

WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

HOMEWORK – CHAPTER 05

COURSE No.: ME-593N, Fall 2000
COURSE NAME: Laser Metrology and Nondestructive Testing (NDT)
<http://www.wpi.edu/~cfurlong/ME-593N>
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NOTE: In all solutions, state explicitly every assumption and/or approximation made, explain every procedure, and justify its use. Dimensional analyses are absolutely necessary. All results must be expressed in appropriate units. PLEASE, ALWAYS SHOW ALL WORK, while writing your results only on one side of the sheet(s) of paper; start each problem on a new sheet. *Attach this sheet to your solution. Show your work using a clear and professional style.*

PROBLEM 5.1. Consider the complex amplitude of a wave

$$U(\mathbf{r}) = A \exp(-j\mathbf{K} \cdot \mathbf{r})$$

in which A is the complex envelope, \mathbf{K} is the wave vector (or *direction of propagation vector*), and \mathbf{r} is the position vector. Demonstrate that the magnitude of vector \mathbf{K} is equal to the wave number ($k = 2\pi / \lambda$), so that $U(\mathbf{r})$ is an appropriate wave function representation.

PROBLEM 5.2. Refer to Problem 5.1. Demonstrate that the complex amplitude, as shown in Problem 5.1, represents a plane wave and that its wavefronts are parallel planes perpendicular to the direction of propagation vector \mathbf{K} .

PROBLEM 5.3. Demonstrate that the spherical wave characterized by the complex amplitude

$$U(r) = \frac{A}{r} \exp(-jkr)$$

where r is the position vector in spherical coordinates and k is the wave number, satisfies the Helmholtz equation, and therefore, it is a valid wave function representation. Note that the use of spherical coordinates can simplify the demonstration. Also, demonstrate that $U(r)$ propagates with spherical wavefronts.

READING 5.1. Chapter 6 (handout) from: C. Furlong, *Hybrid, experimental and computational, approach for the efficient study and optimization of mechanical and electro-mechanical components*, Center for Holographic Studies and Laser micro-mechaTronics, Worcester Polytechnic Institute, Mechanical Engineering Department, Worcester, MA, 1999.
