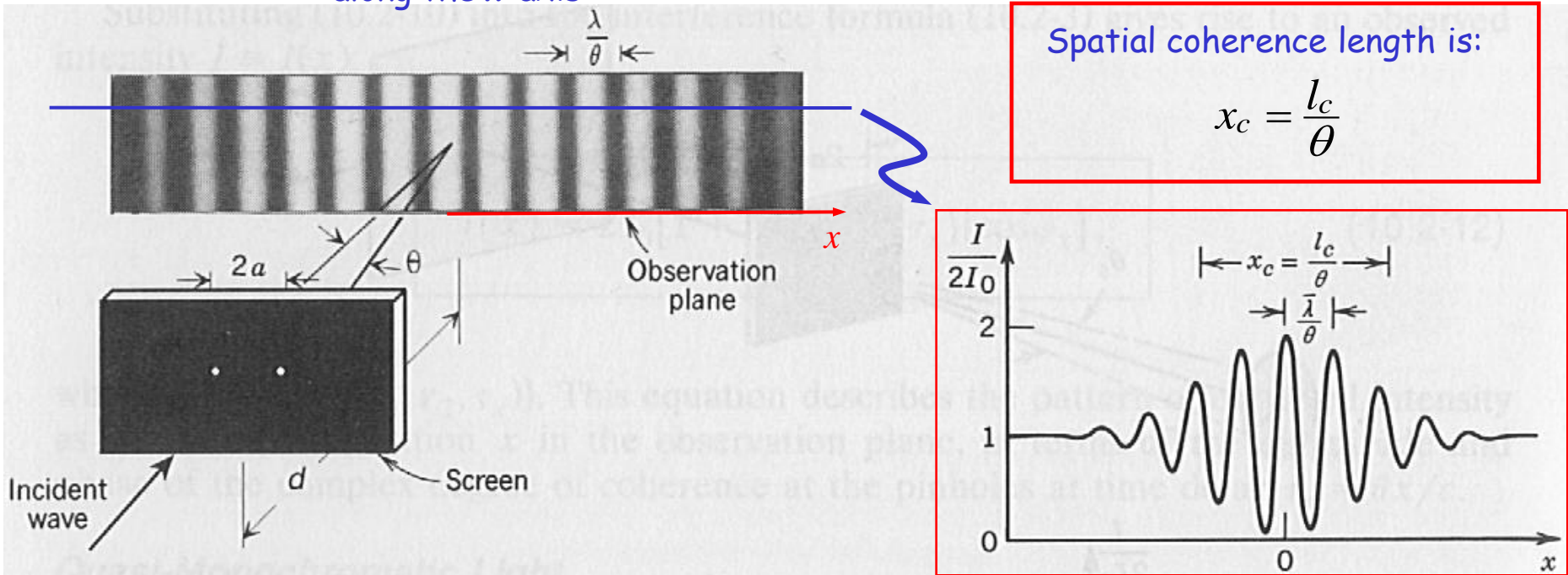
Mechanical Engineering Department/NEST - NanoEngineering, Science, and Technology

CHSLT - Center for Holographic Studies and Laser micro-mechanics

Wave optics: coherence

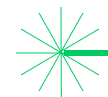
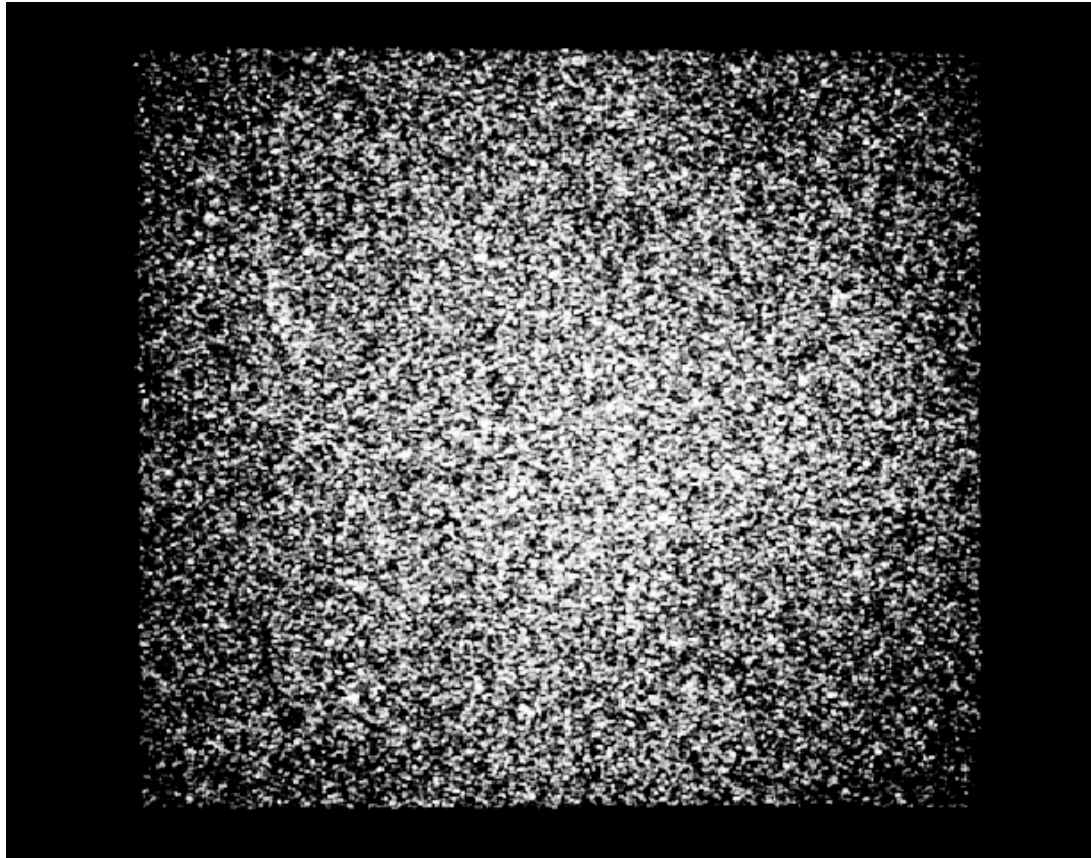
Spatial coherence

Actual fringe pattern: note variation of intensity along the x -axis



Wave optics: light scattering

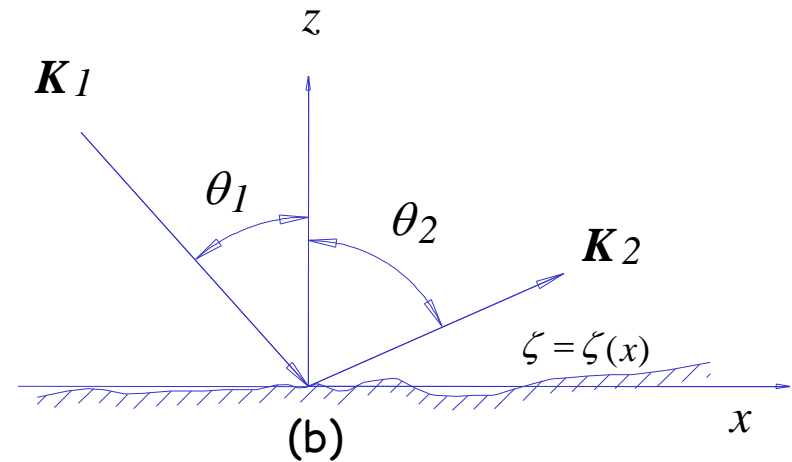
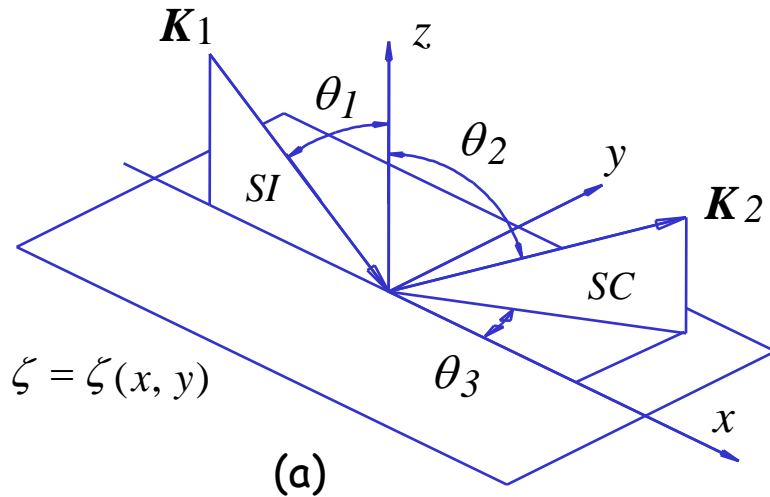
Speckle pattern observed on the flat surface of an object illuminated with a coherent light source with characteristic wavelength $\lambda = 994.1$ nm and horizontally polarized. Average surface roughness is $5 \mu\text{m}$, object distance is 30 mm, and f/2.8 aperture is used. The surface is illuminated and observed normal to the plane where the surface lies.



Wave optics: light scattering

Illumination and scattering geometry from surfaces defined as $\zeta = \zeta(x, y)$ and $\zeta = \zeta(x)$.

Vectors of illumination and scattering: $\mathbf{K}_1 = k \hat{\mathbf{K}}_1$, $\mathbf{K}_2 = k \hat{\mathbf{K}}_2$



Wave optics: light scattering

Scattering from a smooth surface

- The complex amplitude of the scattered light, F_o , at point p , can be predicted using the Kirchoff integral theorem

$$F_o = \frac{1}{4\pi} \left\{ \iint_s \frac{1}{r} \exp(-jkr) \nabla U \cdot d\mathbf{S} - \iint_s U \nabla \left[\frac{1}{r} \exp(-jkr) \right] \cdot d\mathbf{S} \right\} \quad (5)$$

which is derived from the Helmholtz equation.

- For a smooth surface, $\zeta(x) \approx 0$, extending from $[-L, L]$:

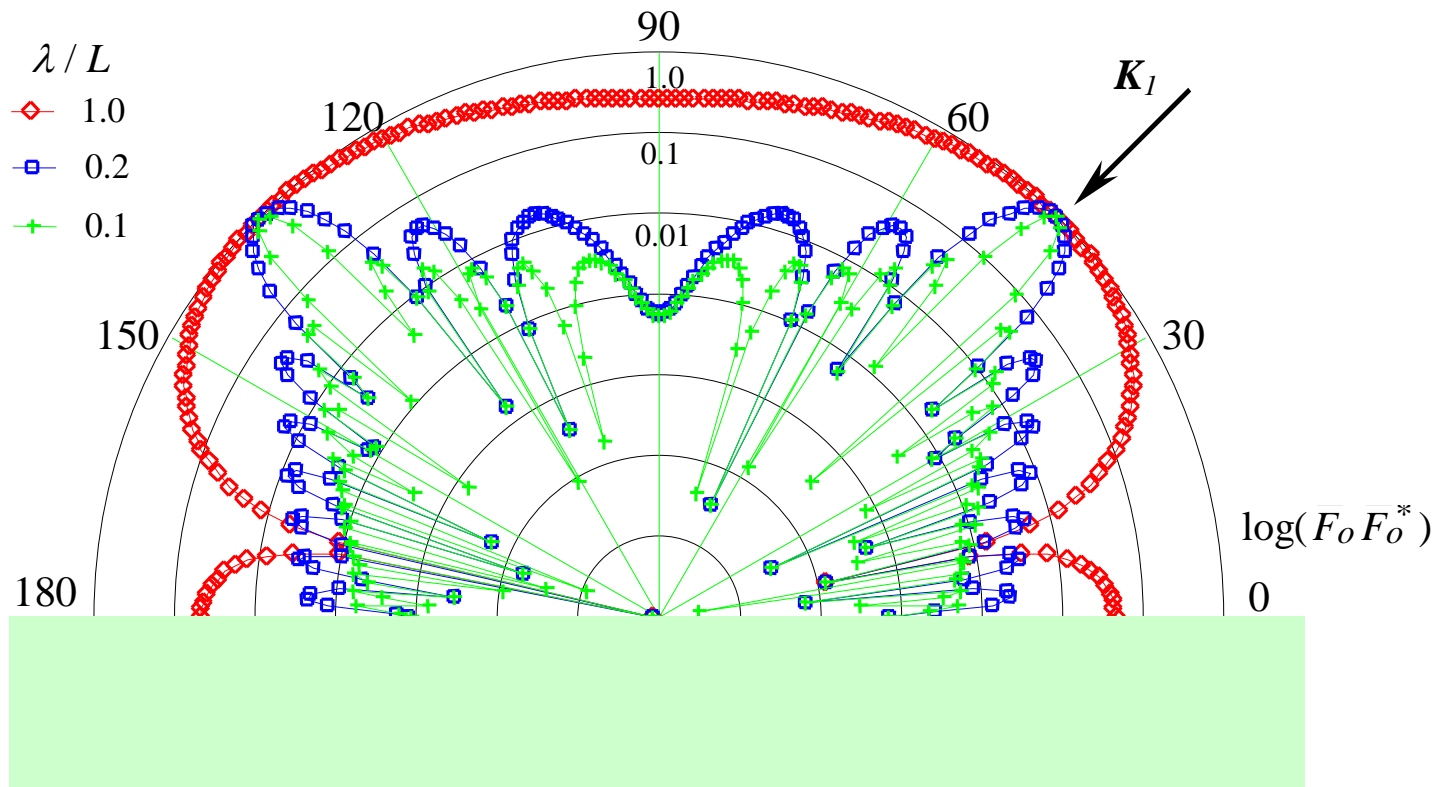
$$\begin{aligned} F_o = F_o(\theta_1, \theta_2) &= \frac{\sin[kL(\sin \theta_1 - \sin \theta_2)]}{kL(\sin \theta_1 - \sin \theta_2)} \quad (6) \\ &= \text{sinc} \left[\frac{2\pi}{\lambda} L(\sin \theta_1 - \sin \theta_2) \right] \end{aligned}$$



Wave optics: light scattering

Scattering from a smooth surface

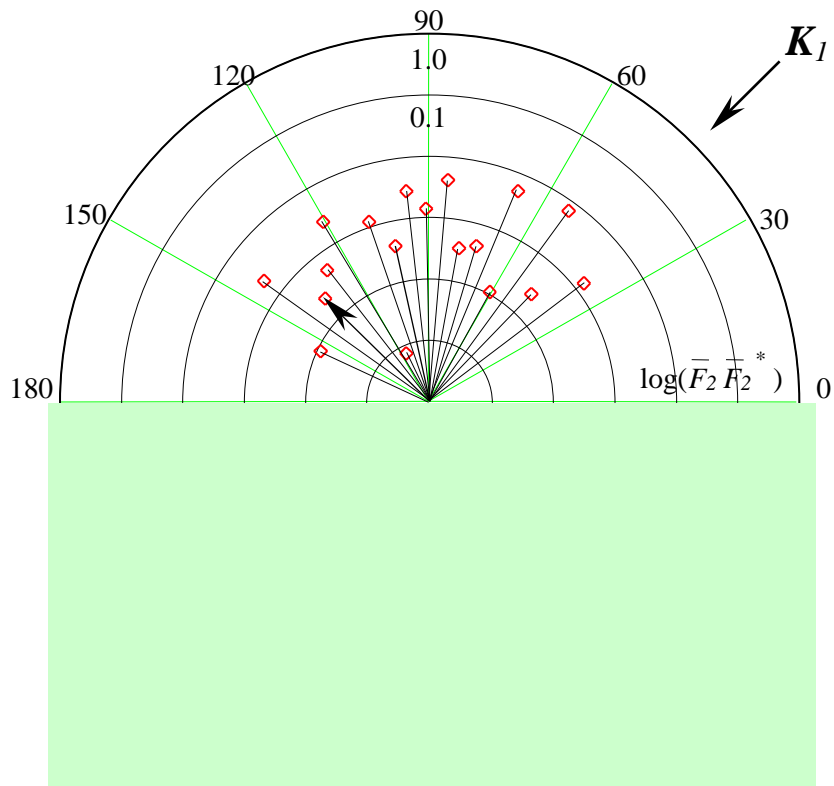
- Scattering diagram for a smooth surface characterized by the ratio λ/L .
- Note that when $\lambda/L \rightarrow 0$ scattering of light concentrates in the direction of specular reflection



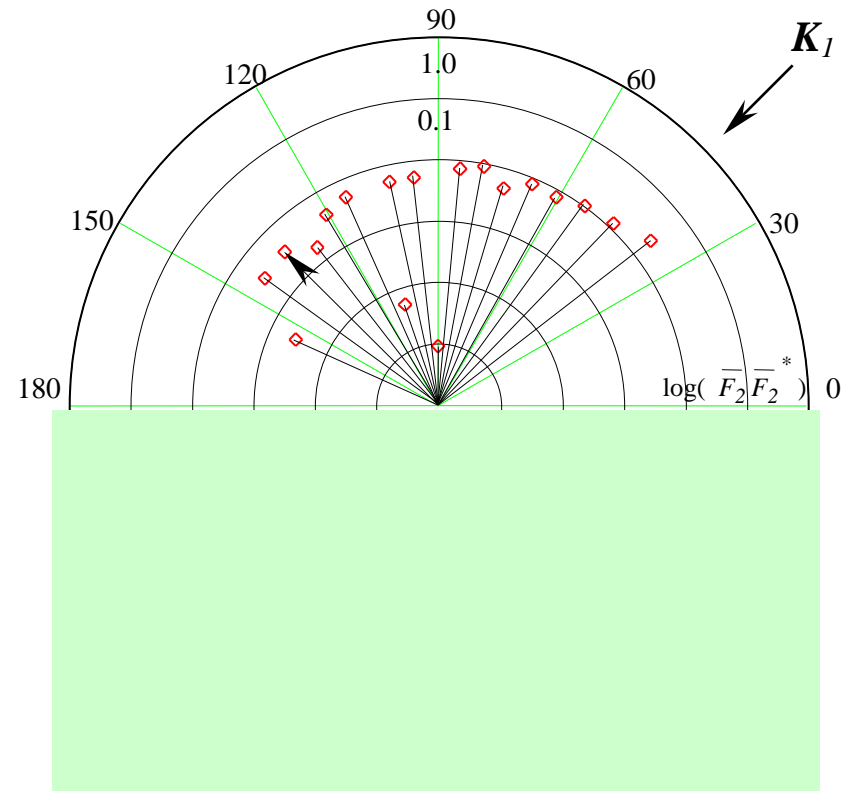
Wave optics: light scattering

Scattering from a surface characterized by a periodic function:

$$\Lambda = 10\lambda, \quad \theta_i = \pi/4, \quad \lambda/h = 0.033$$



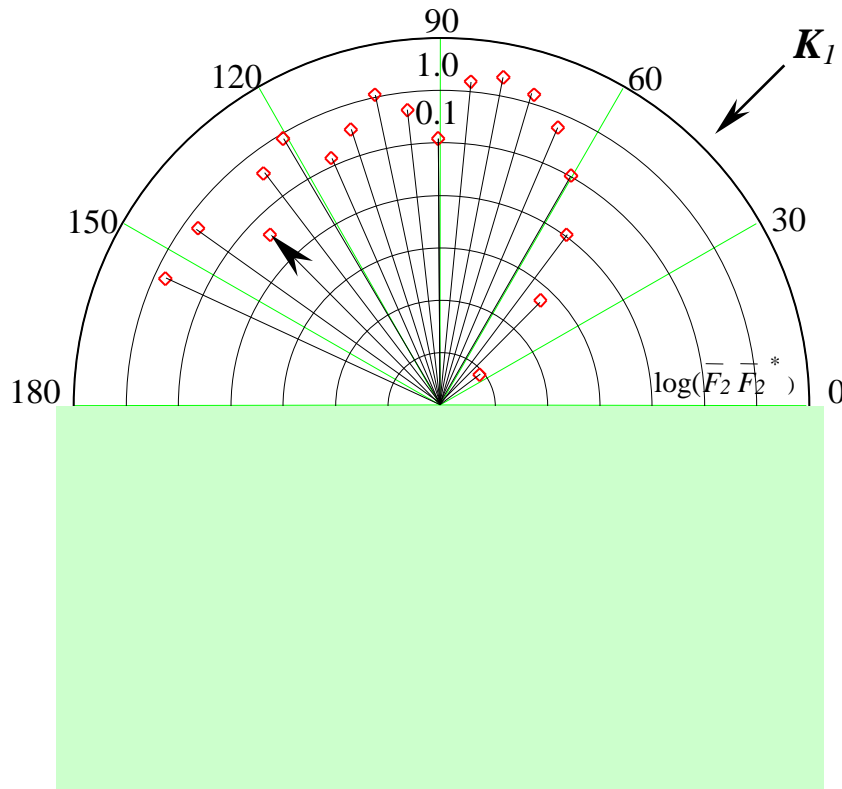
$$\Lambda = 10\lambda, \quad \theta_i = \pi/4, \quad \lambda/h = 0.066$$



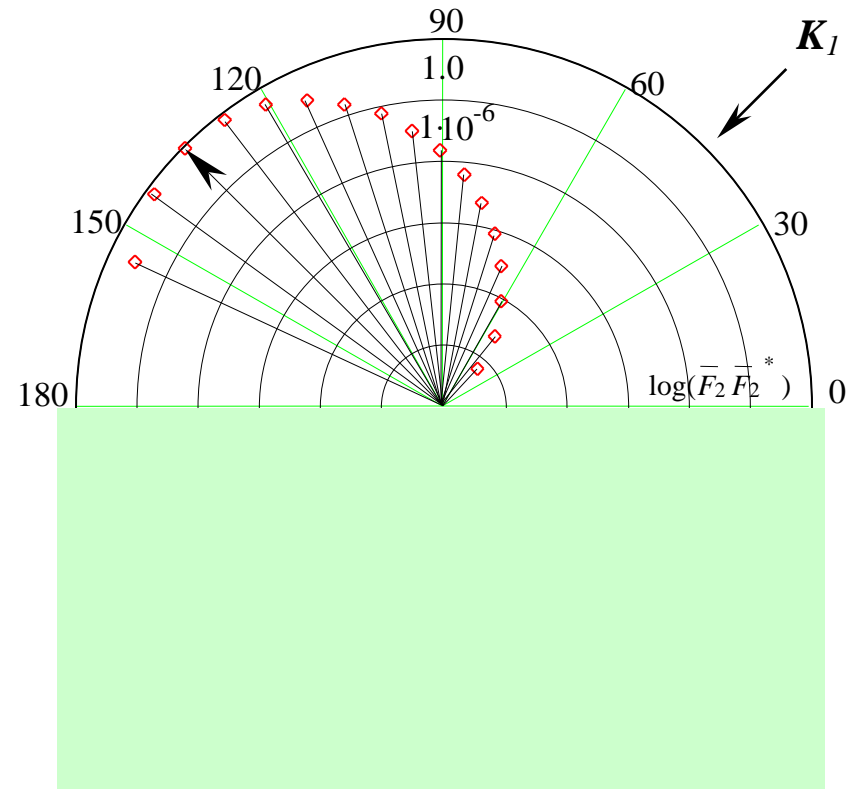
Wave optics: light scattering

Scattering from a surface characterized by a periodic function

$$\Lambda = 10\lambda, \quad \theta_i = \pi/4, \quad \lambda/h = 1.0$$



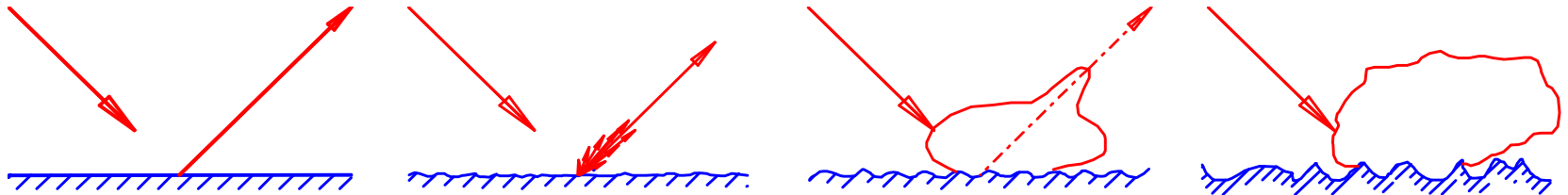
$$\Lambda = 10\lambda, \quad \theta_i = \pi/4, \quad \lambda/h = 10.0$$



Wave optics: light scattering

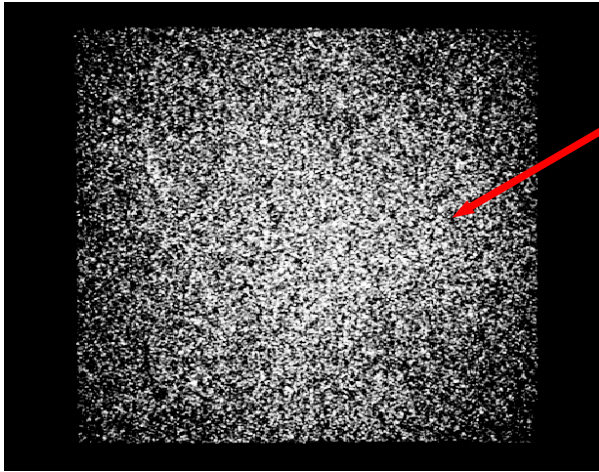
Scattering from a surface characterized by a random function

Transition from specular to diffuse scattering reflection as a function of surface roughness:



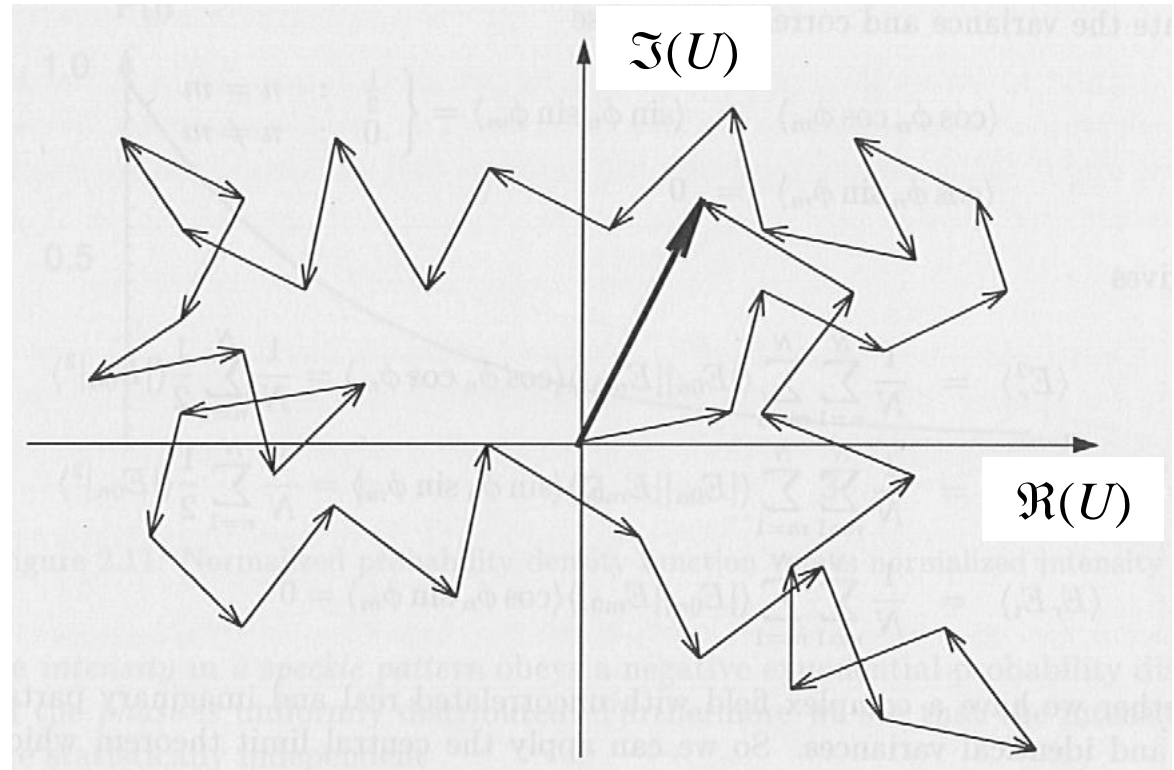
Speckles

Speckle pattern



(x,y,z)

Adding contribution of scattered components at point (x,y,z)



Speckles

Speckle properties: first order statistics

- Sum of N complex amplitude components:

$$U(x, y, z) = \frac{1}{\sqrt{N}} \sum_{k=1}^N |A_k| \exp[j\phi_k(x, y, z)] \quad (7)$$

- Real and imaginary components:

$$\Re\{U\} = \frac{1}{\sqrt{N}} \sum_{k=1}^N |a_k| \cos(\phi_k) \quad (8)$$

$$\Im\{U\} = \frac{1}{\sqrt{N}} \sum_{k=1}^N |a_k| \sin(\phi_k)$$

- Amplitude and phase are statistically independent.



Speckles

Speckle properties: first order statistics

- Probability distribution for the intensity is (mean = $2\sigma^2$, variance = $\langle I^2 \rangle$)

$$P_I(I) = \int_{-\pi}^{\pi} P_{I,\phi}(I, \phi) d\phi = \begin{cases} \frac{1}{2\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right) & ; I \geq 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (9)$$

- Probability distribution for the intensity is:

$$P_\phi(\phi) = \int_0^{\infty} P_{I,\phi}(I, \phi) dI = \begin{cases} \frac{1}{2\pi} & ; -\pi \leq \phi \leq \pi \\ 0 & ; \text{otherwise} \end{cases} \quad (10)$$

Speckles

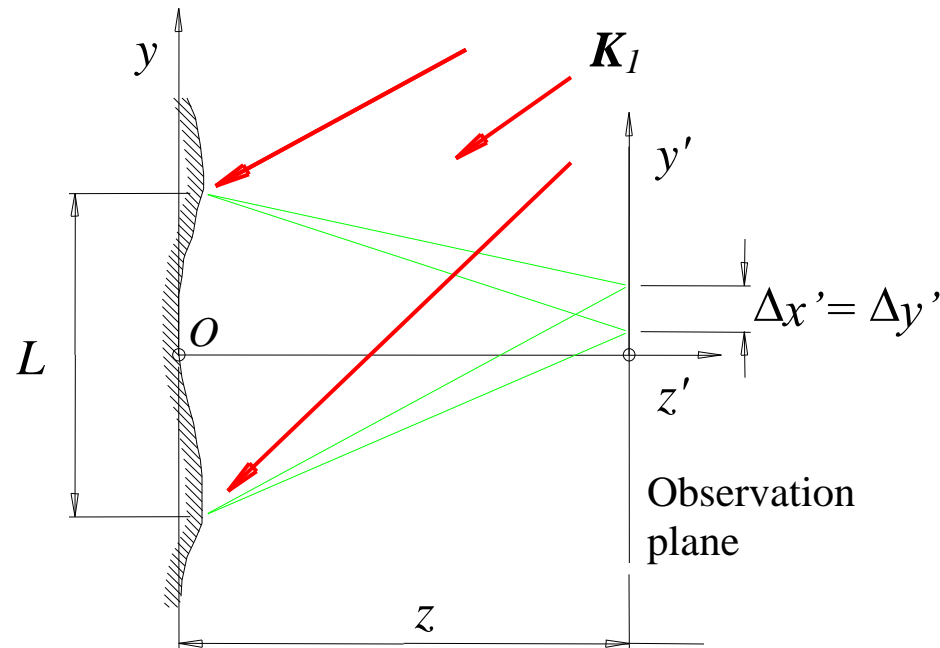
Speckle properties: second order statistics

- Observed speckle size is (without imaging system):

$$\delta x_s = 2 \frac{\lambda z \pi}{L}$$

(11)

Speckle field formation without imaging system



Speckles

Speckle properties: second order statistics

- Observed speckle size is (with imaging system):

$$\delta r_s = 2.44 \frac{\lambda z}{D}$$

(12)

Speckle field formation with imaging system

